中國機械工程學刊第三十九卷第六期第 619~635 頁(民國一百零七年)
Journal of the Chinese Society of Mechanical Engineers, Vol.39, No.6, pp 619~635 (2018)

A New Framework of Disturbance Observer with Vidyasagar's Structure

Yu-Xian Chen, Ruei-Yu Huang, Shih-Chung Chang, Yi-Ren Pan and An-Chen Lee*

Keywords: Robust stability, Decoupling control, Disturbance-observer controller, Doubly coprime factorization.

ABSTRACT

This paper proposes a novel two degree of freedom (TDOF) control structure: a combination of Vidyasagar’s structure (VS) and the doubly coprime factorization based disturbance observer (DCFDOB). The DCFDOB-VS framework is aimed at providing a control structure and design procedure for a MIMO system with good characteristics of disturbance attenuation, tracking, and decoupling property. The structure can deal with stable/unstable and minimum phase/non-minimum phase plants. The advantage of the method is that two parameters can be designed for different purposes, independently. The parameter H stabilizes the system and improves tracking property. Once the parameter H has been designed, then the Q_Y can be implemented via a parameter Q for rejecting disturbances and improving the system robustness.

INTRODUCTION

In order to simultaneously achieve different requirements of a control system, such as attaining a desired response, tracking, stabilizing, decoupling, increasing robustness and attenuating disturbances, a control configuration with a two degree of freedom (TDOF) compensator is necessary. A two degree of freedom configuration is first discussed by Horowitz (1963). For multiple input multiple output (MIMO) system, Vidyasagar (1985) had a discussion of a two-parameter compensator using a factorization approach and Skogestad et al. (1996).

Paper Received July, 2018. Revised October, 2018. Accepted November, 2018. Author for Correspondence: An-Chen Lee.

** Professor, Department of Mechanical Engineering, National Chiao Tung University, Hsinchu, Taiwan 30010, ROC*

It also discussed TDOF design in classical control and the limitation when existing right-half plane zeros or poles.Therefore, there has been much research on the design and construct a two degree of freedom control system. Youla et al. (1985) designed all TDOF stabilizing controllers and also discussed the tradeoff between optimum performance, tracking-cost sensitivity, and stability margins. Sugie et al. (1986) parameterized a TDOF compensator with two free parameters to achieve the robust tracking with internal stability. Hara et al. (1988) also parameterized a TDOF compensator to achieve the robust tracking with internal stability, but results can deal with systems where its output vector is not coincident with the measurement vector. Sugie et al. (1989) formulated a general result for TDOF with robust tracking problem which include the case where the controlled output not available directly. Also, Umeno *et al.* (1993) proposed a robust servo system design method based on the TDOF controller to apply in advanced motion control for a robot manipulator.

An important special case of the two parameter compensator is the observer controller structure proposing by Viswanadham (1981) and Vidyasagar (1985). The Vidyasagar’s structure (VS) use an observer-controller structure to observe the partial state and has a controller with a H parameter. And the H parameter is a unit over the set of proper and stable real rational functions. The VS structure has been extensively discussed in control problems. Baños (1996, 1998) investigated the stabilization of a plant with an observer-controller structure and developed the parameterization of nonlinear stabilizing observer- controller compensators of a given nonlinear system. Huang *et al.* (2007) discussed the relationships among the Youla-Kucera parameterization, VS structure and both expanded parameterizations with generalizing all stabilizing compensators for finite-dimensional linear systems. Lee *et al.* (2010) proposed a robust observer-controller compensator design with the parameter H which gives flexibility in tracking control. Thus, The VS can provide a stable control configuration within our proposed structure.

In addition, to eliminate disturbances and reduce the effect of uncertainties of a system, the concept of so-called disturbance-observer structure (DOB) can effectively handle these problems and provide a better performance. The original DOB structure proposed by Ohnishi (1987) is based on the concept of plant dynamics inversion and many other studies extended DOB to advanced researches. Lee et al. (1996) presented a design of robust digital DOB controller to deal with various uncertainties and external disturbances. Yi et al. (1999) proposed a TDOF controller for hard disk servo systems with DOB and adaptive robust control structure. Kim et al. (2003) proposed an advanced design method of DOB for mechanical positioning systems. Horng *et al.* (2006) proposed a method in designing DOB controller parameters to eliminate limit cycle problems while maintaining the system performance. Although the DOB has simple structure with strong robustness, these researches of DOB structure cannot be directly applied to all kind of plants such as non-minimum phase or unstable plants. On the other hand, for the disturbance rejection problem in the MIMO unstable non-minimum phase plant, Choi *et al.* (1996) provided solutions for DOB in H_∞ framework. Güvenç *et al.* (2010) proposed a robust MIMO disturbance-observer structure which can decouple the plant by treating the multiplicative model uncertainty as the extended disturbance and use the dynamics inversion method proposed by Ohnishi (1987). However, if the plant is strongly coupled or exist an unstable non-minimum phase plant, the studies is not capable of handling the system.

In this paper we propose a novel disturbance-observer compensator which is described in doubly coprime factorization Nett (1984) disturbance observer structure (DCFDOB). Unlike the way of traditional DOB using the inverse of a nominal model, the DCFDOB use factorization approach to construct a compensator with a parameter Q. The DCFDOB can estimate disturbances and the estimated states can be utilized to reject disturbances while providing satisfactory feedback properties such as sensitivity and robust stability in the presence of uncertainties and disturbances. In the meantime, the Vidyasagar's structure is merged, which has the same structure in some parts of loop with DCFDOB. Thus, the whole structure forms a new two degree of freedom structure (DCFDOB-VS) while inheriting the advantages of both sub-structures. That is, the DCFDOB-VS can be used to design a two-degree-of-freedom compensator to stabilize all kinds of MIMO plants and achieve desired properties such as tracking, decoupling and disturbance rejection.

The organization of the rest of this paper is as follows. The proposed DCFDOB-VS structure is introduced in Section 2. The proposed design procedures of parameters $H(s)$ and $Q(s)$ for minimum and non-minimum phase systems are introduced in Section 3. The system robust stability condition with coprime factor uncertainties is analyzed and the relationships between the DCFDOB-VS structure and the Youla-Kucera parameterization are discussed in Section 4. In section 5, two design examples are used to demonstrate the design procedures and the discussion offers a comparison with different parameters design. The paper ends with conclusions in section 6

THE PROPOSED FRAMEWORK

The structure of DCFDOB-VS not only has an observer-controller compensator described in Vidyasagar's structure (VS), but has a disturbance-observer compensator described in doubly coprime factorization based disturbance observer structure (DCFDOB). The concepts and properties of VS and DCFDOB are given in appendix A. In the following, we directly analyze the proposed DCFDOB-VS framework.

Let the nominal plant $P_n = N_n M_n^{-1} = \tilde{M}_n^{-1}\tilde{N}_n$ be the right coprime factorization (RCF) and the left coprime factorization (LCF) of P_n over RH_∞, respectively. By the coprime factorization approach, we have matrices $X_r, Y_r, X_l, Y_l \in RH_\infty$ that satisfy the Bezout identities $X_r M_n + Y_r N_n = I$ and $\tilde{M}_n X_l + \tilde{N}_n Y_l = I$ (McFarlane, 1990). The DCFDOB-VS framework is represented in Fig. 1

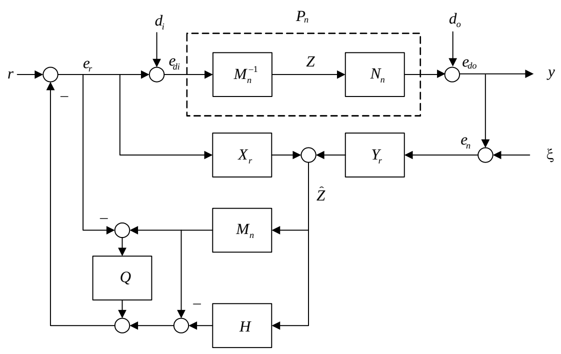

Fig. 1. The DCFDOB-VS framework

The 4×4 transfer function matrix from $\begin{bmatrix} r & d_i & d_o & \xi \end{bmatrix}^T$ to $\begin{bmatrix} e_r & e_{di} & e_{do} & e_n \end{bmatrix}^T$ is obtained as follows equation (1) (details in appendix A)

$$\begin{bmatrix} e_r \\ e_{di} \\ e_{do} \\ e_n \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} \end{bmatrix} \begin{bmatrix} r \\ d_i \\ d_o \\ \xi \end{bmatrix} \quad (1)$$

We replace $H^{-1}(I - Q)Y_l \in RH_\infty$ of the 2nd, 3rd and 4th columns in equation (1) with $Q_Y \in RH_\infty$ and yield equation (2) (details in appendix A) and also form the TDOF control scheme contains two independent parameters.

$$\begin{bmatrix} e_r \\ e_{di} \\ e_{do} \\ e_n \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} \\ R_{21} & R_{22} & R_{23} & R_{24} \\ R_{31} & R_{32} & R_{33} & R_{34} \\ R_{41} & R_{42} & R_{43} & R_{44} \end{bmatrix} \begin{bmatrix} r \\ d_i \\ d_o \\ \xi \end{bmatrix} \quad (2)$$

To ensure the internal stability of the system, it is necessary and sufficient to test whether each of sixteen transfer matrices in equation (2) is in RH_∞. Because of the matrices $M_n, N_n, \tilde{M}_n, \tilde{N}_n, X_r, Y_r, X_l, Y_l \in RH_\infty$ are all stable, if we check all transfer matrices of equation (2), the DCFDOB-VS is internally stable if and only if the parameter $Q_Y \in RH_\infty$, i.e., $Q \in RH_\infty$ and the parameter $H \in \mathbf{U}(RH_\infty)$, i.e., $H(s), H^{-1}(s) \in RH_\infty$ where the notation $\mathbf{U}(RH_\infty)$ denotes a unit over RH_∞. These two parameters can be designed for different purposes, independently. The parameter H stabilizes the system and improves tracking property. Once the parameter H has been designed, then the Q_Y can be implemented via a parameter Q for rejecting disturbances and improving the system robustness.

