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A Comparative Study of Physical and Data-Driven Models for Systems with Friction and Application to Engine Actuators

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ABSTRACT

In automotive applications, especially for the development of Electronic Control Units (ECU), Hardware-in-the-Loop (HiL) simulation plays a significant role, which requires high-precision models to simulate vehicles with key components like engine actuators. Most actuators are mechatronic systems with significant friction effects such that modeling these systems is not an easy task. This paper presents a comparative study of physical, Sliding-Mode-Observer-based (SMO) and data-driven models for systems with friction and application to mechatronic throttle. Case studies with automotive actuators show that proposed methods can achieve higher model quality, which is sufficient for the HiL simulation.

INTRODUCTION

The development and optimization of modern combustion engines faces constantly increasing requirements to reduce emissions and fuel consumption while showing similar or even improved driving performance. Once dynamic system models are available, efficient, model-based analysis and design procedures can be developed. An important application is the Hardware-in-the-Loop (HiL) simulation, which provides a simulated driving system in which control devices can be tested. With a HiL environment, testing in extreme driving conditions requires less efforts. Models with the required prediction quality in the HiL environment are necessary. However, modeling of dynamic systems is not an easy task in the case of nonlinear effects like friction. This limits the widespread utilization of model-based applications in industrial practice (Ren and Guo, 2023).

Friction plays a crucial role in mechatronic applications like vehicle powertrains or robotics. It is a complex nonlinear phenomenon and occurs at contacting surfaces with relative motion and directly influences the dynamic behavior. The dominant friction components are Coulomb friction (constant friction force depending on the velocity sign), viscous friction (proportional to the velocity) and static friction (maximal required force to initiate movement). It causes problems like the stick-slip effect and pre-sliding displacement and can be position-dependent. A large number of friction models can be found in the literature, see e.g. (Armstrong-Héouvry et al., 1994; Wit et al., 1995; Olsson et al., 1998; Marques et al., 2016; Pennestri et al., 2016; Gagnon et al., 2020; Ren and Guo, 2023). Often, these models are classified as static or dynamic models. Static models usually describe the friction effects in steady-state like static, Coulomb, viscous and Stribeck friction. Unlike static models, dynamic models take steady-state behaviors and dynamic effects into account by introducing state variables.

Although physical models can capture the main friction effects, some challenges remain, such as the required detailed knowledge of physics. These challenges motivated using data-driven modeling. Takagi-Sugeno (TS) fuzzy models and piecewise-affine (PWA) models have attracted considerable interest due to their high approximation capability. Both TS fuzzy and PWA models are composed of a number of local models. The difference between both models is that local models of a TS fuzzy model are smoothly connected using membership functions, while PWA models have hard partition boundaries. Globally, TS fuzzy and PWA models can approximate nonlinear systems, and locally, the mapping from the regressor to the output space allows to transfer linear theory directly to nonlinear systems. The identification of TS fuzzy and PWA models is usually a challenging problem but in past decades, plenty of methods were proposed (Ferrari-Trecate et al., 2003; Roll et al., 2004; Juloski et al., 2005; Nakada et al., 2005; Vaezi and Izadian, 2015; Breschi et al., 2016; Wang et al., 2020;

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Ren and Guo, 2023). Among these methods, clustering-based ones are becoming the mainstream. These assign data points to sub-models using clustering algorithms. Parameters of the sub-models and the partition boundaries are estimated simultaneously or subsequently. A major drawback of such methods is that design parameters like the number of sub-models and the model orders should be tuned properly before. Overestimated model order could lead to limited model quality. In addition, local parameters are estimated with the least squares method by minimizing least-square errors of the series-parallel model, which are generally not bias-free and can lead to limited prediction quality for parallel evaluation like simulation.

This work presents and compares several representative physical, SMO-based and data-driven models for mechatronic systems with friction, which can be used for real-time simulation. Firstly, the system characterization will be introduced. Secondly, physical, SMO-based and data-driven methods are discussed. For physical modeling of friction, representative static and dynamic models are presented. As two typical data-driven approaches, a PWA and a TS modeling method are presented. Furthermore, these methods are applied to the real-time simulation of engine actuators like throttles and different models are compared.

SYSTEM CHARACTERIZAION

Friction effects

As aforementioned, friction is a nonlinear, complex and stochastic phenomenon in mechanical systems like gearboxes with contact surfaces under relative movement. In general, friction can be divided into dry and viscous friction: dry friction consists of sliding and static friction and often occurs if there is no lubricating film between the surfaces. Static friction is the force to be overcome before motion occurs and initiates the transition from stand-still to slide motion. A stick-slip effect often occurs at low speed when the body alternately sticks or is slides. If the contacting surface between the two bodies is lubricated, viscous friction can occur, which is proportional to the speed. Usually, the significant friction phenomena mainly include the effects, which are recorded in table 1.

Example systems with friction

The available methods for modeling systems with friction will be applied to the mechatronic test case.\ In the case of automotive mechatronic actuators like throttles (see Fig.1), they are mainly composed of a DC servo motor, a gearbox, a return spring and a potentiometer position sensor. The DC servo motor drives the actuator and the angle sensor provides for an output signal with a value range between 0.5 V and 4.5 V for the position of the valve. Friction occurs mainly between gears and bearing/shaft in the gearbox. In the context of physical models, it is usually modeled as a nonlinear function of the angular velocity.

