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A General Method for Developing Different Types of 3-DOF and 6-DOF Isotropic Manipulators

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Keywords：Parallel manipulator; 3-DOF; 6-DOF; Isotropic

ABSTRACT

This paper first presents design parameters for developing six types of spatial 3-DOF isotropic parallel manipulators. Many manipulators that can simultaneously reach isotropic position and orientation can be easily obtained using optimization methods. An isotropic design can be developed in about ten seconds, and at least three dimensions can even be specified to obtain designs with desired shapes or sizes. The 3-DOF manipulators are then used as modules to develop 6-DOF or redundant isotropic manipulators. The modules or the modules and other serial or parallel designs can be joined in parallel or in series to create different types of 6-DOF, redundant or hybrid isotropic manipulators. Some dimensions of manipulators of higher degrees of freedom can also be given in the development process.

INTRODUCTION

Isotropic manipulators are generally regarded as designs with optimum dexterity. An isotropic manipulator can be obtained if the condition number gives the optimum value of one. The condition number, however, is only applicable to 2-DOF or 3-DOF manipulators with dimensionally homogeneous Jacobian matrices. For 6-DOF manipulators whose first and last three rows of the Jacobian matrix have different physical units, a dimensionally homogeneous Jacobian matrix can be developed using natural length or characteristic length (Zanganeh et al, 1997; Fattah et al, 2002; Fassi et al, 2005). The 6-DOF isotropic manipulators can also be developed using three isotropic conditions (Klein et al, 1991). The method is independent of the physical units of the Jacobian matrix. Many 6-DOF isotropic manipulators can be easily obtained using an isotropic generator that comprises six straight lines obtained by solving the system of nonlinear equations developed from the isotropic conditions (Tsai et al, 2003). Optimization methods can also be employed to develop symmetrical isotropic manipulators (Tsai et al, 2008). In general, the existing methods can only be used for 6-DOF serial manipulators or Stewart-Gough parallel manipulators, and only one design parameter can be specified to determine the size of the manipulators.

For other types of manipulators, it is extremely complicated to express the required equations as functions of link parameters, and directly solving a large number of nonlinear equations might get impractical designs with strange shapes or unacceptable dimensions. Compared with 6-DOF manipulators, 3-DOF isotropic parallel manipulators are much easier to develop. However, most manipulators, developed by most existing methods, are special planar, orientational or translational isotropic designs (Kircanski et al, 1994; Gogu et al, 2004; Kuo et al, 2011; Tsai et al, 2000; Wang et al, 2003). The 3-DOF parallel manipulators developed by the proposed method are spatial designs that can simultaneously reach isotropic position and orientation, and they can be used to develop isotropic manipulators of higher degrees of freedom. This work first presents design parameters that can be used to develop 3-RPS, 3-CS, 3-PPS, 3-US, 3-RRS and 3-UPU isotropic manipulators. By giving the Jacobian matrix, an isotropic design can be easily developed in about ten seconds using optimization methods. For each type of manipulators, hundreds of sample manipulators are developed to explore how many dimensions can be specified in the development process. The results show that at least three parameters can be specified for developing manipulators with desired shapes or sizes. The 3-DOF designs are then used as modules to develop

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6-DOF or redundant isotropic manipulators. Two modules can be joined in parallel into a 3-limb or 6-limb isotropic parallel manipulator or they can be joined in series to create different types of hybrid or redundant isotropic manipulators. Several design parameters can also be specified in the development of 6-DOF or redundant isotropic manipulators. Many numerical examples are provided for illustration.

JACOBIAN MATRICES

The Jacobian matrices for some 3-DOF parallel manipulators can be found in the literature (Gosselin et al, 1989; Huanga et al, 2002; Li et al, 2002). The twist of the moving platform of a 3-DOF parallel manipulator can be expressed as a linear combination of instantaneous twists:

$$\begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{v} \end{bmatrix} = \dot{q}_{ij}\,\mathbf{\$}_{\mathbf{ij}}\,, \tag{1}$$

where $\boldsymbol{\omega}$ is the angular velocity of the moving platform, $\mathbf{v}$ is the linear velocity of a reference point on the platform, $\mathbf{\$}_{\mathbf{ij}}$ represents the j[th] screw on limb i, and $\dot{q}_{ij}$ denotes the corresponding joint rate. Joshi and Tsai presented a general method for developing the Jacobian matrix of limited-DOF parallel manipulators (Joshi et al, 2002). The general equations for velocity analysis can be expressed as

$$\begin{bmatrix} \mathbf{M_1} & \mathbf{M_2} \\ \mathbf{M_3} & \mathbf{M_4} \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{v} \end{bmatrix} = \begin{bmatrix} \dot{\mathbf{q}} \\ \mathbf{0} \end{bmatrix}, \tag{2}$$

where $\mathbf{M_i}$ for i = 1, 2, 3, 4 are 3×3 matrices and $\dot{\mathbf{q}} = [\dot{q}_1, \dot{q}_2, \dot{q}_3]^t$ with $\dot{q}_i$ denotes the actuated joint rate for limb i. The equations can be developed using reciprocal screws. The last three equations, $\mathbf{M_3}\boldsymbol{\omega} + \mathbf{M_4}\mathbf{v} = \mathbf{0}$, give

$$\boldsymbol{\omega} = -\mathbf{M}_3^{-1}\mathbf{M_4}\mathbf{v}\,, \tag{3}$$

$$\mathbf{v} = -\mathbf{M}_4^{-1}\mathbf{M}_3\boldsymbol{\omega}\,. \tag{4}$$

Substituting Eqs. (4) and (3) into the first three equations of Eq. (2), $\mathbf{M}_1\boldsymbol{\omega} + \mathbf{M}_2\mathbf{v} = \mathbf{q}$, respectively yields

$$\boldsymbol{\omega} = \left(\mathbf{M_1} - \mathbf{M_2}\mathbf{M}_4^{-1}\mathbf{M_3}\right)^{-1}\mathbf{q} \equiv \mathbf{J}_{\boldsymbol{\omega}}\mathbf{q}\,, \tag{3}$$

$$\mathbf{v} = \left(\mathbf{M}_2 - \mathbf{M}_1\mathbf{M}_3^{-1}\mathbf{M}_4\right)^{-1}\mathbf{q} \equiv \mathbf{J}_{\mathbf{v}}\mathbf{q}\,. \tag{4}$$

3-DOF isotropic manipulators can be developed using the two Jacobian matrices, $\mathbf{J}_{\boldsymbol{\omega}}$ and $\mathbf{J}_{\mathbf{v}}$. Let κ_ω and κ_v denote the reciprocal of the condition numbers of $\mathbf{J}_{\boldsymbol{\omega}}$ and $\mathbf{J}_{\mathbf{v}}$ respectively. Then a manipulator can reach both an isotropic orientation and an isotropic position if $\kappa_\omega = 1$ and $\kappa_v = 1$.