The closed-loop transfer function from r to y is represented by

$$y(s) = N_n(s)H^{-1}(s)r(s) \quad (3)$$

If one can design the parameter H appropriately, then the tracking response can be improved obviously. The property of parameter H has been discussed in detail by Vidyasagar (1985) and Huang (2007). Therefore, the system is internally stable if and only if the parameter $H \in \mathbf{U}(RH_\infty)$.

The transfer functions of input/output sensitivity functions are represented as follows:

$$S_i = M_n(X_r + Q_Y \tilde{N}_n) \quad (4)$$

$$S_o = I - N_n(Y_r - Q_Y \tilde{M}_n) = N_n(X_r + Q_Y \tilde{N}_n)\tilde{N}_n^{-1}\tilde{M}_n \quad (5)$$

Thus, one can design Q_Y and make the same term $(X_r + Q_Y \tilde{N}_n)$ in both input sensitivity function S_i and output sensitivity function S_o as small as possible. The smallness means to minimize frequency-dependent singular values $\bar{\sigma}(X_r + Q_Y \tilde{N}_n)$ in a certain range of low frequencies. Then the effects of both input and output disturbance are simultaneously eliminated over that frequency range.

THE DESIGN OF PARAMETERS

The design of parameter *H(s)*

A. Minimum phase case

For a minimum phase square plant, an inverse idea can be used to design the parameter $H(s)$ as follows. Let the plant $P_n = N_n M_n^{-1}$, where $N_n(s) \in RH_\infty$ is a $n \times n$ matrix. Then, the parameter $H(s)$ is selected to be

$$H(s) = \alpha(s) \cdot N_n(s) \quad (6)$$

where $\alpha(s) = diag\{\alpha_1(s), \alpha_2(s), \ldots, \alpha_n(s)\}$ in which $\alpha_i(s)$ for $i = 1 \sim n$ are polynomials:

$$\alpha_i(s) = \alpha_{i,n}s^n + \alpha_{i,n-1}s^{n-1} + \cdots + \alpha_{i,1}s + 1 \quad (7)$$

The roots of $\alpha(s)$ are all in the open left-half plane such that $\alpha(s)N_n(s) \in \mathbf{U}(RH_\infty)$. In Fig. 1, the closed-loop transfer function from r to y is represented by

$$y(s) = N_n(s)H^{-1}(s)r(s) \quad (8)$$

Then in the case we have $y(s) \approx \alpha^{-1}(s)r(s)$. Obviously, the system response is determined by the pole locations of $\alpha^{-1}(s)$ and thus the tracking can be improved. It is also easy to see that if N is square, then $N_n H^{-1}$ can always be made into a diagonal matrix to achieve decoupling for $\alpha(s)$ is a diagonal matrix. Note that the degree of polynomial $\alpha(s)$ depends on the relative degree of $N_n(s)$.

B. Non-minimum phase case

For a non-minimum phase square plant $P_n = N_n M_n^{-1}$, where $N_n(s) \in RH_\infty$ is a $n \times n$ matrix with zeros in the open right-half plane. Then the inverse idea cannot be directly used for $N_n^{-1}(s) \notin RH_\infty$. In Fig. 1, the closed-loop transfer function from r to y is

$$y(s) = N_n(s)H^{-1}(s)r(s) = G(s)r(s) \quad (9)$$

The obvious way to design $H(s)$ is using spectral factorization technique (Francis, 1987) on $N_n^{-1}(s)$ to extract the stable and minimum-phase component from the rest, and applying the method discussed above to this component while leaving the rest part. However, the whole system does not have decoupled property by this approach. Here, we consider the case with a less stringent criterion, static decoupling, which involves only the steady-state behavior of the output response (Wang, 2003). The system form equation (9) is said to be statically decoupled if it is stable and its static gain matrix $G(0)$ is diagonal and nonsingular, i.e., $G(s) = diag\{g_1(s), g_2(s), \cdots, g_n(s)\}$ with that $g_i(0) \neq 0$, $i = 1 \sim n$. Now, we have

$$\lim_{t \to \infty} y_i(t) = g_i(0) r_i(t), \; i = 1 \sim n \quad (10)$$

In general, every input r_i may affect all of the output transient response, but equation (10) guarantees that each output y_i will be unchanged in a steady state. Assume that $N_n(0)$ is nonsingular and $G(0) = I$ for tracking and decoupling are satisfied such that

$$G(0) = N_n(0) H^{-1}(0) = I \quad (11)$$

The equation (11) implies that the parameter $H(0)$ equals to $N_n(0)$. Furthermore, the system is internally stable if and only if the parameter $H \in \mathbf{U}(RH_\infty)$. Then, the parameter $H(s)$ is selected to be a unimodular matrix and to have a static gain matrix $H(0) = N_n(0)$.

Let n_{ij} be the static gain of elements of $N_n(0)$. Also let $p_1, p_2, \cdots p_m$ and $z_1, z_2, \cdots z_m$ be the open left-half plane poles and zeros of elements of $H(s)$, respectively. The relative location of these poles and zeros will affect the system response. We have

$$N_n(0) = \begin{bmatrix} n_{11} & n_{12} & \cdots & n_{1n} \\ n_{21} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ n_{n1} & \cdots & \cdots & n_{nn} \end{bmatrix} \quad (12)$$

and

$$H(s) = \begin{bmatrix} n_{11}h_{11}(s) & n_{12}h_{12}(s) & \cdots & n_{1n}h_{1n}(s) \\ n_{21}h_{21}(s) & \ddots & & \vdots \\ \vdots & & \ddots & \\ n_{n1}h_{n1}(s) & \cdots & & n_{nn}h_{nn}(s) \end{bmatrix} \quad (13)$$

where $h_{ij}(s) = \prod_{k=1}^{m} K_k \dfrac{s+z_k}{s+p_k} = \prod_{k=1}^{m} \dfrac{\eta_{z,k}s+1}{\eta_{p,k}s+1}$

and $h_{ij}(0) = 1$, $i = 1 \sim n, j = 1 \sim n$.

For $H \in \mathbf{U}(RH_\infty)$, the numbers of poles and zeros of an element of $H(s)$ should be the same which can be arbitrarily assigned, but one should also consider the limitation of bandwidth of the system and the tracking to be improved.

The design of parameter *Q(s)*

A. Minimum phase case

For a minimum phase square plant, suppose plant $P_n = N_n M_n^{-1}$ is a $n \times n$ matrix. In Fig. 1, the transfer function from d_i to e_{di} is represented by

$$e_{di}(s) = M_n(X_r + Q_Y \tilde{N}_n) \cdot d_i \quad (14)$$

Also, the transfer function from d_o to e_{do} is represented by

$$\begin{aligned} e_{do}(s) &= I - N_n(Y_r - Q_Y \tilde{M}_n) \cdot d_o \\ &= N_n(X_r + Q_Y \tilde{N}_n)\tilde{N}_n^{-1}\tilde{M}_n \cdot d_o \end{aligned} \quad (15)$$

where $Q_Y = H^{-1}(I - Q)Y_l$.

These two transfer function are so called as the input sensitivity matrix S_i and the output sensitivity matrix S_o, respectively. To reject the disturbance, the simplest way in designing Q_Y can be done as follows. Suppose that $Q_Y(s) = -X_r J \tilde{N}_n^{-1}$, where J is a $n \times n$ diagonal matrix which is composed of low-pass filters, i.e., $J(s) = diag\{j_1(s), j_2(s), \cdots, j_n(s)\}$. Then equation (14) and equation (15) can be rewritten as

$$e_{di}(s)=M_n(X_r-X_rJ)\cdot d_i=M_nX_r(I-J)\;d_i \tag{16}$$

and

$$\begin{aligned} e_{do}(s)&=N_n(X_r-X_rJ)\tilde{N}_n^{-1}\tilde{M}_n\cdot d_o\\ &=N_nX_r(I-J)\tilde{N}_n^{-1}\tilde{M}_n\cdot d_o \end{aligned} \tag{17}$$

If we design the matrix $J(j\omega)\approx I$ in a certain range of low frequencies, then the effects of both input and output disturbance are simultaneously eliminated over that frequency range. So, the parameter Q can be obtained as

$$Q(s)=I+HX_rJ\tilde{N}_n^{-1}Y_l^{-1} \tag{18}$$

where the parameter H is given in foregoing design. Note that the relative degree of each element of the low-pass filter $J(s)$ depends on the relative degree of $X_r(s)$, $\tilde{N}_n(s)$ and $Y_l(s)$ so that $X_rJ\tilde{N}_n^{-1}Y_l^{-1}$ is proper or strictly proper. Since $H\in\mathbf{U}(RH_\infty)$, then the parameter Q is realizable.

B. Non-minimum phase case

For a non-minimum phase square plant $P_n=N_nM_n^{-1}$, where $N_n(s)\in RH_\infty$ is a $n\times n$ matrix with zeros in the open right-half plane. Just as in the design of the parameter H, the inverse idea cannot be directly used for $N_n^{-1}(s)\notin RH_\infty$. In Fig. 1, the transfer function of input and output sensitivity functions are represented as equation (4-5). For the sensitivity minimization, our goal is to find a stable parameter Q to reject the input and output disturbance. Thus, one can design the parameter Q and make the same term of both the input sensitivity function S_i and output sensitivity function S_o as small as possible. The notion of smallness for these two transfer matrices especially in a certain range of low frequencies $0\le\omega\le\omega_b$ can be made using frequency dependent singular values $\bar{\sigma}(X_r+Q_Y\tilde{N}_n)$, where the matrix $X_r+Q_Y\tilde{N}_n$ is the common term of S_i and S_o with $Q_Y=H^{-1}(I-Q)Y_l$. Then, the effects of both input and output disturbance are simultaneously eliminated over that frequency range. To solve the problem, the objective function can be described as

$$\min_{Q\in RH_\infty}\left\|W\cdot[X_r+H^{-1}(I-Q)Y_l\tilde{N}_n]\right\|_\infty<1 \tag{19}$$

where the weighting function $W(s)$ is a stable and minimum phase transfer function with the properties

$$|W(j\omega)|\approx 1,\;\; 0\le\omega\le\omega_b \tag{20}$$

and

$$|W(j\omega)|\ll 1,\;\; \omega>\omega_b \tag{21}$$

For practical purpose, a possible choice of $W(s)$, by choosing suitable M_s, ω_b and ε to satisfy the performance specifications (Zhou, 1998), is obtained as

$$W=\frac{s/M_s+\omega_b}{s+\omega_b\varepsilon} \tag{22}$$

where ω_b and ε are related to the bandwidth of the disturbance rejection and the steady-state error of the system response, respectively. For MIMO, the weighting function can be designed as a square matrix which is diagonal such as $W(s)=diag\{W_1(s),W_2(s),\cdots,W_n(s)\}$ with each term $W_i(s), i=1\sim n$ chosen in the form as equation (22). Now, we rewrite equation (19) as

$$\min_{Q\in RH_\infty}\left\|W\cdot[X_r+H^{-1}(I-Q)Y_l\tilde{N}_n]\right\|_\infty \tag{23}$$

$$=\min_{Q\in RH_\infty}\left\|WX_r+WH^{-1}Y_l\tilde{N}_n-WH^{-1}QY_l\tilde{N}_n]\right\|_\infty \tag{24}$$

$$=\min_{Q\in RH_\infty}\left\|T_1+T_2QT_3]\right\|_\infty \tag{25}$$

where $T_1=WX_r+WH^{-1}Y_l\tilde{N}_n$, $T_2=-WH^{-1}$ and $T_3=Y_l\tilde{N}_n$.