Table 1. Overview of friction effects.

Friction effect	Description
Coulomb friction	constant friction opposing motion
Static friction	maximal friction before movement
Viscous friction	friction due to viscosity of lubricants
Stribeck effect	friction drop from static to sliding
Stick-slip effect	permanent change between stiction and sliding
Presliding	micro-motion after overcoming static friction
Hysteresis	visible effect during reversal of movement
Asymmetry	direction-dependent friction force

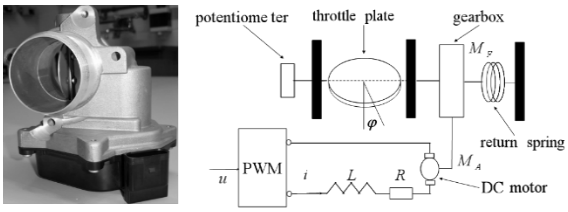


Fig. 1 Throttle and its technology scheme

PYSICAL MODELING

Physical modeling

Normally, the models for electronic and mechanical components usually have a simple structure so that the total complexity of modeling systems with friction is reduced. Modeling the other components, including the DC-motor and the return spring, is described in (Ren and Guo, 2023). When combining the friction torque M_R, the motor torque M_A and the return spring torque M_F with the torque due to inertia J, the model has the following form:

$$\begin{aligned} J \cdot \ddot{y} &= M_F - M_A + M_R \\ &= k_F \cdot (y_o - y) + M_0 - k_M \cdot u + k_{EMK} \cdot \dot{y} + M_R \end{aligned} \quad (1)$$

where k_F is the spring stiffness, M_o is the spring torque at the open position y_o, k_M is the motor constant, u is the duty cycle and k_{EMK} is the coefficient of back EMF. Parameter values in (1) for an example mechatronic throttle are provided in (Ren and Guo, 2023).

Static friction models

Static friction models mainly describe the steady-state friction effects, which could be measured as the static friction-velocity map.

- Type a Coulomb friction: This friction moment M_R can be modeled as:

$$M_R = M_G \cdot \mathrm{sgn}(\dot{\varphi}) \quad (2)$$

where M_G is the Coulomb friction, proportional to the normal contact force.

- Type b Coulomb friction + viscous friction: Besides the constant friction moment, the speed-dependent friction due to lubricant occurs during sliding is adding up to:

$$M_R = M_G \cdot \mathrm{sgn}(\dot{\varphi}) + k_R \cdot \dot{\varphi} \tag{3}$$

where k_R is the viscosity friction coefficient and $\dot{\varphi}$ is the angular velocity.

- Type c Coulomb friction + viscous friction + static friction: The model including Coulomb friction + viscous friction + static friction is formulated as:

$$M_R = \begin{cases} M_{ext}, & \text{if } \dot{\varphi} = 0 \text{ and } |M_{ext}| < M_H \\ M_G \cdot \mathrm{sgn}(\dot{\varphi}) + k_R \cdot \dot{\varphi}, & \text{otherwise} \end{cases} \tag{4}$$

where M_{ext} is the external moment and M_H is the maximum static friction moment.

- Type d Coulomb friction + viscous friction + static friction + Stribeck effect (general model): By summarizing the aforementioned friction models follows as a general static friction model:

$$M_R = \begin{cases} M_{ext}, & \text{if } \dot{\varphi} = 0 \text{ and } |M_{ext}| < M_H \\ (M_H - M_G) \cdot e^{(-|\frac{\dot{\varphi}}{\dot{\varphi}_S}|)} \cdot \mathrm{sgn}(\dot{\varphi}) & \\ + M_G \cdot \mathrm{sgn}(\dot{\varphi}) + k_R \cdot \dot{\varphi}, & \text{otherwise} \end{cases} \tag{5}$$

where $\dot{\varphi}_S$ is the Stribeck speed.

- Type e (Karnopp model): The Karnopp model can be written as:

$$M_R = \begin{cases} M_{ext}, & \text{if } |\dot{\varphi}| < DV \text{ and } |M_{ext}| < M_H \\ M_G \cdot \mathrm{sgn}(\dot{\varphi}) + k_R \cdot \dot{\varphi}, & \text{otherwise} \end{cases} \tag{6}$$

where DV is the defined velocity interval.

Dynamic friction models

Over the past years, various dynamic friction models were presented in the literature to illustrate the dynamic friction effects. As reported in (Marques et al., 2016), the dynamic models can theoretically better capture the friction phenomena compared to the static models. In the following section, three representative dynamic models will be briefly introduced:

- Type a (Dahl model): The mathematical representation of the stress-strain curve is modeled by a differential equation as follows:

$$\begin{aligned} &M_R = \sigma_0 \cdot z \\ &\text{with } \dot{z} = \dot{\varphi} \cdot \left(1 - \frac{\sigma_0 \cdot z}{M_G} \cdot \mathrm{sgn}(\dot{\varphi})\right)^{\alpha} \end{aligned} \tag{7}$$

where α defines the shape of the stress-strain curve, σ_0 is the stiffness coefficient and z denotes the internal state variable.

