

A New Wave Energy Converter Extreme Dynamic Analysis Method

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transformation kernel density estimation,
long term extremes

ABSTRACT

One goal of this research is to find better ways to extrapolate and compute the tails of distributions of sea state parameters. In order to achieve this goal, this paper suggests using a Kernel Density Estimation (KDE) transformation method. The main findings are provided below. The method is applied to forecast the probability distribution tails of four ocean wave datasets measured at three National Data Buoy Center Stations. When compared to the parametric method's prediction results, the suggested method's accuracy is confirmed. Two of the previously described wave datasets are then used in conjunction with the transformation KDE approach to derive precise 50-year environmental contour lines. In order to forecast the PTO (Power-Take-Off) force values for a point absorber Wave Energy Converter (WEC) over the course of 50 years, the determined environmental contour lines together with other contour lines acquired by a parametric contour technique are then used. With a predicted 50-year design PTO force value of 3362900 N derived from one of the suggested transformation KDE contours—whose accuracy has been demonstrated in this paper—the WEC PTO system will undoubtedly be more reliable and safe.

INTRODUCTION

If you want your wave energy converter (WEC) to last the 50 years or more it's supposed to, you need to know the details of the worst possible sea conditions that can arise during that time. Statistics tabulating significant wave height (H_s) and energy period (T_e) built from data collected over a specific number of years at a particular ocean site is the most

often used sea severity information. Determining a probability distribution that adequately depicts the data is the initial step in making predictions about upcoming extreme significant wave height or energy period at a given location, say, in 50 years.

Within the body of research that is currently available, the three-parameter Weibull model (Coe et al. (2016); Haselsteiner and Thoben (2020); Haselsteiner et al. (2017); Mathisen and Bitner-Gregersen (1990); Ochi (2005); and so on) has been studied extensively and represents the distribution that is utilized the most frequently for the significant wave height (H_s). A typical pattern that can be observed is that the Weibull model with three parameters is able to provide accurate estimations in the primary portion of the distribution of the sea state parameter (H_s). It is important to note that this model is typically incapable of providing an accurate estimation of the distribution tails of the sea state parameter (H_s), which are essential for forecasting extreme values. In addition, the Weibull model with three parameters has a location parameter that has a minimum value that is not zero, and it is challenging to explain the physical significance of this smallest significant wave height in the distribution. It is necessary to select the sample space of significant wave height between zero and the infinity, where a significant wave height with a value of zero symbolizes calm water, which is an essential component of the distribution. Concerns have been raised regarding the use of the three-parameter Weibull model for the analysis of significant wave height data.

Using the data from the parametric marginal distributions of H_s and the conditional distribution of T_e , an environmental contour line for a given time period (say, 50 years) is typically generated using the IFORM (Inverse First Order Reliability Method) with

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Rosenblatt transformation. This method has been documented in various works (e.g., Winterstein et al. (1993), Wang (2020a), Rosenblatt (1952), and more). Unfortunately, the Rosenblatt-IFORM contour technique has its flaws, as pointed out in certain articles in the current literature. As an example, Haselsteiner et al. (2019) described the procedures and rules for an open-comparison-focused collaborative benchmarking exercise where researchers might create and share their own methods for contour derivation using publicly available information and data. Additionally, Haselsteiner et al. (2019) introduced the computed contour baseline findings. Haselsteiner et al. (2019) used the parametric marginal distribution of H_s and the conditional distribution of T_e according to DNV rules to demonstrate the technique of the benchmark and display baseline results. From the fitted models, the authors proceeded to develop Rosenblatt-IFORM contour outlines. They did note, however, that the acquired environmental contours are outstripped by a number of data points, and they noted that this outcome is not always anticipated given that the return period of the contour exceeds the span of the sample dataset. Additionally, Mackay and Haselsteiner (2021) noted that in standard normal space, the maximum values of the standard normal variables have marginal exceedance probability α for a Rosenblatt-IFORM contour at exceedance probability α . Nonetheless, this does not hold true for the highest possible values of the original parameter space's original random variables along the IFORM contour. In order to estimate inverse second-order (ISORM) contours, Chai and Leira (2018) expand the IFORM technique. The Rosenblatt transformation of the parametric marginal model of H_s , upon which the ISORM contour approach is still based, however, has erroneous probability distribution tails.

In an effort to circumvent the shortcomings of parametric methods, Wang (2020a) sought to derive two-dimensional environmental contours at two offshore sites using a non-parametric methodology known as ordinary Kernel Density Estimation (KDE). The choice of kernel and kernel parameters (band width parameters) substantially influences the tail, hence it is important to use extreme caution when describing fitted marginal distribution tails with kernel density models (Rosset al., 2020). As a matter of fact, the standard Kernel

Density Estimation (KDE) technique is notoriously bad at generalizing and extrapolating to the tail region of probability distributions. To create the joint probability distribution of significant wave height (H_s) and energy period (T_e), Eckert-Gallup et al. (2016) suggested utilizing bivariate kernel density estimation (KDE) with adaptive bandwidth selection. On the other hand, their methodology was only tested using a single dataset, specifically 122,467 hourly measurements of significant wave height (H_s) and energy period (T_e) recorded at NDBC (National Data Buoy Center Station) 46022, a buoy off the coast of Northern California. The authors acknowledged in their research conclusion that in their future endeavors, "this method will be applied to a diverse array of data sets to assess its overall adaptability." One approach to creating environmental contours that Eckert-Gallup et al. (2021) brought up was the constant bandwidth kernel density estimation (KDE) method. One limitation of this constant bandwidth KDE technique is that it does not provide a good description of the fitted marginal distributions' tails. A joint probability density function of significant wave height and wind speed was suggested by Haselsteiner et al. (2019) using the constant bandwidth kernel density estimation (KDE) approach. Having said that, Haselsteiner et al. (2019) only used hindcast data to validate their KDE technique; they did not use measured environmental data to substantiate their method. By employing a constant bandwidth kernel density estimation (KDE) technique, Manuel et al. (2018) were able to derive environmental contour lines and generate non-parametric marginal distributions for significant wave height (H_s) and peak period (T_e). Nonetheless, the obtained environmental contours in Manuel et al. (2018) do not exhibit smoothness over areas with sparse data when non-parametric distributions are used for data fitting.

In this study, we will propose a new transformational kernel density estimation (KDE) method to improve the accuracy of calculating the tails of the distribution of sea state parameters and to enable better extrapolation, in an effort to address the drawbacks of the previously mentioned parametric approaches and the ordinary KDE method. Since this study aims to improve the prediction of the distribution tails of sea state parameters, we herewith discuss relevant published studies with similar objectives in this section and will also compare the

results between our proposed new method and those existing methods in section 3 and section 4 of this paper. In order to improve the prediction of the distribution tails of significant wave height (H_s), Mathisen & Bitner-Gregersen (1990) applied a lognormal distribution model. Ochi (1978) also applied a lognormal distribution model to try to improve the prediction of the H_s probability distribution tails. In order to more precisely calculate the tails of the probability distribution for the significant wave height (H_s), Wang (2024) used a Generalized Gamma model.

We suggest beginning the implementation of our proposed novel method by transforming the original dataset into a new sample with a density g that can be more easily estimated using the usual KDE method. This transformation would be accomplished using the Box-Cox formulation. Once we have an estimate of g , we can "back-transform" it to find the original dataset's probability density distribution value. By applying our new transformation KDE approach to forecast the probability distribution tails of three ocean wave datasets collected at three different National Data Buoy Center Stations, we will be able to prove that it is accurate. In addition, using two of the above measured wave datasets, this new transformation KDE approach will be used to derive correct 50-year environmental contour lines. Predicting the 50-year design PTO (Power-Take-Off) force values for a point absorber Wave Energy Converter (WEC) will be done utilizing the derived environmental contour lines and other contour lines generated using a parametric contour technique. At last, the engineering importance of employing a more trustworthy environmental contour line, like the one obtained by the transformation KDE approach, will be emphasized after a thorough analysis and comparison of the computed results.