3-DOF ISOTROPIC MANIPULATORS

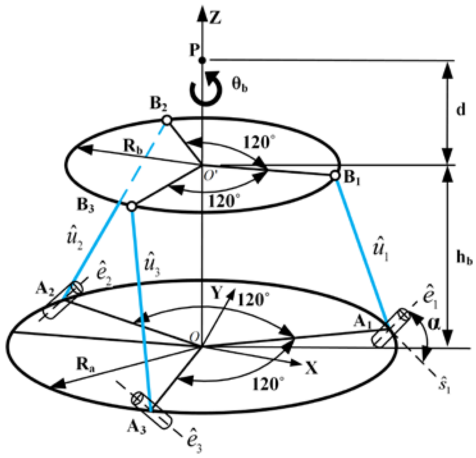


Fig. 1. Design parameters for 3-RPS, 3-CS and 3-PPS manipulators

This section presents design parameters and kinematic constraints for developing different types of 3-DOF isotropic manipulators. Optimization methods are employed to search for the designs with $\kappa_\omega = 1$ and $\kappa_v = 1$. Some parameters can be provided to specify the shape of a desired manipulator.

The first pattern of straight lines for developing symmetrical isotropic manipulators is shown in figure 1. All the design parameters are generated by rotating one limb about the z-axis by 120° and 240° respectively. Since manipulators developed from the parameters have one special characteristic in that two out of three singular values of the Jacobian matrix are equal, an isotropic design (with three equal singular values) is relatively easy to develop using the proposed parameters. In this work, the local coordinates of reference point P_i for i = 1, 2, 3 are defined by $\mathbf{p}'_\mathbf{i}$ = $(R_p\cos(30° + (i-1)*120°)$ $R_p\sin(30°+(i-1)*120°)$ $0)$ and the global coordinates of the points are determined by

$$\mathbf{p_i} = \mathbf{rot}(\hat{\mathbf{z}}, \theta_p)\mathbf{p}'_\mathbf{i} + \begin{bmatrix} 0 \\ 0 \\ h_p \end{bmatrix}, \tag{7}$$

where $\mathbf{rot}(\hat{\mathbf{z}}, \theta_p)$ denotes the rotation matrix about the z-axis by an angle of θ_p . For the reference points in the figure, the coordinates of point A_i are determined by parameter R_a with $\theta_a = h_a = 0$, the coordinates of B_i being functions of R_b , θ_b and h_b , and $\mathbf{p}$ = (0, 0, d $+h_b$) for the position of the tool center point (TCP). The unit vectors in the figure are defined by $\hat{\mathbf{u}}_i = \frac{\mathbf{b}_i - \mathbf{a}_i}{\|\mathbf{b}_i - \mathbf{a}_i\|}$, $\hat{\mathbf{s}}_i = \frac{\hat{\mathbf{u}}_i \times \hat{\mathbf{z}}}{\|\hat{\mathbf{u}}_i \times \hat{\mathbf{z}}\|}$ with $\hat{\mathbf{z}}$ = $[0\ 0\ 1]^t$ and $\hat{\mathbf{e}}_i = \mathbf{rot}(\hat{\mathbf{u}}_i, \alpha)\hat{\mathbf{s}}_i$. The pattern generated by six parameters, R_a, R_b, d, h_b, θ_b and α, can be employed to develop 3-RPS, 3-CS and 3-PPS isotropic manipulators. For a 3-RPS manipulator, $\hat{\mathbf{e}}_i$ and $\hat{\mathbf{u}}_i$ respectively define the direction of the first revolute joint on the base and the direction of the second prismatic joint on limb i that connects A_i and B_i. Vectors $\hat{\mathbf{e}}_i$ and $\hat{\mathbf{u}}_i$ specify the directions of the two prismatic joints on limb i of a 3-PPS manipulator, and $\hat{\mathbf{e}}_i$ defines the direction of the cylindrical joint on limb i of a 3-CS manipulator.