- Type b (The reset integrator model): This model reproduces the bonding effects during stiction by a pre-sliding displacement z. The dynamics of z are described as:

$$\dot{z} = \begin{cases} 0, & \text{if } |z| \geq z_0 \text{ and } z \cdot \dot{\varphi} > 0 \\ \dot{\varphi}, & \text{otherwise} \end{cases} \tag{8}$$

The model reproduces the dynamic friction effects by using an integrator with a reset, in which both stick and slip modes are modeled as:

$$M_R = \begin{cases} \sigma_0 \cdot (1+a) \cdot z + \sigma_1 \cdot \dot{z}, & \text{if } |z| < z_0 \\ \sigma_0 \cdot z_0, & \text{otherwise} \end{cases} \tag{9}$$

$\sigma_1 \cdot \dot{z}$ is a damping term introduced physical meaning by having damped oscillations and viscous friction effects. a is a stiction-related coefficient, and σ_0 is the contact stiffness.

- Type c (LuGre model): In the LuGre model, friction is modeled as the average bristle deflection through an internal state variable z. The following equation describes the model:

$$\begin{aligned} &M_R = \sigma_0 \cdot z + \sigma_1 \cdot \dot{z} + k_R \cdot \dot{\varphi} \\ &\text{with } \dot{z} = \dot{\varphi} \cdot \left(1 - \frac{\sigma_o \cdot z \cdot \mathrm{sgn}(\dot{\varphi})}{M_G + (M_H - M_G) \cdot e^{(-\frac{\dot{\varphi}}{\dot{\varphi}_S})^2}}\right) \end{aligned} \tag{10}$$

where σ_0 is the stiffness coefficient, σ_1 is the damping coefficient and k_R is the viscosity friction coefficient.

Estimation of friction parameters

Normally, the identification problem can be solved by minimization of a cost function as following:

$$\hat{\theta} = \arg\min_{\theta} \frac{1}{N} \sum_{k=1}^{N} (\hat{y}(k,\theta) - y(k))^2 \tag{11}$$

θ are the friction parameters, which are iteratively optimized by minimization of the cost function (11).

It is known that most aforementioned physical friction models are nonlinear in the parameters, which leads to a nonlinear estimation problem. In this paper, we used a two-step identification approach: firstly, the regression space was searched using Genetic Algorithm (GA), in order to locate regions containing global/local minimal values; secondly, the minimum value was identified within the located regions based on the Nelder–Mead Simplex Algorithm (NMSA). This Nelder–Mead Simplex Algorithm is a nonlinear iterative optimization method, which is often used for determining the local minima of multivariate functions. The two-step approach has the advantage that the cost function does not need to be differentiable and that it balances exploration and exploitation properties, which is suitable for identifying physical friction models with a discontinuity at zero velocity.

SLIDING-MODE OBSERVER-BASED MODELING

Alternatively, an offline friction-identification method is presented in this paper. This method is based on an online friction estimation approach. Using the sliding-mode observer, the friction can be estimated and combining the velocity (calculated from the angular position), a friction-velocity plot can be generated. In the latter, piecewise linear curves can be estimated by using a robust curve-fitting algorithm. With these estimated piecewise linear curves, the parameters of a static friction model can easily be offline identified.

Model description

In order to design the sliding-mode observers, the physical model equation representation (1) is formed in input/output form:

$$(1)\Rightarrow\begin{bmatrix}\dot{\varphi}(t)\\ \ddot{\varphi}(t)\end{bmatrix}=\begin{bmatrix}0 & 1\\ -\frac{k_F}{J} & \frac{k_{EMK}}{J}\end{bmatrix}\cdot\begin{bmatrix}\varphi(t)\\ \dot{\varphi}(t)\end{bmatrix}+\begin{bmatrix}0\\ -\frac{k_M}{J}\end{bmatrix}\cdot u(t)+\begin{bmatrix}0\\ \frac{k_F}{J}\cdot\varphi_o+\frac{M_o}{J}\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}\cdot\frac{M_R}{J} \quad (12)$$

where the state vector $\mathrm{x}(t)=[\varphi(t)\quad \dot{\varphi}(t)]^T$ consists of the angular position and the angular velocity of the actuators. Because the constant terms in the state space representation are typically not analytically considered in system analysis and controller design, equation (12) is transformed into a minimal state space representation by considering the constant terms as offsets at the input signal $u(t)$. The new minimal state space representation follows as follows:

$$(12)\Rightarrow\begin{bmatrix}\dot{\varphi}(t)\\ \ddot{\varphi}(t)\end{bmatrix}=\begin{bmatrix}0 & 1\\ -\frac{k_F}{J} & \frac{k_{EMK}}{J}\end{bmatrix}\cdot\begin{bmatrix}\varphi(t)\\ \dot{\varphi}(t)\end{bmatrix}+\begin{bmatrix}0\\ -\frac{k_M}{J}\end{bmatrix}\cdot u'(t)+\begin{bmatrix}0\\ 1\end{bmatrix}\cdot\frac{M_R}{J} \quad (13)$$