The structure of this article is as follows: Our proposed new transformation KDE method is introduced in Section 2, along with the standard IFORM and ISORM contour approaches, as well as the Three Parameter Weibull distribution. We give some examples of calculations for environmental contour lines and distributions of sea state parameters in Sections 3 and 4. Section 5 presents several instances of calculations for a point absorber wave energy converter's 50-year extreme design values prediction, and Section 6 summarizes the final

remarks.

THE THEORIES

The Three Parameter Weibull Distribution

One recommendation from the "DNVGL RP-C205" regulations is to fit the marginal distribution function of the significant wave height H_s data with a three-parameter Weibull distribution. This is for usage in the modern marine industry. As an example, consider a three-parameter Weibull distribution and its PDF is:

$$f(x; \beta, \alpha, \gamma) = \frac{\beta}{\alpha} \left(\frac{x - \gamma}{\alpha} \right)^{\beta-1} e^{-\left(\frac{x - \gamma}{\alpha} \right)^\beta} \quad (1)$$

where the values of the significant wave heights are denoted by the letter x , the scale parameter is denoted by the letter α , the shape parameter is named β , and the location parameter is denoted by the letter γ .

While Coe et al. (2016) were creating their computer software toolbox for design response analysis of wave energy converters, they also applied the technique of fitting a three parameter Weibull distribution to H_s . Haselsteiner et al. (2017) used this technique to derive environmental contours from the region with the highest density. The aforementioned studies, on the other hand, have not conducted any research into the accuracy of the three-parameter Weibull model when it comes to fitting the tails of the probability distribution of the H_s sample.

The Underlying Theories of The Classic ISORM and IFORM Contour Methods

In the context of the reliability-based design of WECs, the environmental contour line approach can be applied as a reliable instrument for evaluating long-term response extremes through the utilization of a short-term analysis. Under the premise that both the marginal probability distribution function for the significant wave height (H_s) and the conditional probability distribution of the spectral peak period (T_p) are accessible, the environmental contour line approach may be utilized for an offshore location. It is possible to obtain environmental contour lines that correspond to a particular annual exceedance probability by first transforming the marginal and conditional probability distribution models into a space that is composed of standard Gaussian variables (u_1, u_2) through the application of the Rosenblatt transformation, and then employing

the inverse first-order reliability method. The Rosenblatt transformation can be represented mathematically as follows (for an example, see Manuel et al. (2018)):

$$F_{H_s}(h) = \Phi(u_1) \quad (2)$$

$$F_{T_p|H_s}(t|h) = \Phi(u_2) \quad (3)$$

A marginal probability distribution function for the significant wave height (H_s) is denoted by $F_{H_s}(h)$ in the equations shown above. The probability distribution of the spectral peak period (T_p) conditional on H_s is denoted by $F_{T_p|H_s}(t|h)$, and the cumulative distribution function (CDF) of the standard univariate normal distribution is denoted by $\Phi(\cdot)$. The environmental contour line that corresponds to a particular annual exceedance probability of q will be circles with a radius of $r = \Phi^{-1}(1 - q/2920)$ in the standard normal space. The number 2920 represents the number of three-hour sea states that occur each year. The reliability index, which is also designated by β_F in the extant literature, is the radius r that extends from the origin to the design point. Obtaining the relationship $\Phi(\beta_F) = 1 - \alpha$ is possible if we denote $q/2920$ as α . As a result of applying the following inverse Rosenblatt transformation, the q -probability environmental contour lines can be obtained by transforming these circles in the standard normal space back to the original physical space:

$$H_s = F_{H_s}^{-1}(\Phi(u_1)) = F_{H_s}^{-1}(\Phi(r \cos \theta)) \quad (4)$$

$$T_e = F_{T_e|H_s}^{-1}(\Phi(u_2)|H_s) = F_{T_e|H_s}^{-1}(\Phi(r \sin \theta)|H_s) \quad (5)$$

Varying the angle θ in the equations presented above from 0 to 2π allows for the acquisition of the whole environmental contour line. According to Mackay and Haselsteiner (2021), the maximum values of the standard normal variables have marginal exceedance probability α in the standard normal space given an *IFORM* environmental contour at exceedance probability α . Nevertheless, this does not hold true for the highest possible values of the original random variables in the original physical space along the *IFORM* contour.

For convex failure regions, the *IFORM* approximation is conservative; for concave regions, it is not. An invariably conservative second-order approximation to the failure surface, the Inverse Second Order Reliability Method (ISORM) was proposed by Chai and Leira (2018). The ISORM

approach takes the conventional normal space, with the origin at its center, and assumes that the failure surface encloses a circle. The radius β_{s2} is defined such that the chance of an observation falling outside the circular zone equals α . Chai and Leira (2018) observed that the sum of two independent standard normal variables conforms to a Chi-squared distribution with two degrees of freedom, χ_2^2 , allowing the radius β_{s2} to be expressed as:

$$\chi_2^2(\beta_{s2}^2) = 1 - \alpha \quad (6)$$

The radius, denoted by β_{s2} , is a function of the exceedance probability according to the ISORM specification. The inverse Rosenblatt transformation is applied to the contour in standard normal space in order to obtain the ISORM environmental contour in the original physical space. This is similar to how the *IFORM* environmental contour is obtained.

The Transformation Kernel Density Estimation Method

To model the distribution and extrapolate extreme sea state parameters using parametric approaches, you need specified probability models for the environmental variables you're considering. Having said that, it's important to think about the downside of parametric modeling. A function's rigidity could be an issue if it is restricted to a parametric family. That is, it's possible that the model's specifications are overly stringent, making it impossible to get a good estimate of the actual underlying function. One non-parametric method that can be used to circumvent the excessive sea state parameter distribution rigidity in parametric models is the Kernel Density Estimation (KDE). In the KDE, we use a small data set to draw conclusions about the whole population, which is a basic data smoothing problem. Assume that at each given location $\mathcal{X}$, there exists a univariate distribution with an unknown density f , and that $(X_1, \dots, X_n)$ are independent and identically distributed samples selected from this distribution. Figuring out what this function f looks like is of interest to us. As previously stated by Eckert-Gallup and Martin (2016) and Silverman (1986):

$$\hat{f}(x; h) = \frac{1}{nh} \sum_{i=1}^n K\left(\frac{x - X_i}{h}\right), \quad (7)$$

Throughout this context, K stands for the kernel, a non-negative function that satisfies $\int K(x) dx = 1$, and

$h > 0$ denotes the bandwidth, a parameter for smoothing. Regular kernel functions include normal, Epanechnikov, biweight, triweight, triangular, uniform, and many more. Since the Epanechnikov kernel is ideal from a mean square error perspective, it is employed in this investigation. The following is the mathematical expression of the univariate Epanechnikov kernel (Wand and Jones (1995)):

$$K(x) = \begin{cases} \frac{3}{4}(1-x^2) & \text{for } |x| < 1 \\ 0 & \text{for } |x| \geq 1 \end{cases} \quad (8)$$

Up to this point, we have covered all the theoretical bases for the standard optimum kernel density estimation (KDE). Estimates in the central portions of distributions are typically qualitatively satisfactory when using the standard Kernel Density Estimation method. However, regular KDE may not provide accurate distribution tail estimations and, as a result, may not perform well when extrapolating figures. Over the past few decades, the environmental contour line method—which includes the paired values of two environmental parameters—has been extensively used and improved for extreme value analysis of the dynamic responses of offshore energy systems. Whenever an environmental contour is used in an extreme value analysis, the tails of the distribution of sea state parameters are what really matter. The sea state parameter distributions' tails—exactly what the environmental contour lines depict—are not captured by the standard Kernel Density Estimation method, which makes extrapolation difficult. In light of this, one could argue that environmental contour development, or any extreme value study for that matter, should not make use of the standard Kernel Density Estimation method.