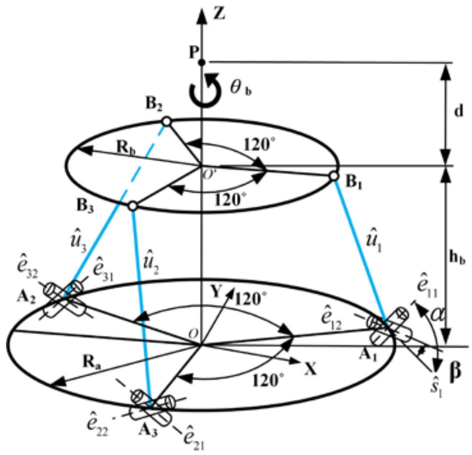


Fig. 2. Design parameters for a 3-US manipulator

The parameters in figure 2 for a 3-US manipulator are the same as those of the pattern in Fig. 1 except that $\hat{\mathbf{e}}_{i1} = \hat{\mathbf{e}}_i$ gives the direction of the first revolute joint and the new unit vector $\hat{\mathbf{e}}_{i2}$ for the second revolute joint on limb i is defined by $\hat{\mathbf{e}}_{i2} = rot(\hat{\mathbf{u}}_i, \beta)\hat{\mathbf{e}}_{i1}$. The coordinates of the three extra points, C_1, C_2 and C_3, on the pattern for 3-RRS manipulators in figure 3 are functions of R_c, θ_c and h_c (with $h_c < h_b$). The unit vector $\hat{\mathbf{e}}_{i2}$ for the second revolute joint on limb i is defined by $\hat{\mathbf{e}}_{i2} = \frac{(\mathbf{c}_i - \mathbf{b}_i)\times(\mathbf{c}_i - \mathbf{a}_i)}{\|(\mathbf{c}_i - \mathbf{b}_i)\times(\mathbf{c}_i - \mathbf{a}_i)\|}$ and $\hat{\mathbf{e}}_{i1} = rot(\hat{\mathbf{u}}_i, \gamma)\hat{\mathbf{e}}_{i2}$ (with $\hat{\mathbf{u}}_i = \frac{\mathbf{c}_i - \mathbf{a}_i}{\|\mathbf{c}_i - \mathbf{a}_i\|}$) gives the direction for the first revolute joint.

For a 3-UPU manipulator, $\hat{\mathbf{e}}_{i1}$ and $\hat{\mathbf{e}}_{i2}$ in figure 4 are the same as those of a 3-US manipulator, $\hat{\mathbf{u}}_i = \frac{\mathbf{b}_i - \mathbf{a}_i}{\|\mathbf{b}_i - \mathbf{a}_i\|}$ defines the direction of the prismatic joint and $\hat{\mathbf{e}}_{13}$ and $\hat{\mathbf{e}}_{14}$ for the other universal joint at point B_i are specified by two sets of Euler angles: (ψ_1, ξ_1) and (ψ_2, ξ_2). Unit vectors for the universal joints on the platform for the second and the third limbs are determined by rotating $\hat{\mathbf{e}}_{13}$ and $\hat{\mathbf{e}}_{14}$ about the z-axis by $120°$ and $240°$, respectively.

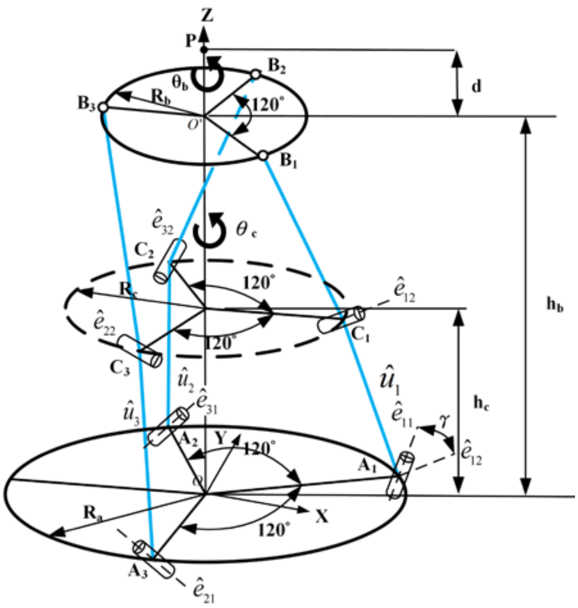


Fig. 3. Design parameters for a 3-RRS manipulator

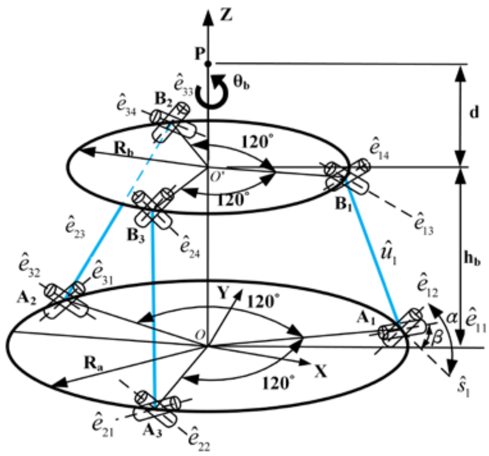
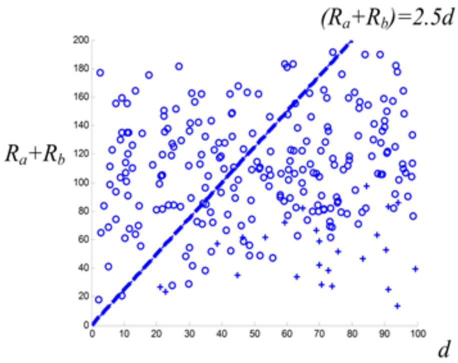
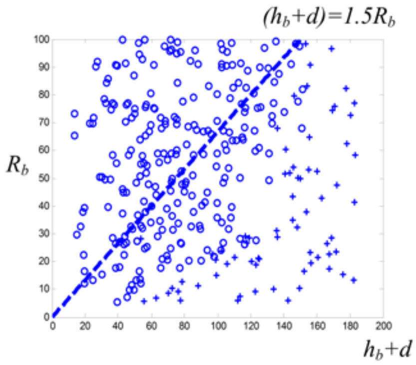


Fig. 4. Design parameters for a 3-UPU manipulator

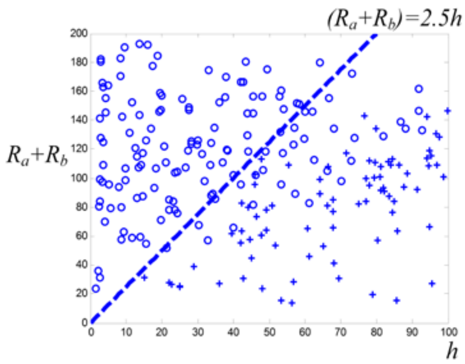
Using the genetic algorithm (from the optimization tool of MatLab), an isotropic design can be developed in about 5 seconds for 3-RPS, 3-CS and 3-PPS manipulators and it takes about 15 seconds to develop an isotropic 3-US, 3-RRS or 3-UPU manipulator using a personal computer (Intel core 2 Quad CPU with 2.66 GHz). For 3-RPS, 3-CS and 3-PPS manipulators, some primary results show that many isotropic designs can be easily developed by specifying two parameters from R_a, R_b, h_b, and d. The dimensions of those designs are analyzed to explore if more dimensions can be provided or some conditions can be developed to facilitate the search for isotropic designs. It is found that the dimensions of three parameters have a similar property that the ratio of the summation of two dimensions and the third dimension is larger or smaller than a certain value. To determine the exact value, additional 150 sample 3-RPS manipulators are developed using random dimensions of R_a, R_b, and d. Figure 5a shows the images of the manipulators and the related dimensions. All the designs that are not isotropic (identified by "+") fall into the lower right region with the straight line defined by $(R_a + R_b) = 2.5d$ as the boundary, so $(R_a + R_b) > 2.5d$ is used as the constraint to develop isotropic manipulators (identified by "o") at the upper left region.