This is known as the model matching problem because to solve it we need to choose the parameter Q such that the matrix $-T_2QT_3$ 'matches' the T_1 as well as possible. In the literature on H_∞ control theory (Glover, 1984 and Limebeer et al., 1987) one can convert the model matching problem into the Nehari extension problem or the Hankel approximation problem. Let we assume the inner-outer factorizations of T_2 and T_3 as $T_2=T_{2i}T_{2o}$ and $T_3=T_{3o}T_{3i}$, respectively. Then the equation (25) can be represented as

$$\min_{Q\in RH_\infty}\left\|T_1+T_2QT_3\right\|_\infty=\min_{Q\in RH_\infty}\left\|R-\tilde{Q}\right\|_\infty \tag{26}$$

Where $R=T_{2i}^{\sim}T_1T_{3i}^{\sim}\in RL_\infty$ and the other condition is $\tilde{Q}=-T_{2o}QT_{3o}\in RH_\infty$. Here we use $X^{\sim}(s)$ to denote $X^T(-s)$. Finally, the Nehari problem of the parameter Q can be solved (Maciejowski, 1989).

The parameter obtained by above procedure may have a higher order. It is useful to reduce as much as possible the parameter order which will simplify the implementation and increase the reliability. To do this one can use Optimum Hankel-norm Approximation method, Frequency Weighted Approximation method, or other controller reduction methods to find a reduced- order parameter (Obinata, 2001), but the stability and performance of the closed-loop system using reduced- order parameter should always be verified.

SYSTEM ROBUSTNESS

In this section, we will investigate the robust stability and the difference between the DCFDOB-VS structure and Youla-Kucera controller structure. The small gain theorem is used to derive robust stability test and the modeling error $\Delta(s)$ is assumed to be stable (McFarlane, 1990). Let the nominal plant $P_n = N_n M_n^{-1}$ with the modeling error $\Delta(s)$ as

$$P = (N_n + \Delta_N)\cdot(M_n + \Delta_M) \tag{27}$$

where $\Delta(s) = \begin{bmatrix} \Delta_N \\ \Delta_M \end{bmatrix}$ and $N_n, M_n \in RH_\infty$.

Then, we can modify Fig. 1 as Fig. 2 (details in appendix A)

To apply the small gain theorem, we modified Fig. 3 as $M_\Delta - \Delta$ loop as shown in the appendix A. According to the small gain theorem, the DCFDOB-VS is guaranteed internally stable for all $\|\Delta\|_\infty < \varepsilon, \varepsilon > 0$ if and only if

$$\left\| \begin{matrix} \Delta_N \\ \Delta_M \end{matrix} \right\|_\infty \cdot \left\| A_L \quad A_R \right\|_\infty \le 1 \tag{28}$$

Where the element $A_L = -Y_r + H^{-1}(I-Q)Y_l\tilde{M}_n$, and element $A_R = -X_r - H^{-1}(I-Q)Y_l M\tilde{N}_n$.

Let $Q_Y = H^{-1}(I-Q)Y_l$ is substituted into $M_\Delta(s)$ of Equation (28), then, we have

$$\|M_\Delta\|_\infty = \left\| A_L \quad A_R \right\|_\infty \le 1$$
$$= \left\| -Y_r + Q_Y\tilde{M}_n \quad -X_r - Q_Y\tilde{N}_n \right\|_\infty \le 1 \tag{29}$$

Obviously, the value of $\|M_\Delta\|_\infty$ is only influenced by the independent parameter Q_Y . Once the parameter H is designed, the parameter Q_Y is determined by the parameter Q. That is, in general, the system is internally stable for all $\|\Delta\|_\infty < 1$ if and only if $\|M_\Delta\|_\infty \le 1$ and the value of $\|M_\Delta\|_\infty$ is determined by the parameter Q_Y. The advantage of DCFDOB-VS is that it will simplify the robustness tuning procedure and disturbances rejection by using only one independent parameter Q_Y.

Furthermore, we can modify Fig. 2 to Fig. 4 and Fig. 5 through I/O equivalence.

In Fig. 4 and Fig. 5 which are shown in appendix A, the loop properties of the DCFDOB-VS structure are the same as those of the well-known Youla-Kucera controller structure (Youla, 1985) if rearranging the equation of controller as

$$[HX_r + (I-Q)Y_l\tilde{N}_n]^{-1}[HY_r - (I-Q)Y_l\tilde{M}_n]$$
$$= (X_r + Q_Y\tilde{N}_n)^{-1}(Y_r - Q_Y\tilde{M}_n) \tag{30}$$

Where $Q_Y = H^{-1}(I-Q)Y_l, Q_Y \in RH_\infty$.

On the other hand, from Fig. 5, we knew that the DCFDOB-VS also can be modified as the Youla-Kucera controller structure with a pre-filter $H^{-1}(s)$. Obviously, the DCFDOB-VS structure is the same as the EYKP (expansion of the Youla-Kucera parameterization (Huang, 2007)) structure which also have the parameter H over $\mathbf{U}(RH_\infty)$. And according to the literature (Vidyasagar, 1985), the DCFDOB-VS structure has a so-called two-parameter compensator. Then, the inverse of parameter H exhibits the pre-filter property. Thus, as discussed in the preceding sections, the DCFDOB-VS structure has two parameters which can be designed for different purpose, independently. The parameter H , which is omitted from the Youla-Kucera parameterization of all stabilizing controller, stabilizes the system and improves tracking property. The parameter Q_Y can be implemented via a parameter Q for rejecting disturbances and improving the system robustness. Moreover, Fig. 5 can explain more clearly why the loop properties, e.g. $\|M_\Delta\|_\infty$, is only influenced by the independent parameter Q_Y.

DESIGN EXAMPLES

To illustrate the design method and the closed-loop behavior of the DCFDOB-VS structure, in the following section, we give two MIMO examples which demonstrate the flexibility of parameter design to deal with unstable minimum phase and unstable non-minimum phase plant respectively.

A. An unstable minimum phase case

Suppose a square MIMO unstable minimum phase plant is given as

$$P_n = \begin{bmatrix} \dfrac{10}{s-5} & \dfrac{2}{s+6} \\ \dfrac{5}{s+7} & \dfrac{3}{s-8} \end{bmatrix} \tag{31}$$

By the coprime factorization approach, we have

$$N_n = \begin{bmatrix} N_{n11} & N_{n12} \\ N_{n21} & N_{n22} \end{bmatrix} \tag{32}$$

$$M_n = \begin{bmatrix} M_{n11} & M_{n12} \\ M_{n21} & M_{n22} \end{bmatrix} \tag{33}$$

where $P_n = N_n M_n^{-1}$ be the RCF of P_n over RH_∞. The Smith-McMillan poles locate at -7, -6, 5, 8 and zeros locate at -24.225 and -1.775. Since the plant is of unstable minimum phase, according to equation (6) and equation (7), we can apply the inverse dynamic method to obtain the parameter $H \in \mathbf{U}(RH_\infty)$ such that

$$H(s) = \alpha(s) \cdot N_n(s) = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \tag{34}$$

Where

$$\alpha(s) = \begin{bmatrix} 0.1s+1 & 0 \\ 0 & 0.1s+1 \end{bmatrix} \tag{35}$$

The details of equation (32-34) are given in appendix A. Although the roots of $\alpha(s)$ can arbitrarily assigned, one should consider the limitation of bandwidth of the system and the tracking behavior to be improved. Furthermore, according to equation (18), the order of the parameter $Q \in RH_\infty$ which contains four elements with transfer function is of order twelve and a reduced order one (Appendix A.) is given as

$$Q = I + HX_r J \tilde{N}_n^{-1} Y_l^{-1} = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \tag{36}$$

where

$$J(s) = \begin{bmatrix} \dfrac{10000}{s^2+141s+10000} & 0 \\ 0 & \dfrac{10000}{s^2+141s+10000} \end{bmatrix}$$ is

composed of low-pass filters and $X_r, \tilde{N}_n, Y_l$ can be obtained by the coprime factorization approach. The closed-loop transient response to the vectors of reference command r and input disturbance d_i is shown in Fig. 6, where

$$r = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} 1, t \geq 1\sec \\ 2, t \geq 2\sec \end{bmatrix},$$

$$d_i = \begin{bmatrix} d_{i,1} \\ d_{i,2} \end{bmatrix} = \begin{bmatrix} 1, t \geq 3\sec \\ 1, t \geq 4\sec \end{bmatrix} \tag{37}$$

In Fig. 6, The DCFDOB-VS can stabilize and decouple the system for an unstable minimum phase plant. On the other hand, the output response influenced by the input disturbance can be rejected and the tracking is well controlled.

The frequency responses from $r = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}$ to $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, d_i to y, and d_o to y are shown in Fig. 7, Fig. 8 and Fig. 9 respectively.