where the new input is defined as $u'(t)=u(t)-k_F\cdot\varphi_0/k_M-M_0/k_M$. The state-space representation of the model for mechatronic throttles is given as:

$$\begin{aligned}\dot{x}(t)&=\mathbf{A}x(t)+\mathbf{B}u'(t)+\mathbf{D}\psi(t)\\ y(t)&=\mathbf{C}x(t)\end{aligned} \quad (14)$$

with the matrices

$$\mathbf{A}=\begin{bmatrix}0 & 1\\ -\frac{k_M}{J} & \frac{k_{EMK}}{J}\end{bmatrix},\ \mathbf{B}=\begin{bmatrix}0 & -\frac{k_M}{J}\end{bmatrix}^T \quad (15)$$
$$\mathbf{C}=\begin{bmatrix}1 & 0\end{bmatrix},\ \mathbf{D}=\begin{bmatrix}0 & 1\end{bmatrix}^T$$

The unknown friction term $\frac{M_R}{J}$ is described by $\psi(t)$. In this study, type c of static friction models (4) is used which has been proven to be able to capture the significant static friction effects.

Sliding-mode observer design

A sliding-mode observer as proposed in (Edwards and Spurgeon, 1998) is used to estimate the friction term $\psi(t)$. The structure of the sliding-mode observer for the mechatronic throttle with $e_y(t)=y(t)-\hat{y}(t)$ is:

$$\begin{aligned}\dot{\hat{x}}(t)&=\mathbf{A}\hat{x}(t)+\mathbf{B}u(t)+\mathbf{G}_l e_y(t)-\mathbf{G}_n v\\ \hat{y}(t)&=\mathbf{C}\hat{x}(t)\end{aligned} \quad (16)$$

where $\mathbf{G}_l,\mathbf{G}_n\in\mathbb{R}^{2\times1}$ are appropriate gain matrices and the discontinuous switched component v(t) induces a sliding motion. With the transformation matrix **T**:

$$\mathsf{A}=\mathbf{TAT}^{-1}=\begin{bmatrix}\mathsf{A}_{11} & \mathsf{A}_{12}\\ \mathsf{A}_{21} & \mathsf{A}_{22}\end{bmatrix}\ \mathsf{B}=\mathbf{TB}=\begin{bmatrix}\mathsf{B}_1\\ \mathsf{B}_2\end{bmatrix} \quad (17)$$
$$\mathsf{D}=\mathbf{TD}=\begin{bmatrix}\mathbf{0}\\ \mathsf{D}_2\end{bmatrix}\ \mathsf{C}=\mathbf{CT}^{-1}=\begin{bmatrix}\mathbf{0} & \mathbf{I}\end{bmatrix}$$

the models in input/output form is transformed into suitable canonical form to design the sliding mode observer:

$$\begin{aligned}\dot{x}_1(t)&=\mathsf{A}_{11}x_1(t)+\mathsf{A}_{12}x_2(t)+\mathsf{B}_1u'(t)\\ \dot{x}_2(t)&=\mathsf{A}_{21}x_1(t)+\mathsf{A}_{22}x_2(t)+\mathsf{B}_2u'(t)+\mathsf{D}_2\psi(t)\\ y(t)&=x_2(t)\end{aligned} \quad (18)$$

with a new coordinate, the system (19) will be used for the sliding-mode observer design:

$$\begin{aligned}\dot{\hat{x}}_1(t)&=\mathsf{A}_{11}\hat{x}_1(t)+\mathsf{A}_{12}\hat{x}_2(t)+\mathsf{B}_1u'(t)-\mathsf{A}_{12}e_y(t)\\ \dot{\hat{x}}_2(t)&=\mathsf{A}_{21}x_1(t)+\mathsf{A}_{22}x_2(t)+\mathsf{B}_2u'(t)-(\mathsf{A}_{22}-\mathsf{A}_{22}^s)e_y(t)+\mathrm{v}(t)\\ \hat{y}(t)&=\hat{x}_2(t)\end{aligned} \quad (19)$$

with $e_y(t)=\hat{y}(t)-y(t)$, A_{22}^s is a stable design matrix and v(t) has the form:

$$\mathrm{v}(t)=\begin{cases}-\rho\|\mathsf{D}_2\|\frac{\mathbf{P}_2e_y(t)}{\|\mathbf{P}_2e_y(t)\|}, & \text{if } e_y\neq0\\ 0, & \text{otherwise}\end{cases} \quad (20)$$

where ρ a positive scalar and $\mathbf{P_2}$ is positive defined satisfying the Lyapunov equation:

$$(\mathsf{A}_{22}^s)^T\mathbf{P}_2+\mathbf{P}_2(\mathsf{A}_{22}^s)=-\mathbf{Q}_2 \quad (21)$$

It follows from the state estimation errors $e_x(t)=\hat{x}_1(t)-x_1(t)$ that:

$$\begin{aligned}\dot{e}_x(t)&=\mathsf{A}_{11}e_x\\ \dot{e}_y(t)&=\mathsf{A}_{21}e_x(t)+\mathsf{A}_{22}^se_y(t)+v(t)-\mathsf{D}_2\psi(t)\end{aligned} \quad (22)$$

In (Edwards and Spurgeon, 1998) it is shown that the error system in (23) is quadratically stable.