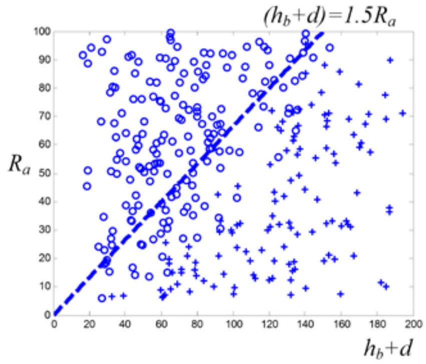
(a)



(b)



(c)



(d)

Fig. 5. Isotropic designs and link dimensions

The images of other 150 sample manipulators generated by random dimensions of h_b, R_b, and d in figure 5b show that $(d + h_b) < 1.5R_b$ can be used as the constraint to develop the isotropic designs at the upper left region. Two other constraints for different sets of three parameters are shown in figure 5c and 5d. Figure 5 also shows that many isotropic designs can still be developed if the related constraint is not satisfied. The parameters that can be specified and the constraints on the dimensions for 3-CS and 3-PPS manipulators are determined using similar approaches. There are more design parameters for 3-US, 3-RRS and 3-UPU manipulators, so many isotropic designs can be developed by specifying at least three parameters. However, the constraints on those parameters are much more difficult to develop. The work first uses 150 sample isotropic manipulators to develop constraint equations. Then additional 150 manipulators are developed using the dimensions that satisfy the constraints. If any of the obtained designs is not isotropic, then modify the constraints until new 150 manipulators developed using the new constraints are all isotropic. The parameters that can be specified and the related constraints on the dimensions for different types of manipulator are listed in Table 1. Over 99% of the manipulators developed using the constraints are isotropic, and a further investigation shows that either the first three joints or the second three joints of each design can be chosen as the actuated joints. For example, either the three revolute joints or the three prismatic joints of a 3-RPS manipulator can be used as the actuated joints.

6-DOF ISOTROPIC MANIPULATORS

This section proposes methods that use 3-DOF modules to create different types of 6-DOF isotropic manipulators. For a 6-DOF manipulator, the closeness to kinematic isotropy can be evaluated by an existing normalized isotropy measure (Tsai et al, 2008):

$$\mu = \left(\frac{\sigma_{o3}}{\sigma_{o1}} * \frac{\sigma_{p3}}{\sigma_{p1}} * \Phi \right)^{\frac{1}{3}}, \tag{8}$$

with

$$\Phi = \sqrt{1 - \left(\frac{\sigma_1 + \sigma_3}{2} \right)^2}, \tag{9}$$

where

σ_{o3}, σ_{o1} smallest and largest singular values, respectively, of the 3×6 submatrix $\mathbf{J_o}$ consisting of the first three rows of the Jacobian $\mathbf{J}$

σ_{p3}, σ_{p1} smallest and largest singular values, respectively, of the 3×6 submatrix $\mathbf{J_p}$ consisting of the last three rows of $\mathbf{J}$

σ_3, σ_1 smallest and largest singular values, respectively, of $\tilde{\mathbf{J}}_\mathbf{o}\tilde{\mathbf{J}}_\mathbf{p}^\mathbf{t}$ in which $\tilde{\mathbf{J}}_\mathbf{o}$ and $\tilde{\mathbf{J}}_\mathbf{p}$ are the two matrices with orthonormal row vectors that span the same row spaces of $\mathbf{J_o}$ and $\mathbf{J_p}$ respectively

Table 1. Design parameters and the related constraints

	Type	parameters	Constraints
1	3-RPS 3-CS 3-PPS	d, h_b, θ_b, R_a, R_b, α	(a) d, h_b, R_b ($d + h_b < 1.5R_b$)
			(b) d, R_a, R_b ($R_a + R_b > 2.5d$)
			(c) d, h_b, R_a ($d + h_b < 1.5R_a$)
			(d) h_b, R_a, R_b ($R_a + R_b > 2.5h_b$)
2	3-US	d, h_b, θ_b, R_a, R_b, α, β	d, h_b, R_a, R_b ($\frac{2}{3}(d + h_b) <$ R_a, R_b $< 2(d+h_b)$)
3	3-RRS	R_a, R_b, R_c, h_b, h_c, d, θ_b, θ_c, γ	R_a, R_b, R_c, h_b, h_c, d (0.9(h_b+d)≦R_a, R_b, R_c≦1.1(h_b+d) and h_c>0.5h_b)
4	3-UPU	θ_b, d, h_b, R_a, R_b, α, β, ψ_1, ζ_1, ψ_2, ζ_2	d, h_b, R_a, R_b ($\frac{2}{3}(d + h_b) <$ R_a, R_b $< 2(d+h_b)$)

An extra condition that two row spaces spanned by $\mathbf{J_o}$ and $\mathbf{J_p}$ are orthogonal (with $\Phi = 1$) is used in the measure for the spatial isotropy for 6-DOF manipulators. The angular velocity $\boldsymbol{\omega}$ and the linear velocity $\mathbf{v}$ are respectively generated by the projection of $\mathbf{q}$ (the vector of input joint rates) onto the row space of $\mathbf{J_o}$ and the row space of $\mathbf{J_p}$. The two projections, denoted by $\mathbf{q_o}$ and $\mathbf{q_p}$, are perpendicular and $\mathbf{q_o} + \mathbf{q_p} = \mathbf{q}$ if the two row spaces are orthogonal. The measure (which is independent of physical units and yields the optimum value of μ = 1 for an isotropic configuration) is used as the objective function to develop 6-DOF or redundant isotropic manipulators.