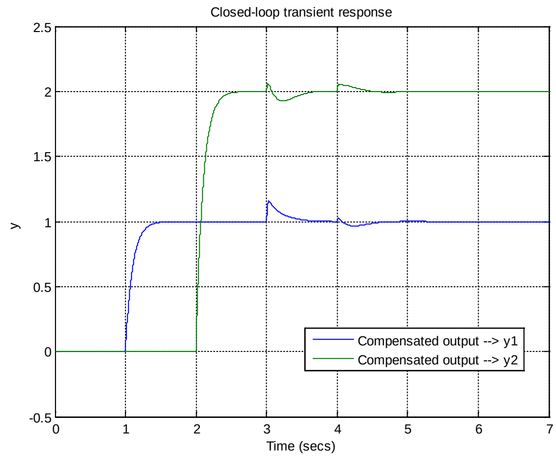

Fig. 6 The closed-loop transient response of an unstable minimum phase case

Fig. 7 represents the frequency response from the command r_1 and r_2 to the output y_1 and y_2, respectively. Obviously, the system has good tracking property in two direct channels as shown in Fig. 7 (a) and (d) and is decoupled for two cross channels as shown in Fig. 7 (b) and (c). The frequency response from input disturbance d_i to output y is shown in Fig.8 and the frequency response from output disturbance d_o to output y is shown in Fig. 9. Both results show that the magnitudes are much less than 1 in low frequency, implying the input and output disturbance can be effectively rejected at a certain range of low frequency.

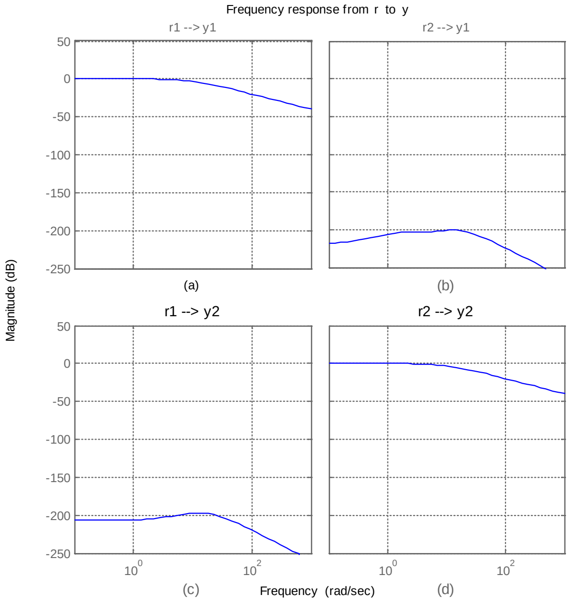

Fig. 7 The frequency response from r to y for an unstable minimum phase case

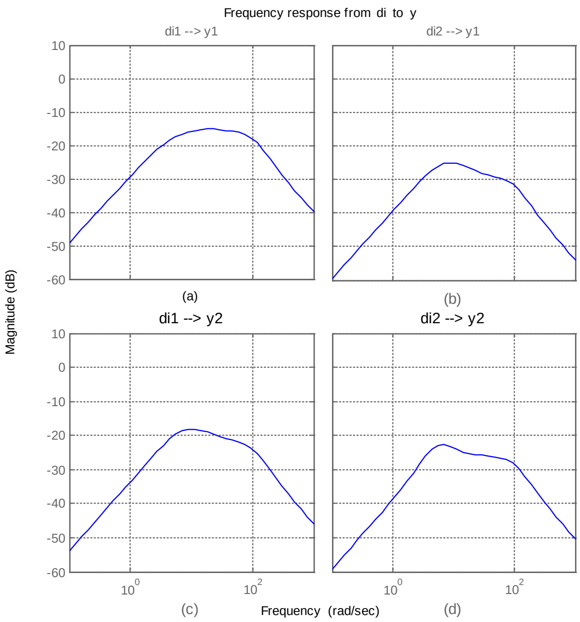

Fig. 8 The frequency response from d_i to y for an unstable minimum phase case

To demonstrate the flexibility of DCFDOB-VS design, such as the tracking and disturbance rejection, we change three kind of the coefficient of $\alpha(s)$ and remain the bandwidth of the low-pass filter $J(s)$ as

$$(a)\alpha = \begin{bmatrix} 0.1s+1 & 0 \\ 0 & 0.1S+1 \end{bmatrix},$$

$$(b)\alpha = \begin{bmatrix} 0.5s+1 & 0 \\ 0 & 0.5S+1 \end{bmatrix}, \text{ and}$$

$$(c)\alpha = \begin{bmatrix} s+1 & 0 \\ 0 & s+1 \end{bmatrix} \tag{38}$$

And

$$J(s) = \begin{bmatrix} \dfrac{10000}{s^2+141s+10000} & 0 \\ 0 & \dfrac{10000}{s^2+141s+10000} \end{bmatrix} \tag{39}$$

As different design, the step responses from $r = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}$ to $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, d_i to y, and d_o to y are shown in Fig. 10. Fig. 11 and Fig. 12, respectively.

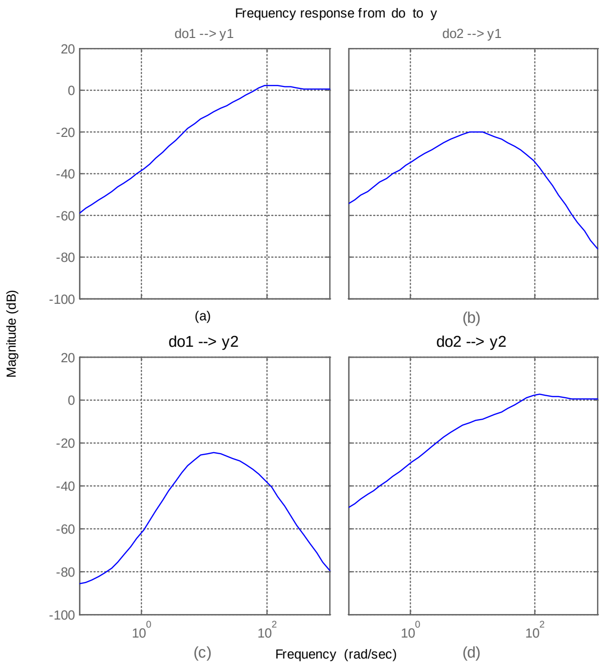

Fig. 9 The frequency response from d_o to y for an unstable minimum phase case

In Fig. 10, the root of α designed for parameter H changes from -10 to -1 and the system transient leads to a slower response. That is, the system response is determined by the pole locations of α^{-1} and the tracking can be improved. On the other hand, the input and output disturbance rejection of the closed-loop system with DCFDOB-VS is shown in Fig. 11 and Fig. 12. We see that the transient response of step output disturbance is larger than the result of step input disturbance. But both results show that the DCFDOB-VS structure have good ability of disturbance attenuation simultaneously. Hence, it is clear that the change of coefficient of $\alpha(s)$ can improve the tracking but have no effect in disturbances attenuation. As mentioned earlier, the parameter H and the parameter Q are designed for different purpose, independently.

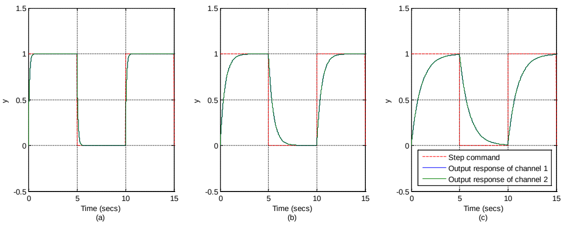

Fig. 10 The step response from r to y for an unstable minimum phase case

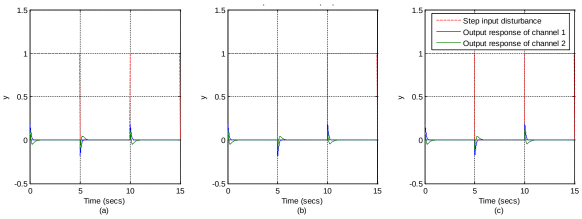

Fig. 11 The step response from d_i to y for an unstable minimum phase case

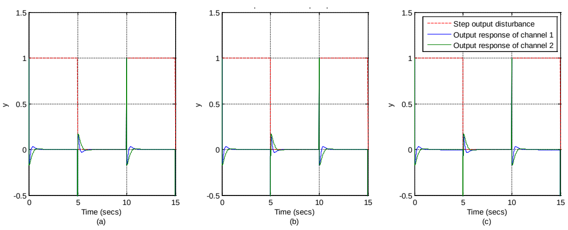

Fig. 12 The step response from d_o to y for an unstable minimum phase case

B. An unstable non-minimum phase case

Suppose a square MIMO unstable minimum phase plant is given as

$$P_n = \begin{bmatrix} \dfrac{s-10}{s^2+s-6} & \dfrac{1}{s+4} \\ \dfrac{1}{s+3} & \dfrac{s-20}{s^2-4s-12} \end{bmatrix} \tag{40}$$

By the coprime factorization approach, we have

$$N_n = \begin{bmatrix} N_{n11} & N_{n12} \\ N_{n21} & N_{n22} \end{bmatrix} \tag{41}$$

$$M_n = \begin{bmatrix} M_{n11} & M_{n12} \\ M_{n21} & M_{n22} \end{bmatrix} \tag{42}$$

where $P_n = N_n M_n^{-1}$ be the RCF of P_n over RH_∞ and the details of equation (41,42) are given in appendix A. The Smith-McMillan poles locate at -3, -3, -2, -4, 2, 6 and zeros locate at 8.6734, -4.4734 and -3. Since the plant is of unstable non-minimum phase, according to equation (12) and equation (13), the parameter $H \in \mathbf{U}(RH_\infty)$ is selected to be a unimodular matrix such that

$$H(s) = \begin{bmatrix} -0.8410\dfrac{0.4s+1}{s+1} & -0.1212 \\ 0.0399 & -0.8508\dfrac{0.4s+1}{s+1} \end{bmatrix} \tag{43}$$

where

$$H(0) = N_n(0) = \begin{bmatrix} -0.841 & -0.1212 \\ 0.0399 & -0.8508 \end{bmatrix} \tag{44}$$

For $H \in \mathbf{U}(RH_\infty)$, the numbers of poles and zeros of an element of H should be the same which can be arbitrarily assigned, but one should consider the limitation of bandwidth of the system and the tracking to be improved.