Friction estimation using sliding-mode observer

During the sliding motion $e_y(t)=0$ and $\dot{e}_y(t)=0$ holds and (22) provides:

$$0=\mathsf{A}_{22}e_{z_x}(t)+\mathrm{v}_{eq}-\mathsf{D}_2\ \psi(t) \quad (23)$$

where v_{eq} is referred to as equivalent output injection signal that maintains the sliding motion. The discontinuous scalar in (20) can be approximated by:

$$\mathrm{v}_{eq}(t)=-\rho\|\mathsf{D}_2\|\frac{\mathbf{P}_2e_y(t)}{\|\mathbf{P}_2e_y(t)\|+\delta} \quad (24)$$

where δ is a small positive number, which will be chosen to compensate the chattering in the sliding motion. Then, the estimation of friction term can be defined as:

$$\frac{M_R}{J}=\hat{\psi}(t)\approx-\rho\|\mathsf{D}_2\|(\mathsf{D}_2^T\mathsf{D}_2)^{-1}\mathsf{D}_2^T\frac{\mathbf{P}_2e_y(t)}{\|\mathbf{P}_2e_y(t)\|+\delta} \quad (25)$$

DATA-DRIVEN MODELING

Besides modelling based on physical principles, mechatronic systems with friction can also be modeled through data-driven methods, which can be applied given limited pre-knowledge and cause moderate experimental efforts.

Model description

As universal approximators, both PWA and TS models are considered, which compose a number of local models with hard (PWA model) and smooth partition boundaries (TS model).

- PWA model: PWA modelling is used mainly for hybrid (e.g. switched) systems. In this paper, a typical structure of the PWA models names PWARX (Piecewise AutoRegressive eXogenous) model will be used for modelling the mechatronic actuators. The model is described as follows (Ren and Guo, 2023):

$$f(\mathrm{x}(k)) = \begin{cases} \theta_1^T \begin{bmatrix} \mathrm{x}(k) \\ 1 \end{bmatrix}, \text{if } \mathrm{x} \in \mathrm{X}_1 \\ \vdots \\ \theta_c^T \begin{bmatrix} \mathrm{x}(k) \\ 1 \end{bmatrix}, \text{if } \mathrm{x} \in \mathrm{X}_c \end{cases} \tag{26}$$

with the regressor:

$$\begin{aligned} \mathrm{x}(k) = [&y(k-1)\; y(k-2) \ldots y(k-n_a) \\ &u(k-1)\; u(k-2) \ldots u(k-n_b)]^T \end{aligned} \tag{27}$$

where n_a and n_b are the numbers of used past outputs and inputs, respectively, $\{\theta_i\}_{i=1}^c$ are the parameter vectors. The regression space is split into c polyhedral partitions $\mathrm{X} \in \mathbb{R}^{n_a+n_b}$ and in each partition a local model is valid.

- TS model: Similar like a PWA model, a TS model is composed of c superposed local models $y_i(k)$, which are weighted by their fuzzy basis function $\phi(k)_i$ (FBF). The model output $y(k)$ is then described by:

$$\begin{aligned} y(k) &= \frac{\sum_{i=1}^{c} \mu_i(\mathrm{x}(k), \theta_{MF,i}) \cdot y_i(\mathrm{x}(k), \theta_{LM,i})}{\sum_{i=1}^{c} \mu_i(\mathrm{x}(k), \theta_{MF,i})} \\ &= \sum_{i=1}^{c} \phi_i(\mathrm{x}(k), \theta_{MF}) \cdot y_i(\mathrm{x}(k), \theta_{LM,i}) \end{aligned} \tag{28}$$

where μ_i is the membership function with parameter vectors $\theta_{MF,i}$ (prototypes) of the i-th local model. In this work, the prototype-based membership function as provided by the Fuzzy-c-means (FCM) clustering are used (Kroll, 2016):

$$\mu_i(\mathrm{x}(k), \theta_{MF,i}) = \left[\sum_{j=1}^{c} \left(\frac{\left\| \mathrm{x}(k) - \mathbf{v}_i \right\|}{\left\| \mathrm{x}(k) - \mathbf{v}_j \right\|} \right)^{\frac{2}{\nu-1}} \right]^{-1} \tag{29}$$

where $\nu > 1$ the fuzziness parameter to adjust the shape of the transitions between local models of TS fuzzy systems and $\mathbf{v_i}, \mathbf{v_j}$ are the prototypes. In this work, the Euclidean distance is used as the distance norm $\|\cdot\|$. The output of the i-th model $y_i(k)$ depends on its local parameter vector $\theta_{LM,i}$ and the regressor variable $\mathrm{x}(k)$. Then, each local model in ARX (autoregressive with exogenous input) configuration is:

$$y(k) = \theta_{LM,i} \cdot \left[\mathbf{X}(k)^T \quad 1 \right]^T \tag{30}$$

Identification of data-driven models

The identification of data-driven systems is conducted with a clustering-based procedure as follows:

Step 1: Choosing feature vectors for clustering

The evaluation of "similarity" of data is important for data clustering. Appropriate features should be defined to achieve a better clustering quality. On the one hand, classes should be well separated and on the other hand, as few features as possible should be used for clustering due to costs and complexity. For mechatronic actuators, the nonlinear effects due to friction should dependent on features. Considering the velocity-dependent friction effects, it makes more sense to use $y(k-1) - y(k-2)$ and $u(k-1)$ as features. Thereby, $y(k-1)$ is not considered because no position-dependent nonlinearity was observed.