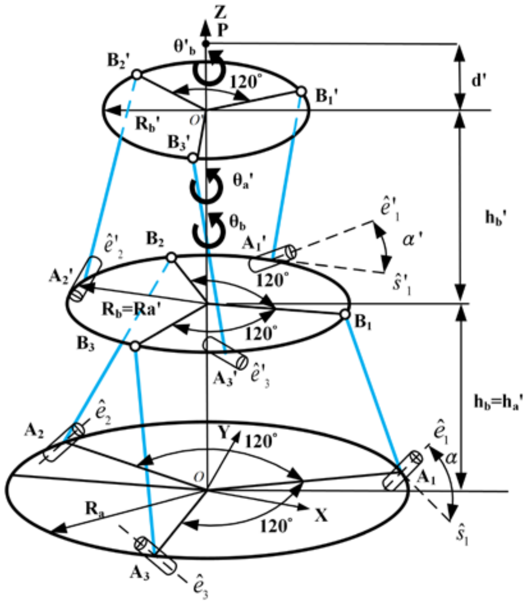


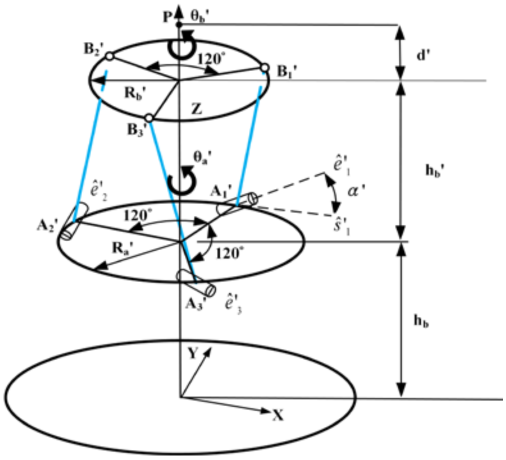
Fig. 6. A 6-DOF hybrid manipulator with R_b=R_a'

The coordinates of the reference points of the 6-DOF manipulator in figure 6 are given by Eq. (7) with R_a and $\theta_a = h_a = 0$ for A_i, (R_b, h_b, θ_b) for B_i, (R'_a, h_b, θ'_a) for A'_i, (R'_b, $h_b + h'_b$, θ'_b) for B'_i, and $\mathbf{p}$ = (0, 0, $d + h_b + h'_b$). With points A_i and B_i replaced respectively by A'_i and B'_i, the joint axes for the module on the top can be determined following similar steps. If a 6-DOF manipulator is developed by placing one 3-DOF module on the platform of another 3-DOF module with the same TCP, then the twist of the platform of the 6-DOF design is the summation of the two twists from the two modules:

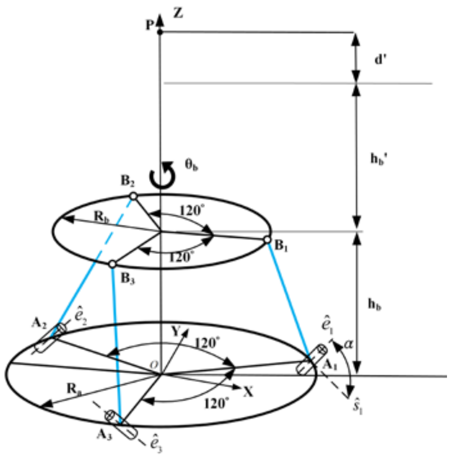
$$\begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{v} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\omega}_1 \\ \mathbf{v}_1 \end{bmatrix} + \begin{bmatrix} \boldsymbol{\omega}_2 \\ \mathbf{v}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{J}_{1\omega} \\ \mathbf{J}_{1v} \end{bmatrix} \dot{\mathbf{q}}_1 + \begin{bmatrix} \mathbf{J}_{2\omega} \\ \mathbf{J}_{2v} \end{bmatrix} \dot{\mathbf{q}}_2 = \begin{bmatrix} \mathbf{J}_{1\omega} & \mathbf{J}_{2\omega} \\ \mathbf{J}_{1v} & \mathbf{J}_{2v} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}}_1 \\ \dot{\mathbf{q}}_2 \end{bmatrix} = \mathbf{J}\dot{\mathbf{q}}, \tag{9}$$

where the four 3×3 submatrices in the 6×6 Jacobian matrix $\mathbf{J}$ are defined in Eqs. (5) and (6). A 6-DOF isotropic design can be generated by the same module or two different modules. Many dimensions can be specified if they satisfy their respective constraints in Table 1. For example, the dimensions of R'_a, R'_b, h'_b, d' for the upper module and the dimensions of R_a, R_b, h_b, $d = h'_b + d'$ R_a, R_b, h_b, $d = h'_b + d'$ for the lower module in figure 7 must satisfy the constraints for the upper module and the lower module respectively.

Since conditions $\kappa_\omega = 1$ and $\kappa_v = 1$ for the two modules are equivalent to $\frac{\sigma_{o3}}{\sigma_{o1}} = 1$ and $\frac{\sigma_{p3}}{\sigma_{p1}} = 1$ for the two factors defined in Eq. (8), a 6-DOF isotropic design can be obtained if the third factor also yields the optimum value of $\Phi = 1$. Many different types of hybrid isotropic manipulators with $R_b = R'_a$ and their approximate values of function μ developed using optimization methods are listed in Table 2. Except for the design that uses two 3-UPU modules, the manipulators have very good dexterity with $\mu \geq 0.96$, which can be further improved for manipulators with $R_b \neq R'_a$ because more free parameters can be used in the optimization methods.