To design parameter Q, according to the equations from (19) to (26), by solving the Nehari extension problem the objective function can be

$$\min_{Q\in RH_\infty} \left\| W\cdot[X_r + H^{-1}(I-Q)Y_l \tilde{N}_n] \right\|_\infty < 1 \tag{45}$$

The weighting function $W = diag\{W_1, W_2\}$ is chosen as

$$W_1 = W_2 = \frac{s/M_s + \omega_b}{s+\omega_b \varepsilon} = \frac{0.002(s+1000)}{s+0.02} \tag{46}$$

where $M_s = 500, \varepsilon = 0.01$ and $\omega_b = 2$. Then, the obtained parameter $Q \in RH_\infty$ contains four elements, each with a 28 orders transfer function and the reduced one is given as

$$Q = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix} \tag{47}$$

Where details of equation (47) are given in Appendix A.

The closed-loop transient response to the vectors of reference command r and input disturbance d_i is showed in Fig. 13 where

$$r = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} 1, t \geq 1\sec \\ 2, t \geq 2\sec \end{bmatrix},$$

$$d_i = \begin{bmatrix} d_{i,1} \\ d_{i,2} \end{bmatrix} = \begin{bmatrix} 1, t \geq 3\sec \\ 1, t \geq 4\sec \end{bmatrix} \tag{48}$$

In Fig. 13, just as the above case, The DCFDOB-VS can stabilize and decouple the system for an unstable non-minimum phase plant, too. Although the overshoot of the response is large, the output response influenced by the input disturbance can be rejected and the tracking is well controlled. Note that, for a non-minimum phase plant, the initial drop of response is due to the right-half plane zeros in the transfer function.

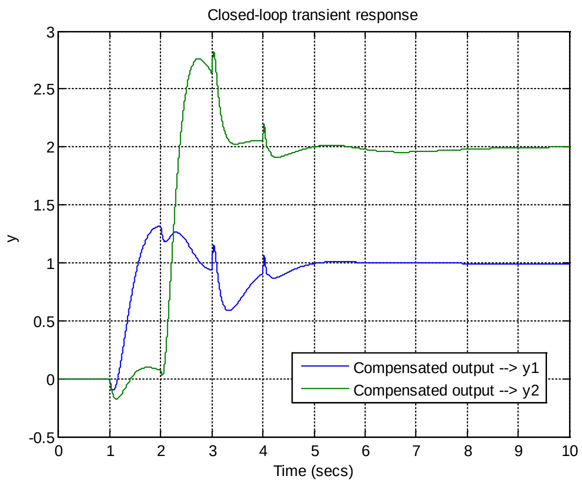

Fig. 13 The closed-loop transient response of an unstable non-minimum phase case

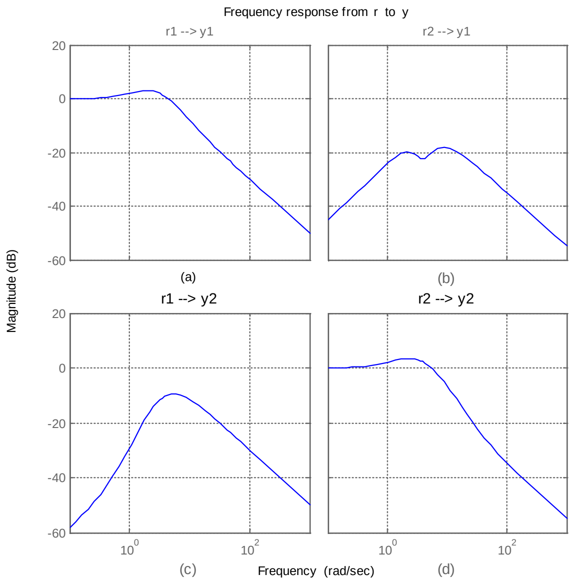

Fig. 14 The frequency response from r to y for an unstable non-minimum phase case

The frequency responses from $r=\begin{bmatrix} r_1 \\ r_2 \end{bmatrix}$ to $y=\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, d_i to y, and d_o to y are shown in Fig. 14, Fig. 15 and Fig. 16, respectively.

Fig. 14 represents the frequency response from the command r_1 and r_2 to the output y_1 and y_2, respectively. Using DCFDOB-VS structure to deal with a unstable non-minimum phase plant, the control system has good tracking property in two direct channels as shown in Fig. 14 (a) and (d) and has less effect in two cross channels as shown in Fig. 14 (b) and (c). The frequency response from input disturbance d_i to output y is shown in Fig. 15 and the frequency response from output disturbance d_o to output y is shown in Fig. 16. Both results show that the magnitudes are much less than 1 in low frequency, implying the input and output disturbance can be effectively rejected at a certain range of low frequency.

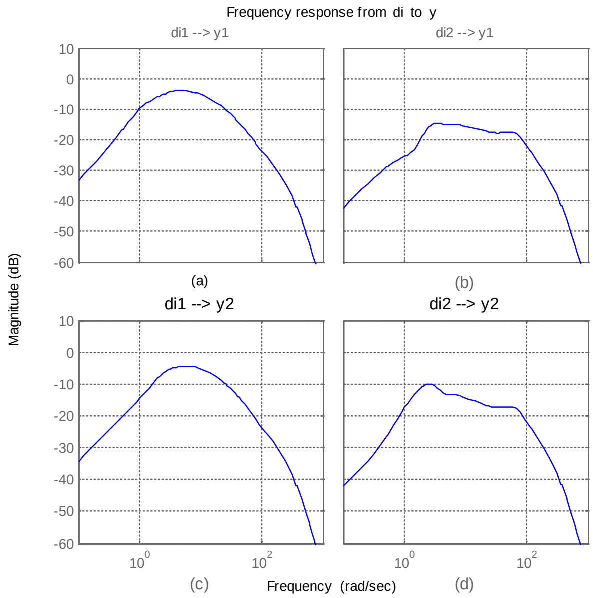

Fig. 15 The frequency response from d_i to y for an unstable non-minimum phase case

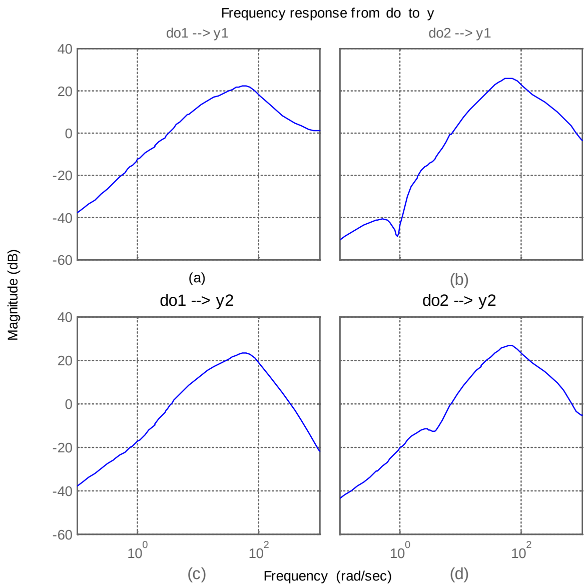

Fig. 16 The frequency response from d_o to y for an unstable non-minimum phase case

To demonstrate the flexibility of DCFDOB-VS design, such as the tracking and disturbance rejection, we design three kind of $H(s)$ and remain the weighting function $W(s)$ as

$$(a)H(s)=\begin{bmatrix} -0.8410\dfrac{0.4s+1}{s+1} & -0.1212 \\ 0.0399 & -0.8508\dfrac{0.4s+1}{s+1} \end{bmatrix},$$

$$(b)H(s)=\begin{bmatrix} -0.8410\dfrac{0.6s+1}{s+1} & -0.1212 \\ 0.0399 & -0.8508\dfrac{0.6s+1}{s+1} \end{bmatrix},$$

$$(c)H(s)=\begin{bmatrix} -0.8410\dfrac{0.9s+1}{s+1} & -0.1212 \\ 0.0399 & -0.8508\dfrac{0.9s+1}{s+1} \end{bmatrix} \quad (49)$$

and

$$W(s)=\begin{bmatrix} \dfrac{0.002(s+1000)}{s+0.02} & 0 \\ 0 & \dfrac{0.002(s+1000)}{s+0.02} \end{bmatrix} \quad (50)$$

For different designs, the step responses from $r=\begin{bmatrix} r_1 \\ r_2 \end{bmatrix}$ to $y=\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, d_i to y, and d_o to y are shown in Fig. 17, Fig. 18 and Fig. 19, respectively.

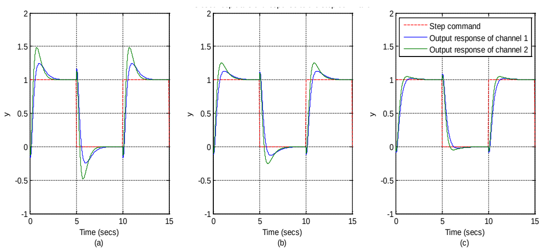

Fig. 17 The step response from r to y for an unstable non-minimum phase case

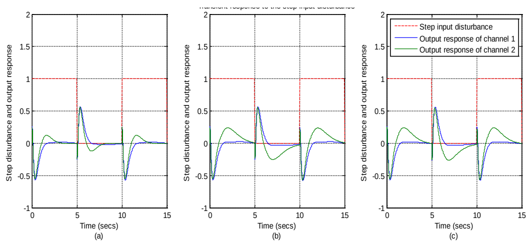

Fig. 18 The step response from d_i to y for an unstable non-minimum phase case

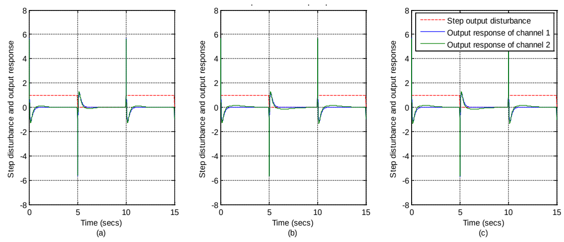

Fig. 17 The step response from d_o to y for an unstable non-minimum phase case

In Fig. 17, the zeros of the element of $H(s)$ change from -2.5 to -1.11 and the poles of the element of $H(s)$ remain at -1. The system transient leads to a slower response. That is, the bandwidth of system is determined by the relative location of zeros and poles of parameter $H(s)$ and the tracking can be improved. To compare these three kinds of design, a faster response will lead to a higher overshoot and undershoot for the nature of second order and non-minimum phase plant. In addition, the responses of the input and output disturbance rejection are shown in Fig. 18 and Fig. 19. We see that the transient response of step output disturbance is much larger than the result of step input disturbance, but both results show that the DCFDOB-VS structure have good ability of disturbance attenuation simultaneously. In this case, it also demonstrate that the property of parameter H can improve the tracking but have no effect in disturbance attenuation. As mentioned earlier, the parameter H and the parameter Q are designed for different purpose, independently.