Step 2: Partition of feature vectors

For similarity-based partitioning of the feature space, clustering methods like c-Means and FCM can be conducted for PWA and TS models. Both clustering algorithms do not automatically determine the number of clusters by itself and cluster validity measures can be used to determine the number of clusters. Such indices indicate the number of clusters for which the clusters are sufficiently well separated. As second criterion, the model quality on test data, should be considered. The results for all sub-criteria will be finally evaluated to choose c.

Step 3: Estimation of hyperplanes in the feature space of PWA system

Based on the data clustering in the feature space using the c-means algorithm for PWA system, the support vector machine (SVM) algorithm can determine the hyperplanes between partitions for the PWA system. The SVM algorithm is a supervised learning algorithm which assigns the given data points to two classes so that a range around the class boundary as wide as possible is free of data.

Step 4: Global parameter estimation

Then the local model parameters are optimized by "parallel identification", which calculates the output from the past input and the predicted output and corresponds to the simulative usage of a model without available outputs. This optimization method causes the estimation problem to no longer be "linear in the parameters", which requires an elaborate iterative nonlinear optimization but provides for higher prediction quality (Ren and Guo, 2023).

EXPERIMENTAL RESULTS

Test stand

In order to collect data for modelling and model validation, a test stand has been built up, which is shown illustrated in Fig. 2. A National Instruments™ system was used for implementing the PWM control signal and for recording the measurement data. Based on the LabVIEW™ software, an application program has been developed to operate the test stand and record the data.

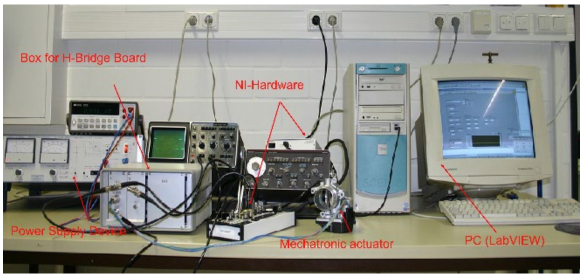


Fig. 2 Test stand

Model quality evaluation

As mentioned above, the model will be used for HiL simulation and should predict the angle position of the actuator precisely to permit functional testing of an ECU. Two criteria were used for the quantitative evaluation of the model quality. They are the maximal absolute prediction error:

$$\|e\|_\infty = \max_k(|\hat{y}(k) - y(k)|) \tag{31}$$

and the NRMSE (Normalized Root Mean Squared Error):

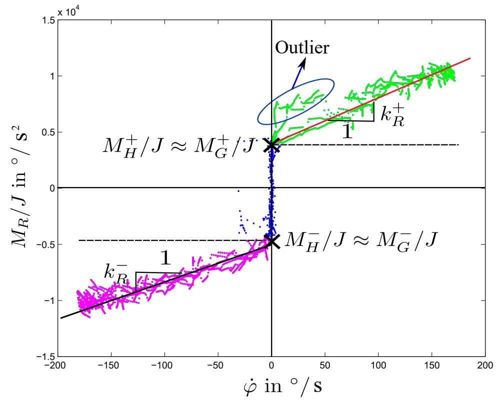
$$\text{NRMSE} = \sqrt{\frac{\sum_{k=1}^{N}\left(y(k)-\hat{y}(k)\right)^2}{\sum_{k=1}^{N}(y(k)-\bar{y})^2}} \tag{32}$$

For the current application, it is required that $\|e\|_\infty \le 5°$ and NRMSE should be as small as possible. Besides the quantitative evaluation, a qualitative evaluation of the time series will also be used for the validation. The identified models will be assessed regarding their performance in parallel evaluation.

Parameter identification

Because the proposed methods allow the separation of identification for friction and non-friction parameters, and the methods are applied to the same mechatronic throttle. Due to space reasons, the complete identification of the whole physical models will not be presented, and more details are provided in (Ren and Guo, 2023). For estimating the unknown parameters of friction models, the two-step approach was used to find the optimal values of the cost function (11).

With the estimated friction term and the angular velocity (calculated from the measured angular position) a friction-velocity data map can be directly presented. Based on this map, piecewise linear curves (see Fig. 3) can be estimated by using the robust curve-fitting algorithm RANSAC. Estimation of the parameters $M_H^+ \approx M_G^+, M_H^- \approx M_G^-, k_R^+, k_R^-$ of the friction model in (4) bases on the crossing points and the slope of two curves. The reconstructed characteristic curve of the identified friction model will be shown in Fig. 3.