This study presents a novel transformation Kernel Density Estimation (KDE) approach to reliably determine the distribution tails of sea state parameters. Applying the Box-Cox transformation to the original data set $(X_1, \dots, X_n)$ yields a new sample $(Y_1, \dots, Y_n)$ with a density g that is better predicted using the standard Kernel Density estimator; this is the first step in applying the new transformation KDE technique. When applied to data sets with nonnormal distributions, the Box-Cox formulation produces data sets with distributions that are very close to normal. In this context, "one-parameter Box-Cox" means:

$$data(\lambda) = \frac{data^\lambda - 1}{\lambda} \quad (9)$$

The algorithm specifies that the optimal λ -value be the one maximizing the Log-Likelihood Function.

Gaining the density g allows us to "backtransform" the g estimate to get the f estimate. The transformation kernel density estimator is the name given to the resulting estimate. Statistical distribution theory yields the usual conclusion when $Y_i = t(X_i)$ is used to represent the Box-Cox transformation process, which is that

$$f(x) = g(t(x))t'(x) \quad (10)$$

where the derivative is denoted by $t'(x)$. It is possible to acquire the transformation Kernel Density Estimator of f by substituting g with $\hat{g}(\cdot; h)$, which is the conventional Kernel Density Estimator of g based on $(Y_1, \dots, Y_n)$, which may be calculated as follows:

$$\hat{g}(\cdot; h) = \frac{1}{nh} \sum_{i=1}^n K\left(\frac{t(x) - t(X_i)}{h}\right) \quad (11)$$

After that, the Kernel Density Estimator of f is attained by employing the equation that is presented below:

$$\hat{f}_T(x; h, t) = \hat{g}(\cdot; h)t'(x) = \frac{1}{nh} \sum_{i=1}^n K\left(\frac{t(x) - t(X_i)}{h}\right)t'(x) \quad (12)$$

The following figure 1 shows a flowchart describing procedures for obtaining f by the transformation kernel density estimation method.

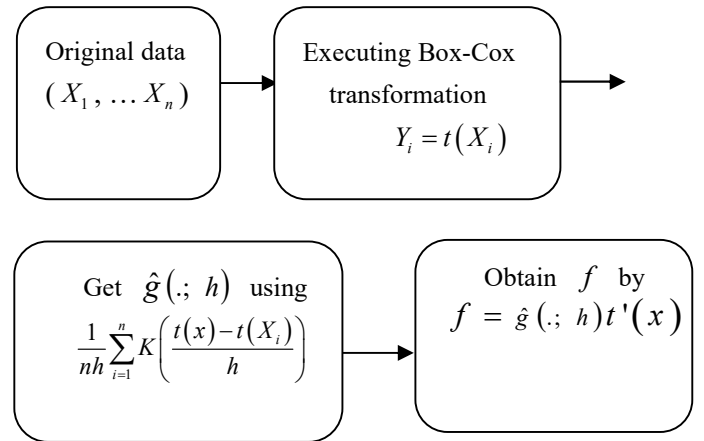


Fig. 1. A flowchart showing procedures for obtaining f by the transformation KDE method

The justification for selecting the Box-Cox transformation for the Kernel Density Estimation

method is as follows: The optimal selection of the transformation $t(\bullet)$ is significantly influenced by the configuration of the density f . If f is a skewed unimodal density, Wand et al. (1991) contended that $t(\bullet)$ should be a convex function on the support of f , as this type of $t(\bullet)$ will diminish the skewness of f in a certain manner. The probability density distributions of sea state characteristics (e.g., significant wave height, spectral peak period) are typically unimodal and significantly skewed. Furthermore, the Box-Cox transformation is a convex function, making it highly appropriate for application in the transformation Kernel Density Estimation method utilized in this study.

The Equations of Motion of a Point-Absorber Wave Energy Converter

According to Wang (2018, 2019, 2020b): The following are the equations of motion for a point-absorber wave energy converter (WEC) in the time domain:

$$\mathbf{M}_{RB} \ddot{\mathbf{x}}(t) = -\mathbf{A}(\infty) \ddot{\mathbf{x}}(t) - \int_0^t \bar{\mathbf{K}}(t-\tau) \dot{\mathbf{x}}(\tau) d\tau + \mathbf{P}_{wave}(t) + \mathbf{P}_{PTO}(t) + \mathbf{P}_{visc}(t) - \mathbf{P}_{hs} \quad (13)$$

in which $\mathbf{M}_{RB}$ represents the WEC matrix for rigid body inertia. The positions of the WEC are in $\mathbf{x}(t)$. For WEC, $\mathbf{A}(\infty)$ stands for the added mass matrix at infinite frequencies. $\bar{\mathbf{K}}(t)$ represent the functions that make up the kernel. The wave excitation load is represented by $\mathbf{P}_{wave}(t)$, the viscous load by $\mathbf{P}_{visc}(t)$, the power-take-off (PTO) load by $\mathbf{P}_{PTO}(t)$, and the hydrostatic load by $-\mathbf{P}_{hs}$.

The PTO loads $\mathbf{P}_{PTO}(t)$ in equation (12) are associated with the power-take-off (PTO) through the subsequent equation:

$$\mathbf{P}_{PTO}(t) = -K_{PTO} \mathbf{x}_{rel} - C_{PTO} \dot{\mathbf{x}}_{rel} \quad (14)$$

In this equation, the variable K_{PTO} represents the stiffness of the power-take-off (PTO) system, while the variable C_{PTO} represents the damping of the PTO system. The relative motion and velocity between the WEC spar and the float are denoted by the variables $\mathbf{x}_{rel}$ and $\dot{\mathbf{x}}_{rel}$.

The Most Probable Maximum Value of The

Dynamic Responses in The Exposure Period

Transforming each time series of the simulated wave energy converter's dynamic responses into the frequency domain is necessary for finding the most likely maximum value of the dynamic responses during the exposure period. In order to carry out this transformation, the autocorrelation function is initially computed using a particular response time series and the following equation (Edwards et al., 2019):

$$R(\tau) = \int_{-T/2}^{T/2} \zeta(t) \zeta(t+\tau) dt \quad (15)$$

in which $R(\tau)$ is the autocorrelation function, which is computed on the time lag τ using the response time series data $\zeta(t)$ and an exposure period T . It is possible to determine the spectral density function $S(\omega)$ by applying the following formula:

$$S(\omega) = \int_{-\infty}^{\infty} R(\tau) e^{-i\omega\tau} d\tau \quad (16)$$

Utilizing the derived spectral density function $S(\omega)$, the most likely maximum value for each response time series throughout the exposure duration T_E can be computed as follows (Edwards et al. (2019)):

$$\zeta_{T_E} = \sqrt{2m_0 \ln(T_E m_1 / 2\pi m_0) (1 - 0.5(1 - m_2^2 / m_0 m_4))} \quad (17)$$

in which m_n represents the n th moment of the response spectral density function $S(\omega)$.

CALCULATION EXAMPLES OF THE SEA STATE PARAMETER PROBABILITY DISTRIBUTIONS

The following illustration of calculations is derived from wave data recorded at the National Data Buoy Center (NDBC) Station 51004, which is managed by the United States National Oceanic and Atmospheric Administration. In the Pacific Ocean, 205 nautical miles southeast of Hilo, Hawaii, USA, lies this data buoy.

The subsequent calculation examples will employ 189,116 hourly observations of significant wave heights (H_s) and energy periods (T_e) collected from 1996 to 2021 at NDBC 51004 (https://www.ndbc.noaa.gov/station_page.php?station=51004).

The contemporary maritime industry

follows the DNV standards "DNVGL RP-C205" which state that the significant wave height (H_s) data should be fitted to the marginal distribution function using the three parameter Weibull distribution. The following equation can be used to calculate the probability density function (PDF) of the Weibull distribution with three parameters:

$$f_{H_s}(x_1) = \frac{\beta}{\alpha} \left(\frac{x_1 - \gamma}{\alpha} \right)^{\beta-1} e^{-\left(\frac{x_1 - \gamma}{\alpha} \right)^\beta} \quad (18)$$

in which the letter α represents the scale parameter, the letter β represents the shape parameter, and the letter γ represents the location parameter. Incorporating a three-parameter model, Coe et al. (2016) utilized the Weibull distribution to H_s in the process of constructing their WDRT toolkit for design response analysis of wave energy converters. Additionally, Haselsteiner et al. (2017) utilized the Weibull distribution to H_s in order to derive environmental contours from the region with the highest density.