(a) Upper module



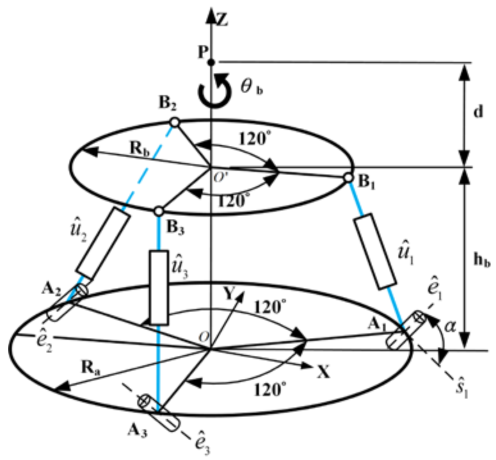
(b) Lower module

Fig. 7. Design parameters for a hybrid manipulator

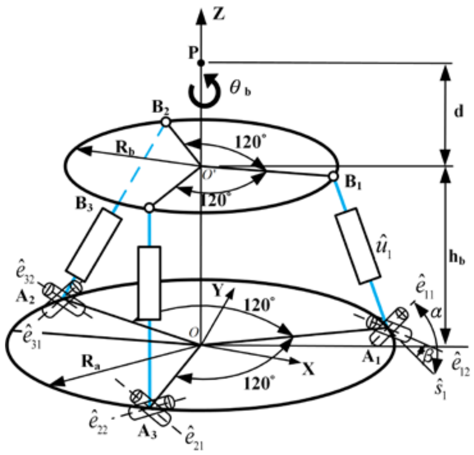
Two modules can be joined in parallel to develop a 3-limb or 6-limb 6-DOF isotropic manipulator. How to develop the Jacobian matrix for a common 6-DOF parallel manipulator can be found in the literature (Tsai et al, 1999). Figure 8a shows the model that joins a 3-CS module with a 3-RPS module into a 3-CPS manipulator with six actuated prismatic joints. The design parameters are the same as those of the 3-CS module except that unit vectors $\hat{\mathbf{u}}_1, \hat{\mathbf{u}}_2$ and $\hat{\mathbf{u}}_3$ in Fig. 1 define the directions of the three additional actuated prismatic joints. The model in Fig. 8b for a 3-UPS manipulator with six actuated revolute joints is developed from the 3-US module with the three unit vectors, $\hat{\mathbf{u}}_1, \hat{\mathbf{u}}_2$ and $\hat{\mathbf{u}}_3$, in Fig. 2 specifying the directions of the three additional passive prismatic joints. A 3-URS manipulator with the six actuated revolute joints (from the three universal joints) can be developed from the model in figure 8c. The directions for the two actuated revolute axes of the universal joint on the first limb are developed using the same way for the direction of the first revolute axis for a 3-RRS module. The two directions are determined by $\hat{\mathbf{e}}_{11} = \mathbf{rot}(\hat{\mathbf{u}}_1, \eta)\hat{\mathbf{e}}_{12}$ and $\hat{\mathbf{e}}'_{11} = \mathbf{rot}(\hat{\mathbf{u}}_1, \xi)\hat{\mathbf{e}}_{12}$, where $\hat{\mathbf{e}}_{12}$ is defined in Fig. 3. Unit vectors for the two universal joints on the other limbs are developed by rotating $\hat{\mathbf{e}}_{11}$ and $\hat{\mathbf{e}}'_{11}$ about the z-axis by $120°$ and $240°$, respectively. Isotropic 3-CPS, 3-UPS and 3-URS parallel manipulators can be developed if the specified dimensions satisfy the constraints of 3-CS, 3-US and 3-RRS modules, respectively.

Table 2. Dexterity of different hybrid manipulators with $R_b = R'_a$

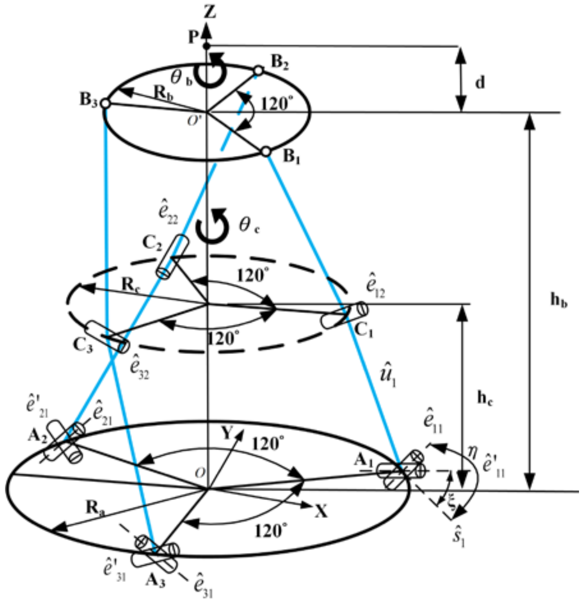
Type	μ
3-CS+3-CS	0.98
3-RPS+3-PPS	0.995
3-RPS+3-CS	0.97
3-RPS+3-RPS	0.99
3-PPS+3-CS	0.985
3-PPS+3-PPS	0.99
3-US+3-RPS	0.985
3-US+3-CS	0.97
3-US+3-US	0.98
3-UPU+3-RPS	0.96
3-UPU+3-CS	0.96
3-UPU+3-PPS	0.96
3-UPU+3-UPU	0.85