$k_R^+ = 0,95 k_R^-$ $\quad$ $M_H^+/J = 0,9 M_G^-/J$

Fig. 3 Velocity-friction data map with straight approximation lines and estimated parameters of the friction model

As mentioned before, the number of sub-models plays an important role in the identification of data-driven models. Both c-Means and FCM algorithms are initialized 10 times for each value within an interval [2,10] and c=8 is the appropriate selection for the PWA model and TS model in this application based on the model quality. Additionally, the fuzziness parameter is chosen as $\nu = 1.1$ for the clustering, estimation and model evaluation of the TS model (Kroll, 2016). For assessment, the identified models are simulated, which means that only the input signal and the initial state are available for the simulation, but the entire resulting time-series for the output will be assessed. Results and discussion are illustrated in the following section.

Results and discussion

With the presented modeling methods, presented modeling methods incl. physical modeling with different friction models and data-driven PWA and TS modeling are applied to a mechatronic throttle as case study. The comparison between measurement and simulation of the mechatronic throttle are compared in Fig. 4. The figure shows that the models can principally capture the main effects like sliding and stiction of the existing system and the model's friction behaviors like static friction. The data-driven model and the real system behavior are quite similar and the TS model achieves a better result than the presented physical models. The difference between measurement and model prediction is subtle and the maximal error with those models is almost within the required threshold of 5°. Models with larger errors could be improved by fine-tuning of model parameters.

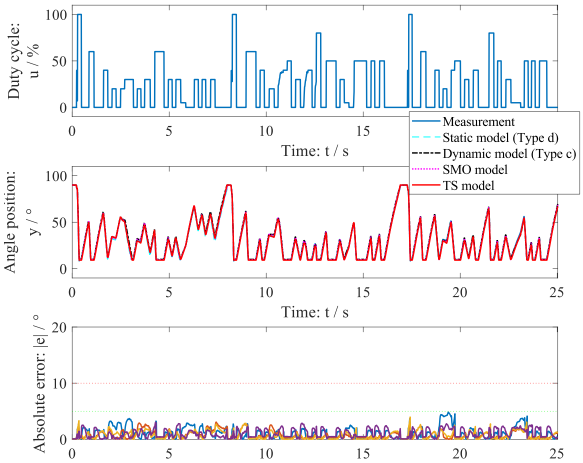


Fig. 4 Measurement and model prediction with different models

In Fig. 5, the stiction effect is visible on the interval [1.9 s; 2.4 s]. The presented models reproduced the stiction effect and they could start to slide if the external moment is bigger than the maximal stiction moment. But the micro-motion like presliding cannot be precisely captured by static models. As mentioned before, dynamic models may help to improve the model quality in micro-motion phase. In the range [1.9 s; 2.4 s], it is possible to observe that the dynamic model (type c) shows a distinct behavior compared with the other models. This difference in the dynamic behavior on this range is mainly due to a viscous component in the model and the prediction of micro-motion like presliding was improved by the dynamic model. Summarizing, the presented models well capture the dynamic and static effects and the model quality is sufficient for testing motor control functions in HiL simulation. It is noted that in Fig. 5 presented models are able to reproduce the stiction effect in the interval [1.9 s; 2.4 s] and the static friction can be well reproduced by the dynamic model (type c).

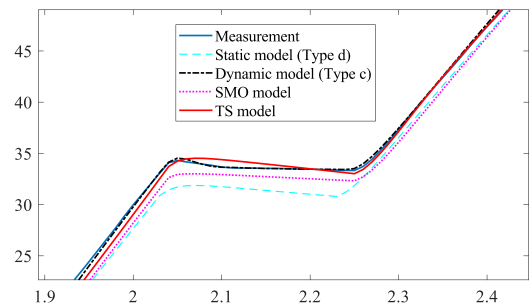


Fig. 5 Measurement and model prediction with static models for range [1.9 s; 2.4 s]

The presented modeling methods are quantitatively compared in table 2. The maximal error with those models is almost within the required threshold of 5°. The NRMSE values for those models are around 0.06. This means that the data-driven model provides the best model quality and the target of $\|e\|_\infty \leq 5^\circ$ is met.

Table 2. Performance comparison of presented modeling methods

Model type	Model	Criterion	
		$\|e\|_\infty$	NRMSE
Static model	Type c	4.93°	0.07°
	Type d	4.82°	0.06
	Type e	5.02°	0.07
Dynamic model	Type a	4.75°	0.07
	Type b	4.31°	0.06
	Type c	3.07°	0.05
SMO-based model	-	3.96°	0.06
Data-driven model	PWA model	3.26°	0.05
	TS model	3.12°	0.05

CONCLUSIONS

This work presented approaches with physical, SMO-based and data-driven models for systems with friction, and these methods were applied to mechatronic actuators of passenger cars. Firstly, the characterization of mechatronic systems with friction is reported, in which a description of friction effects and a mechatronic throttle as a typical application of systems with friction are briefly introduced. In order to understand the physical characteristics, significant friction effects like Coulomb friction, static friction, viscous friction, stick-slip effect, pre-sliding displacement, hysteresis and asymmetry were discussed, which will be reproduced through models. Secondly, presented modeling methods for mechatronic systems with friction were discussed. As typical data-driven modeling methods, the PWA and the TS model with the proposed clustering-based identification methods were briefly presented in the sequel of this section, in which friction effects were addressed in particular by a specific scheduling vector design. Finally, a mechatronic throttle was treated as case study to compare various models.