All told, we have 189,116 H_s -measurements collected at NDBC station 51004 every hour from January 1996 all the way through December 2021. The three-parameter Weibull distribution that was utilized to fit the H_s data at NDBC station 51004 is shown by the red curve in Figure 2a. Its three-parameter Weibull distribution includes the following values for their parameters: $\alpha = 1.9421$, $\beta = 3.0477$ and $\gamma = 0.6084$.

Clearly, the red curve does a poor job of fitting the observed data at either the middle or the right end of the distribution. Figure 2b's zoomed-in tail graphic makes it even more apparent that the right tail region fits the measured data extremely poorly with the three-parameter Weibull distribution. In this area, the fitted Weibull distribution grossly understates the genuine density values from the observed data set for each probability density value (corresponding to a certain support value). No extreme value analysis or environmental contour derivation should be done using this fitted three-parameter Weibull distribution because it does not extrapolate properly at all.

The green curve in Figure 2a illustrates the distribution derived from the Kernel Density Estimation (KDE) approach applied to the recorded 189,116 hours of data at NDBC 51004. The green curve aligns effectively with the measured data in the

center region of the distribution. Moreover, the zoomed-in plot of the tail region in Figure 2b demonstrates that the green curve aligns closely with the measured data in the right tail area. The transformation Kernel Density Estimation (KDE) technique produces a highly smooth estimate in the right tail of the distribution. The calculation results have effectively validated the efficacy of our suggested strategy for estimating the distributions of extreme sea state characteristics.

Since this study aims to improve the prediction of the distribution tails of sea state parameters, we herewith compare the results between our proposed new method and those existing methods in the relevant published studies. Clearly, the blue curve (from the generalized Gamma model) also does a poor job of fitting the observed data at either the middle or the right end of the distribution. Figure 2b's zoomed-in tail graphic makes it even more apparent that the right tail region fits the measured data extremely poorly with the generalized Gamma distribution and the lognormal distribution respectively.

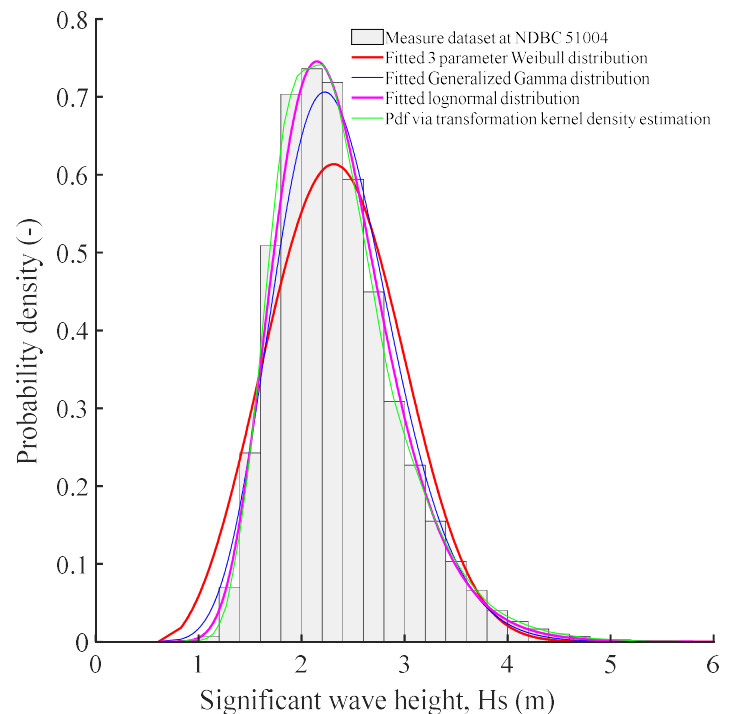


Fig. 2a. Model fit between the measured data at NDBC 51004 and the four considered models respectively

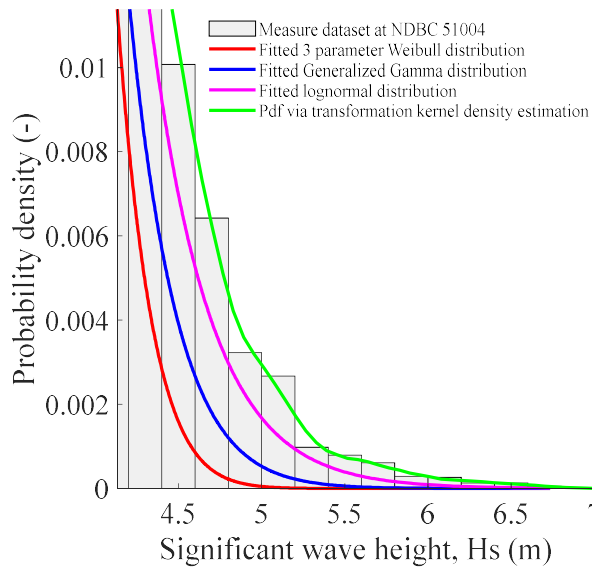


Fig. 2b. Tail region model fit between the measured data at NDBC 51004 and the four considered models respectively

Using just one site in the previous calculation example makes it impossible to draw any broad conclusions about the results or the validity of the procedure. Applying our new approach in multiple places with varying $H_s - T_e$ shapes is necessary to demonstrate its validity. Hence, we have completed an additional calculation example using wave data obtained at the National Data Buoy Center (NDBC) Station 44011 (https://www.ndbc.noaa.gov/station_page.php?station=44011), which is administered by the U.S. National Oceanic and Atmospheric Administration. This data buoy can be found on Georges Bank, approximately 170 nautical miles east of Hyannis, Massachusetts, USA. This calculation example will make use of 177819 hourly data of significant wave heights (H_s) and energy periods (T_e) recorded at NDBC 44011 from 1996 to 2023, as illustrated in Figure 3. We can see that the data that was used in the study reveals waves that are less than half a meter in height.

The three-parameter Weibull distribution was utilized to fit the H_s data at NDBC station 44011, as shown by the red curve in Figure 4a. We can see that the red curve does not adequately fit the measured data, and this holds true both in the middle and at the right end of the distribution. A zoom-in graphic at the tail region in Figure 4b shows that the three-parameter Weibull distribution fits the observed data very poorly at the right tail region. Here we see the connection between the two much more clearly.

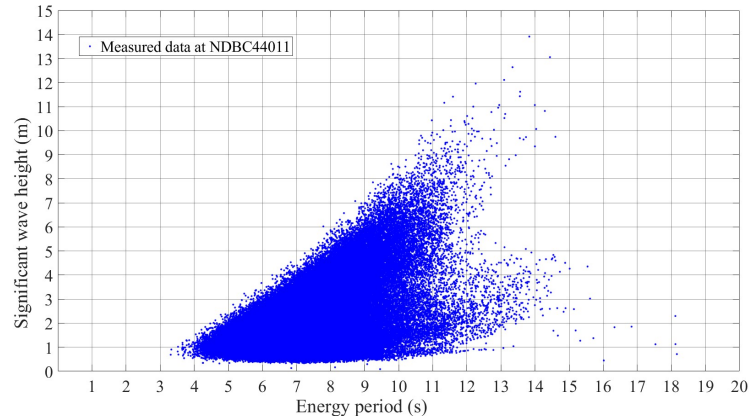


Fig. 3. Measured data at NDBC 44011

Here, the fitted Weibull distribution tends to grossly understate the probability density values (corresponding to certain support values) by a significant margin when contrasted with the actual density values (derived from the observed data set) for those values. Neither extreme value analysis nor the creation of environmental contours should make use of this fitted three-parameter Weibull distribution, which should come as no surprise.