(a) A 3-CPS manipulator



(b) A 3-UPS manipulator



(c) A 3-URS manipulator

Fig. 8. 3-limb 6-DOF parallel manipulators

Two 3-UPS modules can be joined in parallel into a Stewart-Gough manipulator with six actuated prismatic joints. The 12 reference points in figure 9 are determined using Eq. (7) with R_a and $\theta_a = h_a = 0$ for A_i, (R_b, h_b, θ_b) for B_i, ($R_a, h'_a = 0, \theta'_a$) for A'_i, (R_b, h_b, θ'_b) for B'_i, and $\mathbf{p} = (0, 0, d + h_b)$. The model is similar to an existing model for developing isotropic Stewart-Gough manipulators. The two approaches can only specify one dimension in R_a, R_b, h_b or d, and the manipulators obtained all have the same value of μ = 0.9667. Likewise, a 6-URS isotropic manipulator can be developed using two 3-URS modules. Besides the 12 points in Fig. 9, the six extra points for the revolute joints in the middle (as shown in Fig. 8c) are defined by (R_c, h_c, θ_c) for C_i and (R_c, h_c, θ'_c) for C'_i with $h_c < h_b$. The steps for the joint axes on each limb are the same as those for a 3-URS manipulator and the first axis of the universal joint on each limb is chosen as the actuated axis. The constraints on the specified dimensions of a 6-URS manipulator are the same as those of the 3-RRS module. The approximate values of the dexterity measure μ for 3-limb or 6-limb manipulators are given in Table 3. Other 6-DOF or redundant isotropic manipulators can also be developed using the presented modules and other existing serial or parallel manipulators.

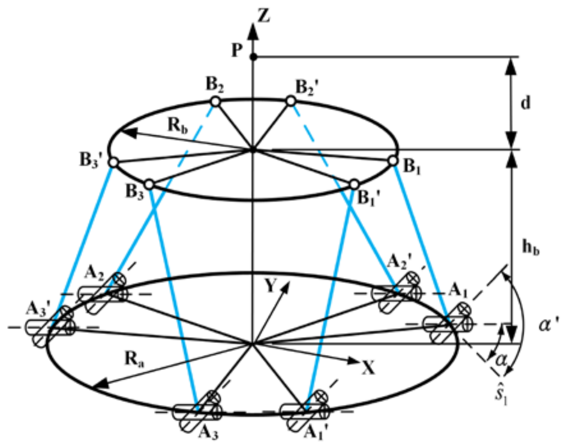


Fig. 9. A 6-UPS parallel manipulator

Table 3. Dexterity of 3-limb or 6-limb parallel manipulators

Type	μ
3-CPS	0.95
3-UPS	0.92
3-URS	0.98
6-UPS	0.9667
6-URS	0.965

NUMERICAL EXAMPLES

For a 3-UPU parallel manipulator, four dimensions can be provided if they satisfy the constraint in Table 1. Given R_a = 17, R_b = 13, h_b =10, and d =5 such that condition 6 with $\frac{2}{3}(h_b + d) < R_a, R_b < 2(h_b + d)$ is satisfied, the optimization software gives $\kappa_\omega = 1$, $\kappa_v = 1$ with $\alpha = 290.3336°$, $\beta = 273.8301°$, $\theta_b = 342.5454°$, $\psi_1 = 235.2954°$, $\psi_2 = 44.0975°$, $\zeta_1 = 8.1006$ and $\zeta_2 = 8.2613$. To develop a 6-DOF hybrid manipulator with a 3-RPS module on the platform of a 3-CS module, three dimensions: $R'_a = 15$, $h'_b = 5$ and $d' = 3$ (that satisfy condition 1(c)) are given for the 3-RPS module on the top. Next, let R_a = 18 and R_b = 15 such that condition 2(b): $R_a + R_b > 2.5\,(d' + h'_b)$ is satisfied for the 3-CS module on the ground. The optimization results give μ = 0.9998 with h_b = 8.0253,

$\theta_b = 0.3549°$, $\alpha = 123.2173°$ for the 3-CS module and $R_b' = 13$, $\theta_a' = 51.3387°$, $\theta_b' = 59.8362°$, $\alpha' = 3.3473°$ for the 3-RPS module. The constraints for a 6-DOF, 3-CPS manipulators are the same as those for a 3-CS module. Given $R_a = 17$, $R_b = 13$ and $h_b = 10$ with $R_a + R_b > 2.5\,h_b$ (condition 2(d)), the optimization method yields the optimal design with $\mu = 0.9576$ with d = 5, $\alpha = 211.798°$ and $\theta_b = 73.0327$. The specified dimensions for a 6-URS manipulator must satisfy the constraints for a 3-RRS module. A larger R_c is suggested to avoid possible link interactions. With $R_a = 17$, $R_b = 15.5$, $R_c = 18$, $h_b = 13$, $h_c = 8$ and d = 4, the optimization software gives $\mu = 0.9931$ with $\eta = 138.1725°$, $\xi = 24.3610°$, $\theta_a' = 7°$, $\theta_b = 0.8060°$, $\theta_b' = 14.7989°$, $\theta_c = 6.1077°$ and $\theta_c' = 59.7822°$.

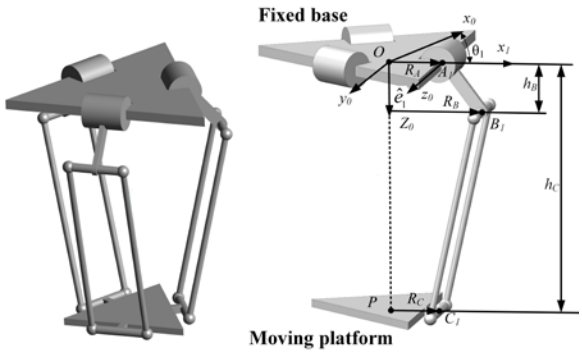
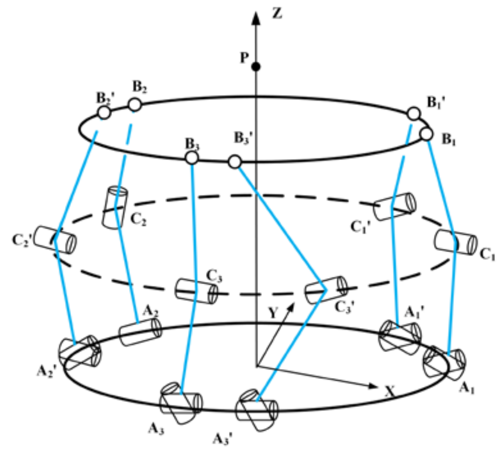
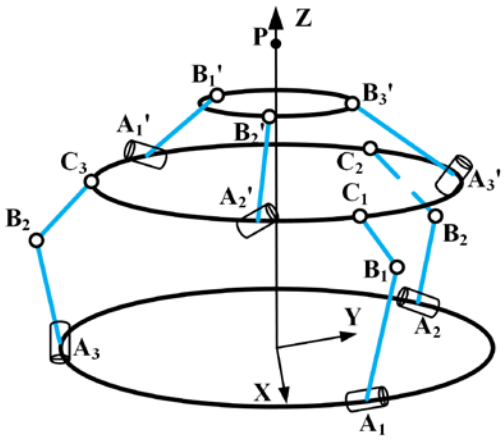