Results for different models were discussed, and it was shown that the dynamic response of the mechatronic system could be reproduced, and the significant effects be captured with the presented modeling methods. Some models show significant differences compared to others, and a detailed analysis based on validation results was carried out. Results show that presented methods can efficiently yield models of high quality, which are sufficient for the functional tests of ECU in HiL simulation. The proposed methods can also be transferred to other mechatronic systems with friction. Future research will address the systematic design of identification signals and the model-based controller design.

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REFERENCES

Armstrong-Héouvry, B., Dupont, P., Wit, C.D., "A survey of models, analysis tools and compensation methods for control of machines with friction," *Automatica,* Vol. 30, pp.1038-1138 (1994).

Breschi, V., Piga, D., Bemporad, A., "Piecewise affine regression via recursive multiple least squares and multicategory discrimination," *Automatica*, Vol. 73, pp. 155–162 (2016).

Edwards, C., Spurgeon, S., *Sliding Mode Control: Theory and Applications,* Taylor and Francis, London (1998).

Ferrari-Trecate, G., Muselli, M., Liberati, D., Morari, M., "A clustering technique for the identification of piecewise affine systems," *Automatica*, Vol. 39 pp. 205–217 (2003).

Gagnon, L., Morandini, M., Ghiringhelli, G. L., "A review of friction damping modeling and testing," *Arch Appl Mech*, Vol. 90, pp. 107–126 (2020).

Grepl, R., Lee, B., "Modeling, parameter estimation and nonlinear control of automotive electronic throttle using a rapid-control prototyping technique," *Experimental Techniques*, Vol. 11, pp. 601–610 (2010).

Juloski, A., Weiland, S., Heemels, W. P. M. H., "A bayesian approach to identification of hybrid systems," *IEEE Transactions on Automatic Control*, Vol. 50, pp. 1520–1533 (2005).

Kroll, A., *Computational Intelligence: Methods and Technical Applications,* De Gruyter Oldenbourg, London (2016).

Marques, F., Flores, P., Claro, J. C. P., Lankarani, H. M., "A survey and comparison of several friction force models for dynamic analysis of multibody mechanical systems," *Nonlinear Dynamics*, Vol. 86, pp. 1407–1443 (2016).

Nakada, H., Takaba, K., Katayama, T., "Identification of piecewise affine systems based on statistical clustering technique," *Automatica*, Vol. 41, pp. 905–913 (2005).

Nakano, K., Sawut, U., Higuchi, K., Okajima, Y., "Modeling and observer-based sliding-mode control of electronic throttle systems," *ETCI Transactions on Electrical Engineering, Electronics, and Communications*, Vol. 4, pp. 22–28 (2006).

Olsson, H., Åström, K.J., Wit, C.D., Gävert, M., Lischinsky, P., "Friction models and friction compensation," *European Journal of Control*, Vol. 4 pp.176–195 (1998).

Pavkovic, D., Deur, J., Jansz, M., Peri, N., "Adaptive control of automotive electronic throttle," *Control Engineering Practice*, Vol. 14, pp. 121–136 (2006).

Pennestri, E., Rossi, V., Salvini, P., Valentini, P. P., "Review and comparison of dry friction force models," *Nonlinear Dynamics,* Vol. 83, pp. 1785–1801 (2016).

Ren, Z., Guo, J., " On Semi-Automated Modeling of Systems with Friction for HiL-Simulation and Application to Electromechanical Actuators of Passenger Car", *Journal of the Chinese Society of Mechanical Engineers*, Vol. 44, No. 5, pp. 455–462 (2023).

Roll, J., Bemporad, A., Ljung, L., "Identification of piecewise affine systems via mix-integer programming", *Automatica*, Vol. 40, pp. 37–50 (2004).

Simoni, L., Beschi, M., Visioli, A., K. J. Åström, "Inclusion of the dwell time effect in the LuGre friction model," *Mechatronics*, Vol. 66, pp. 102345 (2020).

Vaezi, M., Izadian, A., "Piecewise affine system identification of a hydraulic wind power transfer system," *IEEE Transactions on Control Systems Technology*, Vol. 23 pp. 2077–2086 (2015).

Wang, J., Song, C., Zhao, J., Xu, Z., "A pwa model identification method for nonlinear systems using hierarchical clustering based on the gap metric, Computers and Chemical Engineering", Vol. 138, pp. 106838 (2020).

Wit, C.D., Olsson, H., Åström, K.J., P. Lischinsky, P., "A new model for control of systems with friction," *IEEE Transaction on Automatic Control*, Vol. 40, pp. 1189-1190 (1995).

Yuan, X. F., Li, T., Wang, Y. N., Sun, W., Wu, L.H., "Parameter identification of electronic throttle using a hybrid optimization algorithm," *Nonlinear Dynamics*, Vol. 63, pp. 549–557 (2011).