Overall, for an unstable minimum phase or an unstable non-minimum phase MIMO plant, the DCFDOB-VS structure can provide a two steps design method to yield satisfactory performance. By appropriately choosing two parameters H and Q, one can design a two-degree-of-freedom controller to stabilize the plant and provide desired properties.

CONCLUSION

The paper presented a new framework that combines the proposed DCFDOB structure with the Visyasagar's structure which has the subset of stabilizing solutions of the Youla-Kucera parameterization. By sharing the common observer-controller configuration to form the DCFDOB-VS structure, we obtain two parameters, i.e., $H(s)$ and $Q(s)$, for different control purposes, i.e., the DCFDOB-VS inherits the advantages of both structures. Therefore, the proposed DCFDOB-VS can deal with tracking, decoupling and input/output disturbance rejection objective, respectively. A two-step design method is proposed to yield satisfactory performance. By appropriately choosing two parameters, one can design a two-degrees-of-freedom compensator to stabilize a MIMO plant and provide desired properties. In particular, the DCFDOB-VS not only can stabilize an unstable minimum phase MIMO plant, but can deal with an unstable non-minimum phase MIMO plant, too.

REFERENCE

Horowitz IM., "*Synthesis of Feedback Systems*", Academic Press: New York, (1963).

Vidyasagar M., "*Control System Synthesis: A Coprime Factorization Approach*", MIT Press: Cambridge, MA, (1985).

Skogestad S, Postlethwaite I., "*Multivariable Feedback Control: Analysis and Design*", John Wiley & Sons Ltd: New York (1996).

Youla DC, Bongiorno JJ., "A Feedback Theory of Two-Degree-of-freedom Optimal Wiener Hopf Design", *IEEE Transactions on Automatic Control*; **30**(7):652—665 (1985).

Sugie T, Yoshikawa T., "General solution of robust tracking problem in two-degree-of-freedom control systems", *IEEE Transactions on Automatic Control*; **31**(6):552—554 (1986).

Hara S, Sugie T., "Independent parameterization of two-degree-of-freedom compensators in general robust tracking systems", *IEEE Transactions on Automatic Control*; **33**(1):59—67 (1988).

Sugie T, Vidyasagar M., "Further results on the robust tracking problem in two degree of freedom control", *Systems and Control Letters*; **13**(2):101—108 (1989).

Umeno T, Kaneko T, Hori Y., "Robust servo system design with two degrees of freedom and its application to novel motion control of robot manipulators", *IEEE Transactions on industrial electronics*; **40**(5):473—485 (1993).

Viswanadham N, Vidyasagar M., "Stabilization of linear and nonlinear dynamical systems using an observer-controller configuration", *Systems and Control Letters*; **1**(2):87—91 (1981)

Baños A., "Stabilization of nonlinear systems based on a generalized Bezout Identity", *Automatica* 1996; **32**(4):591—595 (1996).

Baños A., "Parameterization of nonlinear stabilizing controllers: the observer-controller configuration", *IEEE Transactions on Automatic Control*; **43**(9):1268—1272 (1998).

Huang YY, Lee AC., "Generalization of all stabilizing compensators for finite dimensional linear systems", *Journal of the Franklin Institute*; **344**(8): 1075—1090 (2007).

Lee AC, Pan YR, Huang YY., "Robust observer-controller compensator design using the loop shaping design procedure and the algebraic method", *International Journal of Robust and Nonlinear Control*; **20**: 1176—1196 (2010).

Ohnishi K., "A new servo method in mechatronics", *Transactions of Japanese Society of Electrical Engineers*; **107**(D):83-86 (1987).

Lee HS, Tomizuka M., "Robust motion controller design for high-accuracy positioning systems", *IEEE Transactions on Industrial Electronics*; **43**(1):48-55 (1996).

Yi L, Tomizuka M., "Two-degree-of-freedom control with robust feedback control for hard disk servo systems", *IEEE/ASME Transactions on Mechatronics*; **4**(1):17-24 (1999).

Kim BK, Chung WK., "Advanced Disturbance Observer Design for Mechanical Positioning Systems", *IEEE Transactions on Industrial Electronics* 2003; **50**(6):1207—1216 (2003).

Horng RH, Chou HL, Lee AC., "Rejection of limit cycles induced from disturbance observers in motion control", *IEEE Transactions on Industrial Electronics*; **53**(6):1770-1780 (2006).

Choi Y, Chung WK, Youm Y., "Disturbance observer in H_∞ frameworks", *Proceedings of the 23th IEEE IECON*, Taipei, Taiwan, 1394—1400, (1996).

Güvenç BA, Güvenç L, Karaman S., "Robust MIMO disturbance observer analysis and design with application to active car steering", *International Journal of Robust and Nonlinear Control*; **20**:873—891 (2010).

Nett CN, Jacobson CA, Balas MJ., "A connection between state-space and doubly coprime fractional representations", *IEEE Transactions on Automatic Control*; **29**(9):831—832 (1984).

McFarlane DC, Glover K., "*Robust Controller Design Using Normalized Coprime Factor Plant Descriptions*", Springer-Verlag: New York, (1990).

Zhou K, Doyle JC., "*Essentials of Robust Control*", Prentice-Hall: New Jersey, (1998).

Wang QG., "*Decoupling Control*", Springer: New York, (2003).

Albertos P, Sala A., "*Multivariable Control Systems: An Engineering Approach*", Springer: New York, (2004).

Nehari Z., "On Bounded Bilinear Forms", *Annals of Mathematics*; **65**: 153-162 1957.

Glover K., "All optimal Hankel-norm approximations of linear multivariable systems and their L^{∞} - error bounds", *International Journal of Control*; **39**(6):1115—1193 (1984).

Francis BA., "*A course in* H_{∞} *Control Theory*", Springer-Verlag: New York, (1987).

Limebeer DJN, Hung YS., "An analysis of the pole-zero cancellations in H_{∞} - optimal control problems of the first kind", *SIAM Journal on Control and Optimization*; **25**(6)1457—1493 (1987).

Maciejowski JM., "*Multivariable Feedback Desig*", Addison-Wesley Publishing Company, (1989)

Obinata G, Anderson BDO., "*Model Reduction for Control System Design*" Springer: New York, (2001).

雙互質分解干擾觀測器

陳宇賢 黃瑞宇 張時中 潘怡仁 李安謙
國立交通大學機械工程學系

摘要

本文提出一新穎雙自由度的控制架構，其結合雙互質分解(Doubly coprime)觀測器以及Vidyasagar’s structure。本文之控制方法提供一完善的控制架構以及設計步驟使得一多輸入多輸出系統可以具備良好的抑制干擾效能之外，並給予優異的軌跡追蹤以及方便的解耦性能射表現。本文所提出之雙自由度的控制架構可應用於穩定、非穩定、極小相位及非極小相位等線性系統，文中對於不同系統狀況之內部穩定及穩健性皆有詳細分析，最後本文方法之優點為對於兩個參數的有最佳的獨立設計步驟，其可獨立針對提升系統追蹤性能表現做設計外，亦可透過另一參數提升對干擾之抑制效果以及系統穩健性。

APPENDIX A

The equation (1,2), (32-34), (36) and (41,42) are given in details as below

$$\begin{bmatrix} e_r \\ e_{di} \\ e_{do} \\ e_n \end{bmatrix} = \begin{bmatrix} M_n H^{-1} & M_n[X_r + H^{-1}(I-Q)Y_l \tilde{N}_n] - I & -M_n[Y_r - H^{-1}(I-Q)Y_l \tilde{M}_n] & -M_n[Y_r - H^{-1}(I-Q)Y_l \tilde{M}_n] \\ M_n H^{-1} & M_n[X_r + H^{-1}(I-Q)Y_l \tilde{N}_n] & -M_n[Y_r - H^{-1}(I-Q)Y_l \tilde{M}_n] & -M_n[Y_r - H^{-1}(I-Q)Y_l \tilde{M}_n] \\ N_n H^{-1} & N_n[X_r + H^{-1}(I-Q)Y_l \tilde{N}_n] & I - N_n[Y_r - H^{-1}(I-Q)Y_l \tilde{M}_n] & -N_n[Y_r - H^{-1}(I-Q)Y_l \tilde{M}_n] \\ N_n H^{-1} & N_n[X_r + H^{-1}(I-Q)Y_l \tilde{N}_n] & I - N_n[Y_r - H^{-1}(I-Q)Y_l \tilde{M}_n] & I - N_n[Y_r - H^{-1}(I-Q)Y_l \tilde{M}_n] \end{bmatrix} \begin{bmatrix} r \\ d_i \\ d_o \\ \xi \end{bmatrix}$$

$$\begin{bmatrix} e_r \\ e_{di} \\ e_{do} \\ e_n \end{bmatrix} = \begin{bmatrix} M_n H^{-1} & M_n(X_r + Q_Y \tilde{N}_n) - I & -M_n(Y_r - Q_Y \tilde{M}_n) & -M_n(Y_r - Q_Y \tilde{M}_n) \\ M_n H^{-1} & M_n(X_r + Q_Y \tilde{N}_n) & -M_n(Y_r - Q_Y \tilde{M}_n) & -M_n(Y_r - Q_Y \tilde{M}_n) \\ N_n H^{-1} & N_n(X_r + Q_Y \tilde{N}_n) & I - N_n(Y_r - Q_Y \tilde{M}_n) & -N_n(Y_r - Q_Y \tilde{M}_n) \\ N_n H^{-1} & N_n(X_r + Q_Y \tilde{N}_n) & I - N_n(Y_r - Q_Y \tilde{M}_n) & I - N_n(Y_r - Q_Y \tilde{M}_n) \end{bmatrix} \begin{bmatrix} r \\ d_i \\ d_o \\ \xi \end{bmatrix} \tag{2}$$