Fig. 10. A 3-DOF Delta manipulator

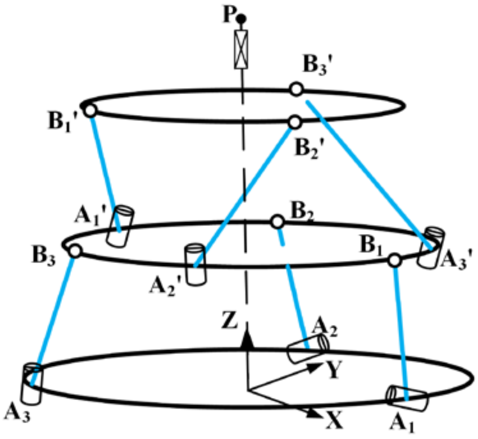
For a hybrid design with a 3-RPS module placed on a XYZ manipulator, $\mathbf{J}_{2\omega}$ in Eq. (9) is a 3×3 zero matrix and $\mathbf{J}_{2v}$ a 3×3 identity matrix. Dimensions $R_a = 17$, $R_b = 13$ and $h_b = 10$ with $R_a + R_b > 2.5\,h_b$ (condition 2(d)) are given to determine the shape of the 3-RPS module. The optimization results give $\mu = 0.9994$ with d = 9.9994, $\alpha = 49.5167°$ and $\theta_b = 75.6767$. If the XYZ manipulator is replaced by a 3-DOF Delta manipulator, $\mathbf{J}_{2\omega}$ is also a zero matrix because Delta is a translational manipulator. The elements of $\mathbf{J}_{2v}$ are functions of link parameters, R_A, R_B, R_C, h_B and h_C, shown in figure 10. Different combinations of specified link dimensions are tried and the results show that isotropic designs can be developed by specifying either three parameters for the 3-RPS module and one parameter for the Delta manipulator or two parameters for both manipulators. Given $R_a' = 16$, $R_b' = 12$ and $h_b' = 12$ for the 3-RPS module and $R_C = 16$ for the Delta manipulator, the optimization method gives $\mu = 0.9997$ with $R_B = 28.2697$, $R_A = 16.0029$, $h_B = 29.9404$, $h_C = 38.6199$ for the Delta manipulator and $d' = 5.247$, $\theta_a' = 210°$, $\theta_b' = 211.1969°$ $\alpha' = 288.5469°$ for the 3-RPS module. An extra prismatic joint is placed on the platform of 3-RPS module of the 3-RPS + 3-CS design to develop a hybrid 7-DOF redundant manipulator. If the joint is in the direction of the z-axis (the reference point on the platform of the 3-RPS module can move up and down along the z-axis of the module in this case), then the 6 x 7 Jacobian matrix can be developed by adding one more column: $[0\ 0\ 0\ 0\ 0\ 1]^t$ to the original 6 x 6 matrix. With the same specified dimensions given above with $R_a' = 15$, $h_b' = 5$, $d' = 3$, $R_a = 18$ and $R_b = 15$, the optimization software gives $\mu = 0.9684$ with $h_b = 4.3203$, $\theta_b = 6.2788°$, $\alpha = 12.0958°$ for the 3-CS module and $R_b' = 13$, $\theta_a' = 178.6226°$, $\theta_b' = 237.4718°$ $\alpha' = 118.5654°$ for the 3-RPS module. Figure 11 shows some of the designs in their initial assembly configurations.



(a) A 6-URS manipulator



(b) A Delta + 3-RPS manipulator



(c) A 7-DOF redundant manipulator

Fig. 11. Different types of isotropic manipulators

CONCLUSIONS

Developing an isotropic manipulator of six or higher degrees-of-freedom is a complicated and time-consuming task that usually involves solving a large number of nonlinear equations and the obtained results might be impractical designs with strange shapes or unacceptable dimensions. This paper presents design parameters and related kinematic constraints for developing isotropic parallel or hybrid manipulators. The proposed method is easy to implement, and can be employed to develop many different types of 3-DOF, 6-DOF or redundant manipulators. Except for a few special cases such as the Stewart-Gough manipulators, at least three design parameters can be specified to determine the shape of the isotropic manipulators. Other types of isotropic manipulators can be developed by the proposed methods if the design parameters of new 3-DOF modules are generated by rotating one limb about the z-axis by 120° and 240° respectively.

An existing dexterity measure is used as the objective function to search for isotropic designs. The measure yields the optimal value of one if a manipulator is isotropic in both position and orientation and the two row spaces defined in the measure are orthogonal (or factor $\Phi = 1$). The two row spaces cannot be orthogonal for 4-DOF and 5-DOF manipulators, so the method is not applicable to those manipulators. The method also cannot be employed to develop isotropic manipulators with different types of actuators. The elements of a row vector of the Jacobian matrix have different physical units in this case so the dexterity measure is not invariant to changes of physical units.

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三自由度及六自由度等向性機器人之設計方法

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摘 要

本文首度提出多種空間型 3-DOF 等向性並聯式機器人之設計方法，以提出之參數配合最佳軟體可於十秒內求得許多同時滿足旋轉及平移等向性之設計，且搜尋各構型之等向性設計時至少可指定三個尺度參數。其次本文以數個 3-DOF 機器人合成 6-DOF、具多餘軸或複合型等向性機器人，且合成之機器人在指定數個尺度參數下亦可得到良好操控性之設計。