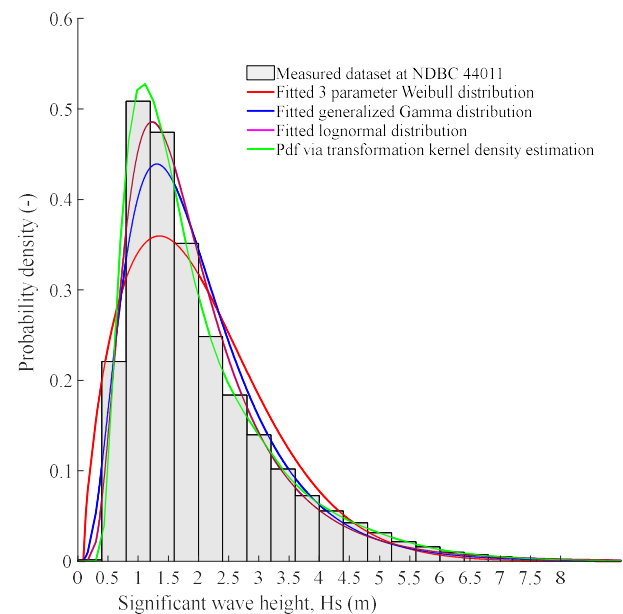


Fig. 4a. Model fit between the measured data at NDBC 44011 and the three considered models respectively

The distribution that was calculated using the transformation Kernel Density Estimation (KDE) method is shown by the green curve in Figure 4b, using the 177819 hours of H_s data observed at NDBC 44011. A good match to the data measured in the center of the distribution is indeed provided by this green curve, as can be seen. Also, as can be seen

from the zoomed-in depiction of the tail region in Figure 4b, this green curve perfectly fits the data that was seen in that area. The fact that the plot is zoomed in clearly shows this. A much smoother estimate is produced by using the transformation Kernel Density Estimation (KDE) method on the right tail of the distribution compared to the ordinary Kernel Density Estimation (KDE) method.

Figure 4b shows the red curve, which represents the distribution determined from the conventional Kernel Density Estimation (KDE) method, according to the H_s data collected at NDBC 44011 for 177819 hours. However, as can be seen from this zoom-in plot in Figure 4b at the tail region, this red curve also fits the data quite poorly at that location. This standard KDE approach gives a much under-smoothed estimate in the right tail and introduces a lot of zigzags and wiggly effects. This means that the red curve generated by the standard KDE method can't extrapolate either, therefore it's not a good choice for making environmental contour lines.

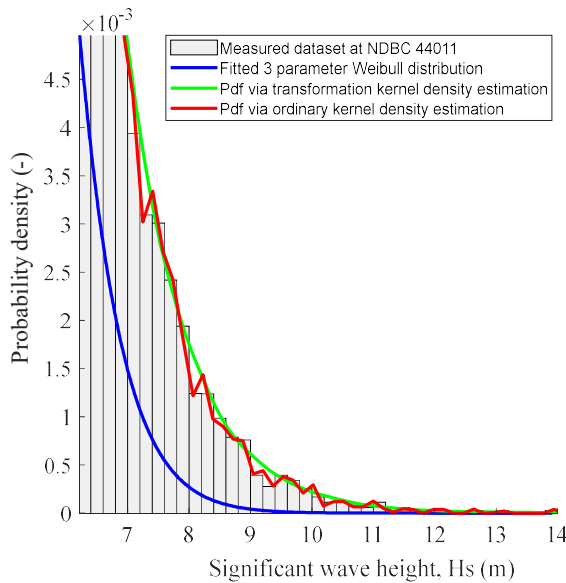


Fig. 4b. Model fit at the tail region between the measured data at NDBC 44011 and the three considered models respectively

Since this study aims to improve the prediction of the distribution tails of sea state parameters, we herewith compare the results between our proposed new method and those existing methods in the relevant published studies. Clearly, the blue curve (from the generalized Gamma model) in Figure 4c also does a poor job of fitting the observed data at the right end of the distribution. Figure 4c's zoomed-in tail graphic makes it very apparent that the right tail

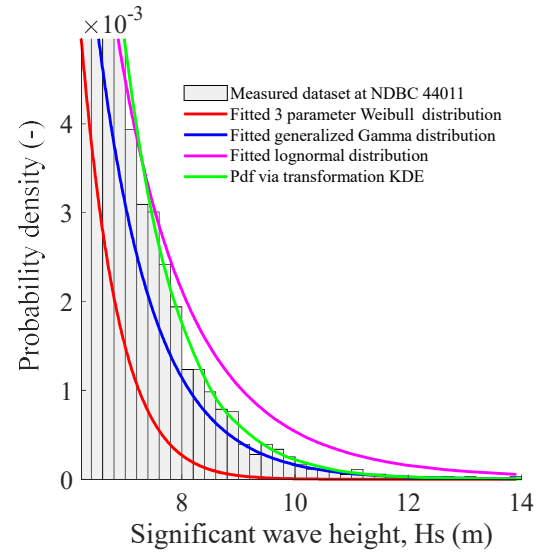


Fig. 4c. Model fit at the tail region between the measured data at NDBC 44011 and the four considered models respectively

region fits the measured data extremely poorly with the generalized Gamma distribution and the lognormal distribution respectively.

The results and the correctness of our suggested transformation KDE method have also been generalized by doing a third calculation example using the wave data obtained at NDBC Station 46026 (https://www.ndbc.noaa.gov/station_page.php?station=46026) with a different shape of $H_s - T_e$. Eighteen nautical miles west of California's San Francisco lies this data buoy. In this example calculation, we will use 237882 hourly observations of significant wave heights (H_s) and energy periods (T_e) recorded at NDBC 46026 from 1996 to 2023. These measurements are shown in Figure 5. Visual inspection of this measured data set reveals waves smaller than half a meter.

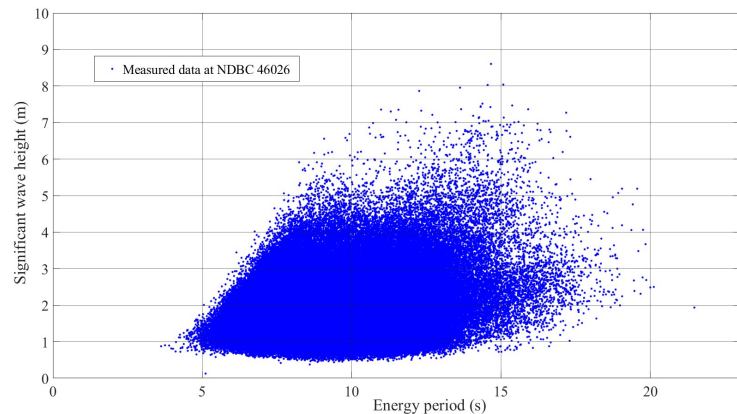


Fig. 5. Measured data at NDBC 46026

To match the H_s data at NDBC station 46026, a three-parameter Weibull distribution was generated and used; Figure 6a shows the red curve representing this distribution. Both the middle and the right-hand end of the distribution show that this red curve does not adequately fit the measured data. Our analysis clearly shows that the red curve does not offer a good fit. Zooming in on the tail region of Figure 6b reveals that the data observed there is not well-fit by the three-parameter Weibull distribution. There is an additional proof of the link between the two in this specific case. The fitted Weibull distribution tends to significantly under-estimate each probability density value (corresponding to a specific support value) at this location compared to the true density value from the measured data set. The fact that this fitted three-parameter Weibull distribution is unfit for environmental contour construction and extreme value analysis should not be surprising.