$$N_n = \begin{bmatrix} \dfrac{10(s+4.065)(s^2+17.59s+85.96)}{(s+13.53)(s+4.162)(s^2+16.06s+73.22)} & \dfrac{2(s-4.31)(s+4.886)(s+8.604)}{(s+13.53)(s+4.162)(s^2+16.06s+73.22)} \\ \dfrac{5\ (s-4.673)\ (s+5.837)\ (s+8.338)}{(s+13.53)(s+4.162)(s^2+16.06s+73.22)} & \dfrac{3(s+2.184)(s^2+22.23s+172.9)}{(s+13.53)(s+4.162)(s^2+16.06s+73.22)} \end{bmatrix} \tag{32}$$

$$M_n = \begin{bmatrix} \dfrac{\text{(s+5.871) (s+7) (s+8.854) (s-5)}}{(s+13.53)(s+4.162)(s^2+16.06s+73.22)} & \dfrac{\text{-0.37027 (s-36.43) (s+7) (s-5)}}{(s+13.53)(s+4.162)(s^2+16.06s+73.22)} \\ \dfrac{\text{-0.37027 (s-24.34) (s-8) (s+6)}}{(s+13.53)(s+4.162)(s^2+16.06s+73.22)} & \dfrac{\text{(s+12.5) (s+6.535) (s+6) (s-8)}}{(s+13.53)(s+4.162)(s^2+16.06s+73.22)} \end{bmatrix} \quad (33)$$

$$H = \begin{bmatrix} \dfrac{(s+4.065)(s+10)(s^2+17.59s+85.96)}{(s+13.53)(s+4.162)(s^2+16.06s+73.22)} & \dfrac{\text{0.2 (s-4.31) (s+4.886) (s+8.604) (s+10)}}{(s+13.53)(s+4.162)(s^2+16.06s+73.22)} \\ \dfrac{\text{0.5 (s-4.673) (s+5.837) (s+8.338) (s+10)}}{(s+13.53)(s+4.162)(s^2+16.06s+73.22)} & \dfrac{0.3(s+10)(s+2.184)(s^2+22.23s+172.9)}{(s+13.53)(s+4.162)(s^2+16.06s+73.22)} \end{bmatrix} \quad (34)$$

$$Q = \begin{bmatrix} \dfrac{(s+4.065)(s+10)(s^2+17.59s+85.96)}{(s+13.53)(s+4.162)(s^2+16.06s+73.22)} & \dfrac{\text{0.2 (s-4.31) (s+4.886) (s+8.604) (s+10)}}{(s+13.53)(s+4.162)(s^2+16.06s+73.22)} \\ \dfrac{\text{0.5 (s-4.673) (s+5.837) (s+8.338) (s+10)}}{(s+13.53)(s+4.162)(s^2+16.06s+73.22)} & \dfrac{0.3(s+10)(s+2.184)(s^2+22.23s+172.9)}{(s+13.53)(s+4.162)(s^2+16.06s+73.22)} \end{bmatrix} \quad (36)$$

$$N_n = \begin{bmatrix} \dfrac{(s-10.1)(s+5.258)(s+3)(s^2+7.922s+17.05)}{(s+3)(s+4.294)(s^2+9.56s+24.69)(s^2+5.732s+10.16)} & \dfrac{(s-5.831)(s+3.571)(s+3)(s^2+4.048s+6.268)}{(s+3)(s+4.294)(s^2+9.56s+24.69)(s^2+5.732s+10.16)} \\ \dfrac{(s+3)(s+4.086)(s+5.78)(s^2+1.209s+1.821)}{(s+3)(s+4.294)(s^2+9.56s+24.69)(s^2+5.732s+10.16)} & \dfrac{(s-20.27)(s+4.17)(s+3)(s^2+5.884s+10.84)}{(s+3)(s+4.294)(s^2+9.56s+24.69)(s^2+5.732s+10.16)} \end{bmatrix}$$

(41)

$$M_n = \begin{bmatrix} \dfrac{(s-2)(s+3)^2(s+3.842)(s^2+9.595s+24.48)}{(s+3)(s+4.294)(s^2+9.56s+24.69)(s^2+5.732s+10.16)} & \dfrac{-0.36171(s+5.723)(s+3)^2(s-2)(s+0.3469)}{(s+3)(s+4.294)(s^2+9.56s+24.69)(s^2+5.732s+10.16)} \\ \dfrac{\text{-0.36171 (s+7.988) (s+4) (s+3) (s+2) (s-6)}}{(s+3)(s+4.294)(s^2+9.56s+24.69)(s^2+5.732s+10.16)} & \dfrac{(s+3)(s+4)(s+2)(s-6)(s^2+6.15s+11.47)}{(s+3)(s+4.294)(s^2+9.56s+24.69)(s^2+5.732s+10.16)} \end{bmatrix}$$

(42)

And each element of equation (47) are shown as below

$$q_{11} = \frac{1162.1189(s+538.8)(s+140.5)(s-29.54)(s^2+0.06484s+1.042)(s^2+13.38s+60.95)}{(s+68.46)(s+16.78)(s+1.08)(s^2+66s+4441)(s^2+608.7s+1.048\times10^5)}$$

$$q_{21} = \frac{752.7588(s+1259)(s+189.7)(s-3.682)(s^2+9.012s+20.73)(s^2+4.589s+20.64)}{(s+578.9)(s+310.8)(s+68.16)(s+16.86)(s+1.001)(s^2+66s+4441)}$$

$$q_{12} = \frac{-1567.8113(s+9.472)(s^2+8.043s+25.67)(s^2+2.818s+73.94)(s^2+259.4s+2.095\times10^4)}{(s+66.51)(s+17.77)(s+3.4)(s^2+65.97s+4441)(s^2+677.1s+1.395\times10^5)}$$

$$q_{22} = \frac{-567.7747(s+15.23)(s^2-2.781s+3.344)(s^2+18.08s+232.6)(s^2+398.7s+5.326\times10^4)}{(s+65.13)(s+19.8)(s+0.9914)(s^2+65.96s+4443)(s^2+445.1s+1.322\times10^5)}$$

And the Fig. 2~5 are given

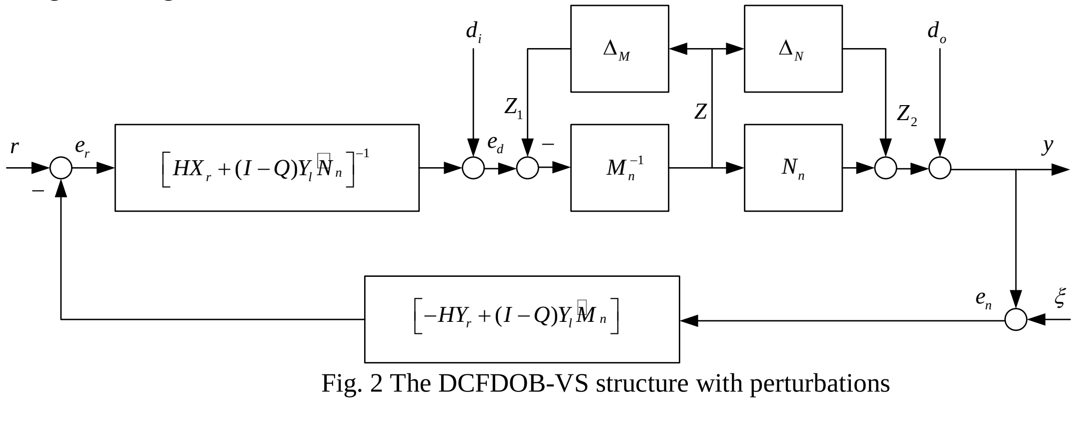

Fig. 2 The DCFDOB-VS structure with perturbations

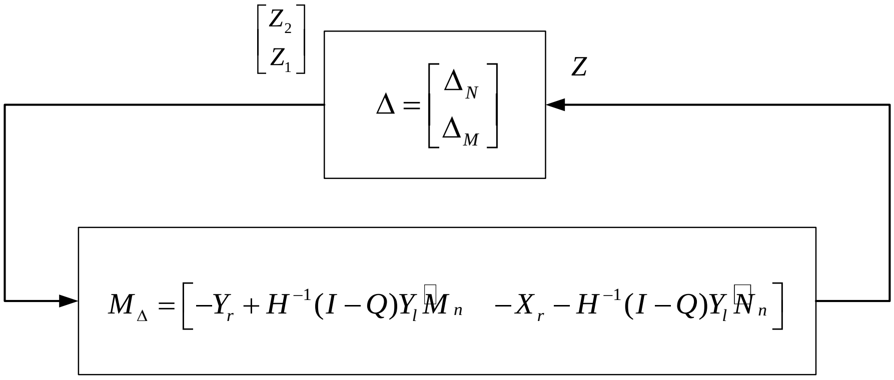

Fig.3 $M_\Delta - \Delta$ loop of DCFDOB-VS structure

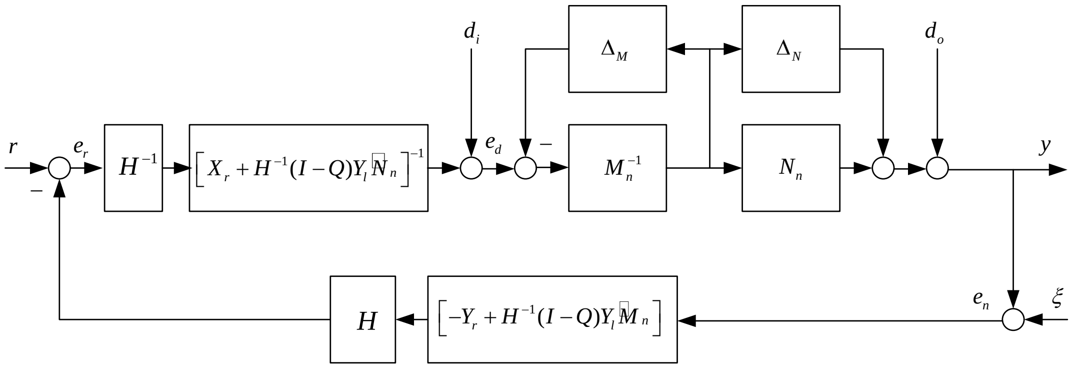

Fig. 4 The modification of DCFDOB-VS structure with perturbations

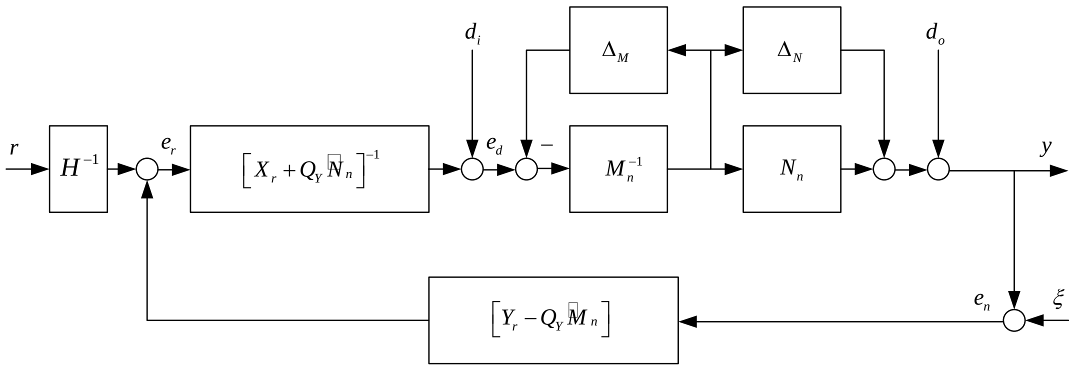

Fig. 5 The equivalent modification of DCFDOB-VS with two independent parameters, $H(s)$ and $Q_Y(s)$

APPENDIX B

An observer-controller compensator described in Vidyasagar's structure (VS) is shown in Figure B.1 (Vidyasagar, 1985). The VS can be equivalent to the well-known Youla-Kucera parameterization, the set of all proper controllers that stable the system, and provides the tracking property when $K_v = H - M_n$, $H \in \mathbf{U}(RH_\infty)$ is applied (Huang, 2007). However, The VS structure only has one parameter to trade-off tracking performance or feedback performance. That is, the VS structure has a one-degree-of-freedom controller. If we use the relationship of $K_v = H - M_n$ to reconstruct the loop path of VS, then an equivalent structure is transformed as Figure B.2.

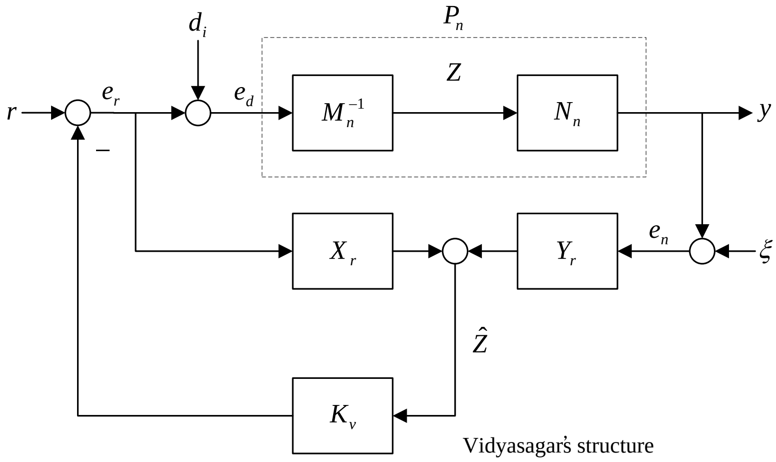

Figure B.1: The Vidyasagar's structure

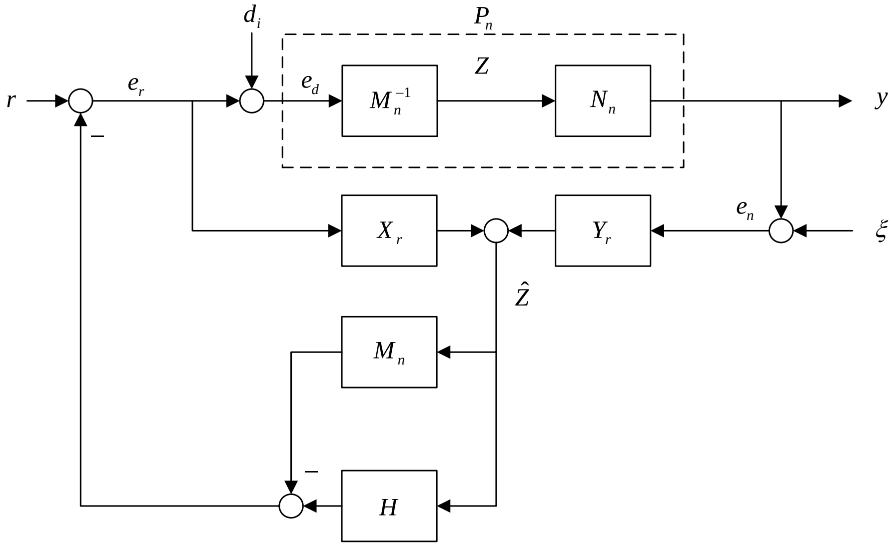

Figure B.2: The transformed Vidyasagar's structure

A disturbance-observer compensator described in doubly coprime factorization based disturbance observer structure (DCFDOB) is shown in Figure A.3.

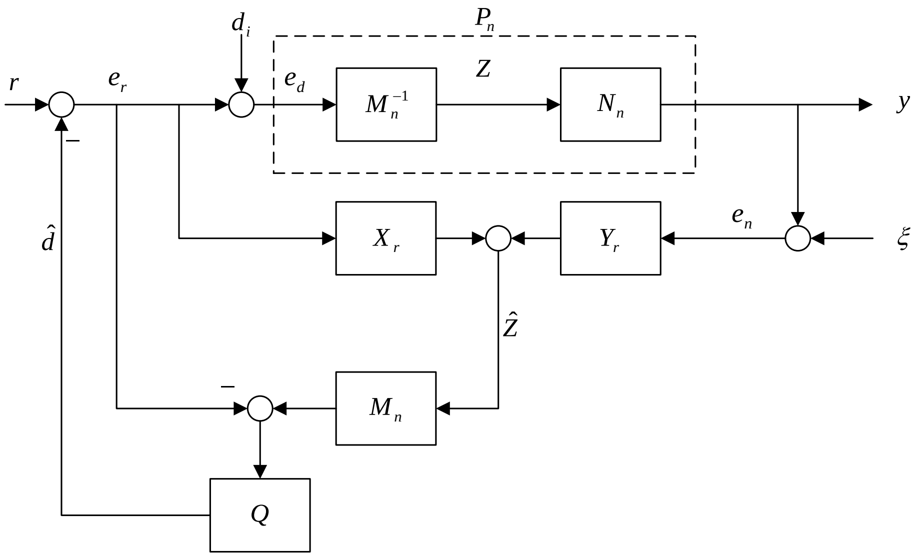

Figure B.3: The DCFDOB structure

Let a plant $P_n = N_n M_n^{-1} = \tilde{M}_n^{-1} \tilde{N}_n$ be the rcf and the lcf of P_n over RH_∞, respectively. By the coprime factorization approach, there exit matrices $X_r, Y_r, X_l, Y_l \in RH_\infty$ that satisfy the Bezout identities. In Figure B.3, the 3×3 transfer function matrix from $\begin{bmatrix} r & d_i & \xi \end{bmatrix}^T$ to $\begin{bmatrix} e_r & e_d & e_n \end{bmatrix}^T$ is obtained as follows.

$$\begin{bmatrix} e_r \\ e_d \\ e_n \end{bmatrix} = \begin{bmatrix} I & -Q(I - M_n X_r) & -QM_n Y_r \\ I & I - Q(I - M_n X_r) & -QM_n Y_r \\ N_n M_n^{-1} & N_n M_n^{-1}(I - Q(I - M_n X_r)) & I - N_n M_n^{-1} Q M_n Y_r \end{bmatrix} \begin{bmatrix} r \\ d_i \\ \xi \end{bmatrix} \tag{B.1}$$

To ensure the internal stability of the DCFDOB, it is necessary and sufficient to test whether each of ninth transfer matrices in equation (B.1) is in RH_∞. For a stable plant $P_n = N_n M_n^{-1}$, i.e. $M_n^{-1} \in RH_\infty$, then the system is internally stable if the parameter $Q \in RH_\infty$ is provided. To investigate the states in the loop, the DCFDOB can estimate disturbances and the estimate states can be utilized to reject disturbances while providing satisfactory feedback properties such as sensitivity and robust stability in the presence of uncertainties and disturbances.

The transfer function from d_i to e_{di} is represented by

$$e_{di}(s) = [I - Q(I - M_n X_r)] \cdot d_i . \tag{B.2}$$

Thus, the transfer function $S_i = I - Q(I - M_n X_r)$ denotes the input sensitivity function from input disturbances d_i to compensated input signals e_{di}. Suppose one can design a parameter $Q \in RH_\infty$ such that the matrix $Q(I - M_n X_r) \approx I$ is obtained or the frequency-dependent singular values $\overline{\sigma}(I - Q(I - M_n X_r))$ as small as possible are existed in a certain range of low frequency. Then, the DCFDOB can effectively eliminate input disturbances over that low frequency range.

The proposed DCFDOB and Vidyasagar's structure are all extended from the basic structure and has the same structure in some parts of loop. Thus, we merge these two structures into a new two-degree-of-freedom structure. To inherit the advantages of both structures, the DCFDOB-VS, provides two parameter $H(s) \in \mathbf{U}(RH_\infty)$ and $Q(s) \in RH_\infty$, can design a two-degree-of-freedom compensator to stabilize unstable plant and achieve desired properties such as tracking, decoupling and disturbance rejection.