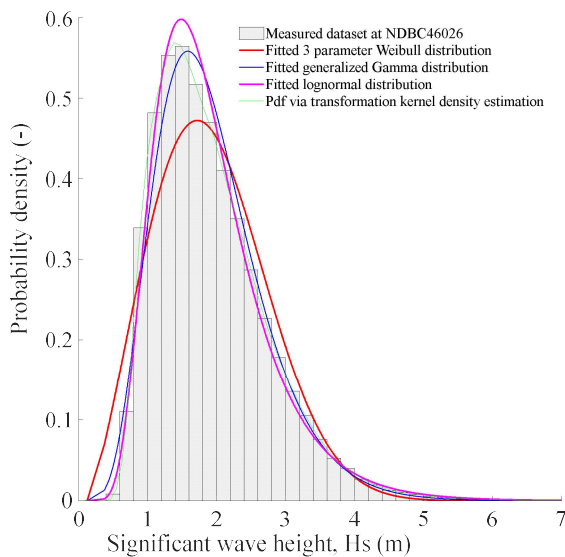


Fig. 6a. Model fit between the measured data at NDBC 46026 and the three considered models respectively

The distribution achieved by using the transformation Kernel Density Estimation (KDE) method is shown by the green curve in Figure 6a, which was derived from the 237882 hours of H_s data collected at NDBC 46026. Determining the distribution was a success with this manner. The fact that we can see this green curve proves that it well fits the data measured at the center of the distribution. Additionally, the zoom-in plot in the tail region of Figure 6b shows that this green curve fit the right tail region data quite well. This is demonstrated by the

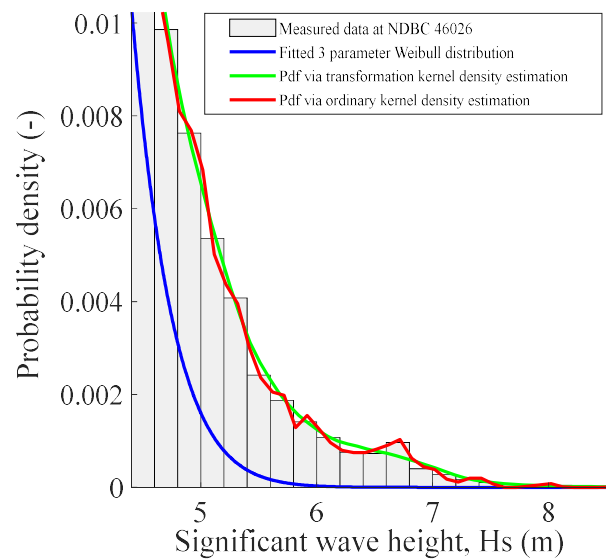


Fig. 6b. Model fit at the tail region between the measured data at NDBC 46026 and the three considered models respectively

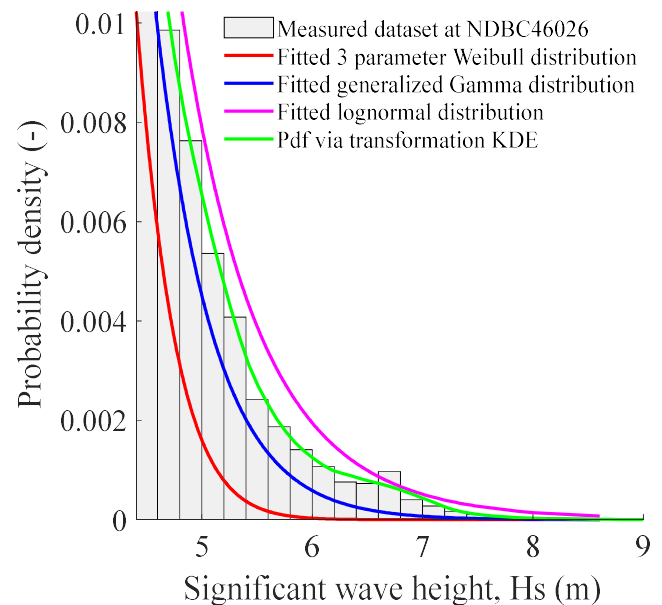


Fig. 6c. Model fit at the tail region between the measured data at NDBC 46026 and the four considered models respectively

fact that the plot is viewed from a closer perspective. Application of the transformation Kernel Density Estimation (KDE) methodology to the right tail region of the distribution yields an estimate that is substantially smoother than the ordinary Kernel Density Estimation (KDE) method. We have proven the correctness of our revolutionary transformation Kernel Density Estimation (KDE) method for estimating the probability distributions of extreme sea state variables using these calculation results, which

have properly shown the procedure's accuracy. We used the computation results to achieve this.

Figure 6b's red curve shows the previously described probability distribution from the ordinary Kernel Density Estimation (KDE) method. This distribution was developed using the H_s data collected at NDBC 46026 over the span of 237882 hours. After taking a closer look, we can observe that the observed data fits the red curve extremely poorly at the right tail region, as shown by the zoom-in graphic in Figure 6b. This is the scenario that the figure represents. Because it is just an ordinary KDE approach, it introduces a lot of zigzags and wiggly effects and produces an estimate that is severely under-smoothed in the right tail. The red curve in Figure 6b produced by the standard KDE method cannot extrapolate correctly either, hence it shouldn't be used to build environmental contours either.

Since this study aims to improve the prediction of the distribution tails of sea state parameters, we herewith compare the results between our proposed new method and those existing methods in the relevant published studies. Clearly, the blue curve (from the generalized Gamma model) in Figure 6c also does a poor job of fitting the observed data at the right end of the distribution. Figure 6c's zoomed-in tail graphic makes it very apparent that the right tail region fits the measured data extremely poorly with the generalized Gamma distribution and the lognormal distribution respectively.

The foregoing studies in this paper analyzes three NDBC stations. NDBC Station 51004 in Hawaii is located in the middle region of the Pacific Ocean. NDBC 44011 in Massachusetts is located on the coast of the Atlantic Ocean. NDBC station 46026 in California is located on the coast of the Pacific Ocean. The selection criteria in this study are clear, i.e., to select data buoys at different world oceans with different climatic conditions. The data record lengths from these three NDBC stations vary slightly (25 years vs. 27 years), the potential impact of this variation on extrapolation accuracy is negligible as demonstrated through analyzing the foregoing computational results in the three calculation examples. These three calculation examples can be deemed as sensitivity studies to show that our proposed new transformation Kernel Density Estimation (KDE) method is very robust in terms of the applicability to different world regions or different climatic conditions.

The results and the correctness of our proposed transformation KDE method have also been generalized by doing a fourth calculation example using the wave data obtained at NDBC Station 46105

(https://www.ndbc.noaa.gov/station_page.php?station=46105) in the Arctic Ocean with a different shape of $H_s - T_e$. Ten nautical miles northwest of Alaska's East Amatuli Light lies this data buoy. In this example calculation, we will use 5108 hourly observations of significant wave heights (H_s) and energy periods (T_e) recorded at NDBC 46026 from 2008 to 2012. These measurements are shown in Figure 7. Visual inspection of this measured data set reveals waves smaller than half a meter.

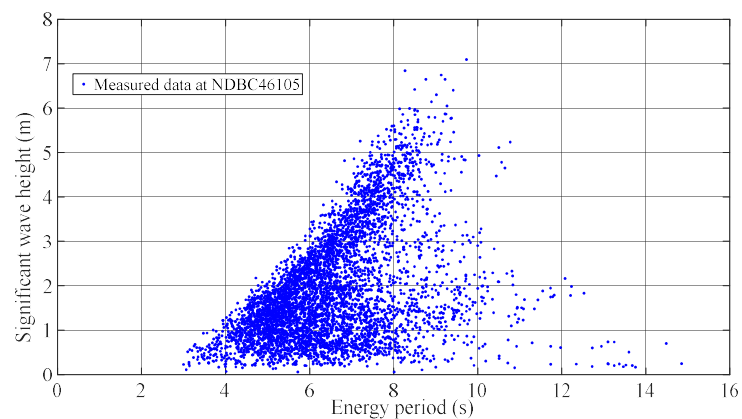


Fig. 7. Measured data at NDBC 46026

Our calculation results regarding the fittings to this dataset with various models are shown in Figure 8a and 8b. Carefully analyzing these results again prove the correctness of our revolutionary transformation Kernel Density Estimation (KDE) method for estimating the probability distributions of extreme sea state variables. Since this study aims to improve the prediction of the distribution tails of sea state parameters, we herewith compare the results between our proposed new method and those existing methods in the relevant published studies. Clearly, the blue curve (from the generalized Gamma model) in Figure 8b also does a poor job of fitting the observed data at the right end of the distribution. The rede curve (from the 3 parameter Weibull distribution model) in Figure 8b also does a poor job of fitting the observed data at the right end of the distribution. Figure 8b's zoomed-in tail graphic makes it very apparent that the right tail region fits the measured data extremely poorly with the generalized Gamma distribution, 3 parameter Weibull distribution and the lognormal distribution respectively.

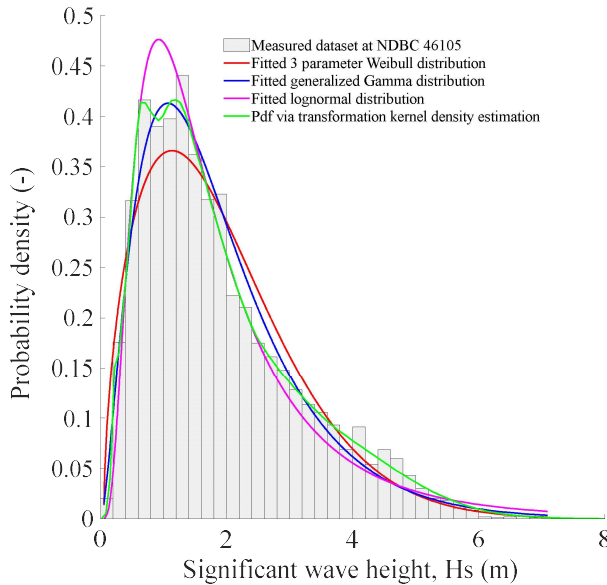


Fig. 8a. Model fit between the measured data at NDBC 46105 and the three considered models respectively

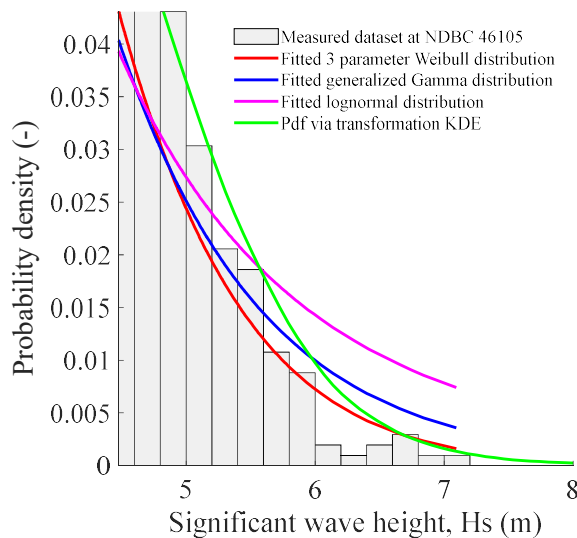


Fig. 8b. Model fit at the tail region between the measured data at NDBC 46105 and the four considered models respectively

CALCULATION EXAMPLES OF THE ENVIRONMENTAL CONTOUR LINES

We provide here the results of our calculations for the 50-year extreme sea state contour lines using the data collected at Station 51004 of the National Data Buoy Center (NDBC) as mentioned before.

The 189116 data points that were measured at NDBC 51004 are represented by the blue "dots" represented in Fig. 9. Using the standard IFORM (Inverse First Order Reliability Method) in conjunction with the Rosenblatt transformation, the

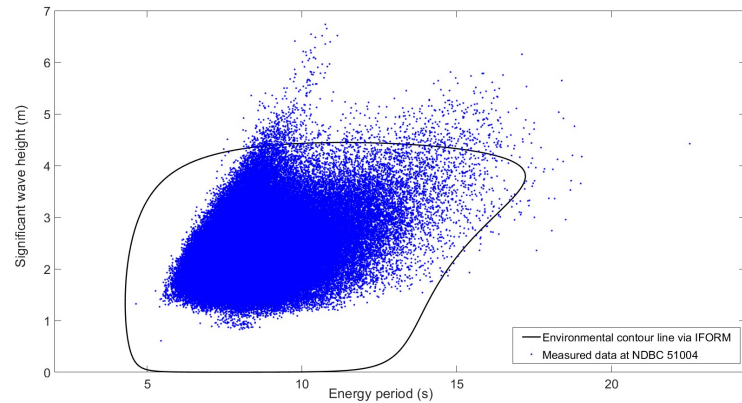


Fig. 9. 50-year extreme sea state contours created by the traditional IFORM method for NDBC 51004.

solid black line in Figure 9 depicts the environment contour line anticipated over the next fifty years. In order to carry out the Rosenblatt transformation, a Weibull distribution with three parameters has been fitted to the marginal distribution function of the significant wave height (H_s) data that has been measured. This was followed by the application of a conditional distribution to the energy period T_e that was conditioned on H_s . It is possible to accomplish this by first ordering the values of H_s , then sorting the matching T_e values into bins, and finally fitting a separate lognormal distribution to the T_e values that are included within each distinct bin.

$$g_{T_e|H_s}(x_2|x_1) = \frac{1}{x_2} \frac{1}{\sigma(x_1)\sqrt{2\pi}} \exp\left(-\frac{(\ln x_2 - \mu(x_1))^2}{2\sigma(x_1)^2}\right) \quad (19)$$

After that, the traditional lognormal parameters, which are the mean $\mu(x_1)$ and standard deviation $\sigma(x_1)$ of the log, were estimated. As a consequence of this, distributions of $\mu(x_1)$ and $\sigma(x_1)$ were produced as a function of the significant wave height. The models that describe the behavior of the sets of lognormal parameters $\mu(x_1)$ and $\sigma(x_1)$ as a function of significant wave height were used to fit the data. The data fitting models that are utilized for the $\mu(x_1)$ and $\sigma(x_1)$ parameters in the DNV regulations are as follows:

$$\mu(H_s) = E[\ln t | H_s = h_s] = a_1 + a_2 h_s^{a_3} \quad (20)$$

$$\sigma(H_s) = Std[\ln t | H_s = h_s] = b_1 + b_2 e^{h_s^{b_3}} \quad (21)$$

Following the fitting of the marginal distribution function for significant wave height (H_s) and the conditional distribution for energy period (T_e), the conventional IFORM utilizing the Rosenblatt transformation was employed to derive the solid black line in Figure 9, which illustrates the 50-year extreme sea state contour.

How the traditional IFORM method performs relative to real-world observed data are explained as follows: With a record duration of approximately 26 years, it is anticipated that the extreme environmental contour line for a 50-year return period will encompass the majority of the observed data. Nevertheless, the black contour line in Figure 9, generated by the Rosenblatt transformation, has overlooked a substantial amount of the collected data. This adverse outcome parallels the findings of Haselsteiner et al. (2019), who similarly observed that their derived Rosenblatt-IFORM environmental contour lines were surpassed by numerous measured data points. It is important to note that Haselsteiner et al. (2019) indicated that their findings may not be anticipated, as the return period of their environmental contour line exceeds the temporal span of the measured sample collection. The black environmental contour in Figure 9 clearly does not conform to the profile of the observed data. The black contour effectively accommodates smaller values of H_s but fails to provide coverage in regions of the input space where H_s values are quite high, suggesting that the resulting Rosenblatt-IFORM contour lacks predictive accuracy. The Rosenblatt-IFORM environmental contour has underestimated the significant wave heights in our case, which will undoubtedly result in the construction of a hazardous offshore structure.

Figure 10 illustrates the environmental contour line that was created using the standard IFORM contour approach, which was used to develop the 50-year extreme sea state contour line using the recorded ocean wave data at NDBC 44011. The record is approximately 28 years long, thus it's safe to assume that most of the data will fall within the extreme contour line for a fifty-year return period. The red contour in Figure 8 was constructed using the Rosenblatt IFORM approach, although a large percentage of the collected data was ignored. Figure

8's red contour, in fact, deviates from the measured data's profile. While this red contour works better with lower H_s values, it won't cover the part of the input space where the H_s values are really high. All signs point to the constructed Rosenblatt-IFORM shape being quite unpredictable. With this information in hand, the Rosenblatt-IFORM environmental contour has predicted dangerously low values for the significant wave heights in our scenario, and the offshore construction will be unsafely designed.

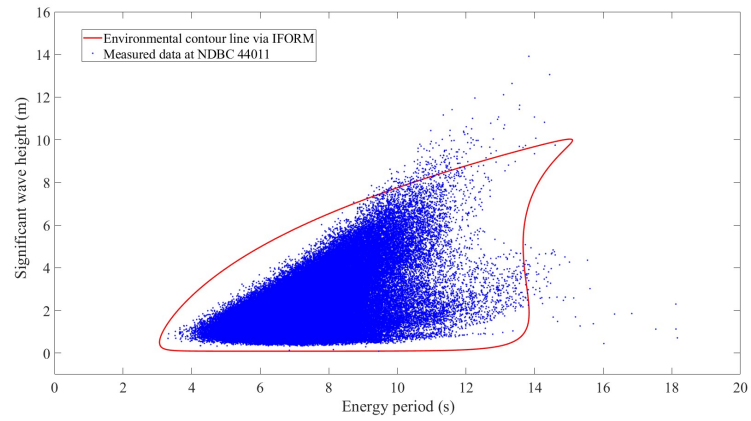


Fig. 10. The 50-year extreme sea state contour created by the traditional IFORM method for NDBC 44011.

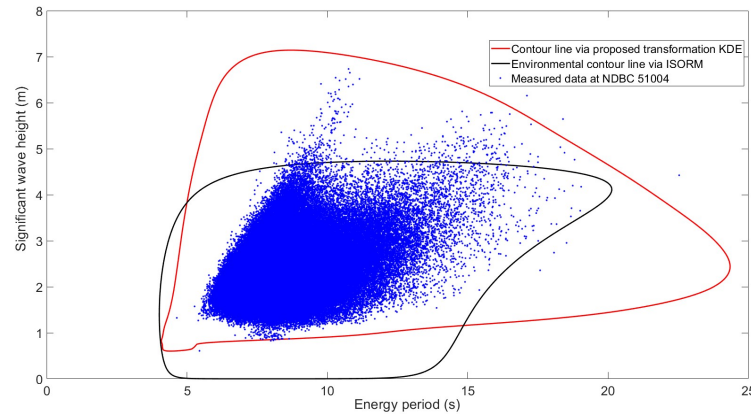


Fig. 11. 50-year extreme sea state contours created by the ISORM method and the proposed new approach presented in this paper for NDBC 51004

How the traditional ISORM method performs relative to real-world observed data are explained as follows: Figure 11 shows the measured 189,116 data points at NDBC 51004. The blue "dots" are used again. The black curve in Figure 11 shows the 50-year environmental contour predicted using the more cautious ISORM (Inverse First Order

Reliability Method), whose theoretical foundations were explained in Section 2.2. The solid black line depicts this forecasted result. Fitting the ISORM contour to the observed data set has been marginally improved, as can be seen. There is less data missing from the ISORM environmental contour line as compared to the IFORM contour in Figure 9. However, coverage in the input space area with very high H_s values is still not possible with the ISORM contour.

How the traditional ISORM method performs relative to real-world observed data are explained as follows: Figure 12 illustrates the blue "dots," which represent the 177,819 data points recorded at NDBC 44011. Figure 12 illustrates the predicted environmental contour over the forthcoming fifty years, shown by the solid red line. This forecast result was generated utilizing the more conservative ISORM (Inverse First Order Reliability Method). We have noted a modest enhancement in the alignment between the ISORM environmental contour line and the measured data set. Unlike the IFORM contour depicted in Figure 10, the ISORM environmental contour line in Figure 12 has overlooked a lesser segment of the recorded data. Nonetheless, the ISORM contour fails to provide coverage in the area of the input space where the H_s values are significantly elevated.

After failing to obtain satisfactory results using the IFORM and ISORM approaches, that rely on fitted parametric sea state parameter distributions,

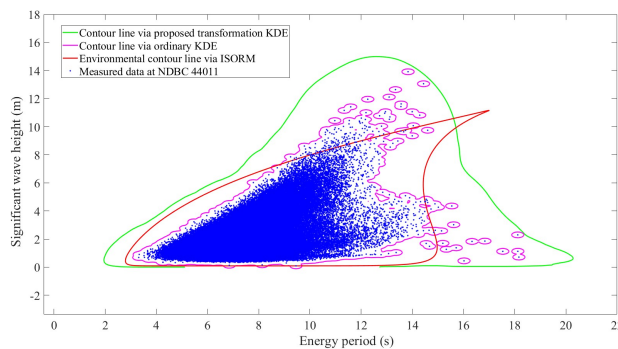


Fig. 12. 50-year extreme sea state contours created by the ISORM method, the ordinary KDE method and the proposed new approach presented in this paper for NDBC 44011

we have attempted to utilize our newly developed Kernel Density Estimation (KDE) method to derive environmental contour lines. In Figure 11 and Figure 12, we have presented the findings of our calculations.

Figure 11 illustrates the solid red line, which

denotes the 50-year environmental contour predicted through the application of the suggested transformation Kernel Density Estimation method, utilizing the observed data from NDBC 51004. Figures 2a and 2b demonstrate that the transformation Kernel Density Estimation method effectively predicts the probability distribution tail of H_s (or T_e) and exhibits strong extrapolation capabilities. Since the solid red contour in Figure 11 is directly derived from these two probability distributions, it is unequivocally more accurate and reliable than the black ISORM contour line, which is based on parametric marginal and conditional models with suboptimal distribution tails. The Rosenblatt-ISORM environmental contour line has underestimated the significant wave heights in our scenario, which will undoubtedly result in the design of an offshore structure that is not safe to use. Clearly, the black contour line will be subordinate to the red contour line in Figure 11 when used for predicting the extreme dynamic responses of wave energy converters. In Section 5, the previously described two environmental contour approaches will be utilized to forecast the 50-year most probable maximum PTO (Power-Take-Off) force values of a wave energy converter, and the results will be carefully studied and compared.

Figure 12 also presents a comparison of the environmental contour line calculations between the conventional KDE and the suggested KDE methods utilizing measured data from NDBC 44011. Figure 12 illustrates a solid green line representing the forecasted environmental contour line for the forthcoming fifty years. This contour line was produced by utilizing the proposed transformation Kernel Density Estimation method on the data collected at NDBC 44011. The green contour line is unequivocally more precise and dependable than the red ISORM contour line, which is directly produced from parametric marginal and conditional models exhibiting suboptimal distribution tails. This is because we have previously shown in Figures 4a 4b, and 4c that the transformation Kernel Density Estimation technique can properly estimate the probability distribution tails of H_s (or T_e) and is proficient in extrapolation. The Rosenblatt-ISORM environmental contour has predicted markedly low values for significant wave heights in our context. This information will certainly lead to the design of an unsafe offshore structure. In the context of anticipating the extreme dynamic responses of wave

energy converters, it is evident that the red contour line will exhibit inferior quality compared to the green contour line depicted in Figure 12. In Figure 12, the magenta curves illustrate the environmental contour derived from the conventional Kernel Density Estimation approach. As demonstrated in Figures 4a 4b, and 4c, the conventional Kernel Density Estimation method fails to accurately predict the probability distribution tails of H_s (or T_e) and lacks effective extrapolation capabilities; thus, the magenta contour is unequivocally inferior to the green contour line derived from the transformation KDE method.

CALCULATION EXAMPLES OF THE EXTREME DYNAMIC RESPONSE VALUES OF A WAVE ENERGY CONVERTER

The chosen wave energy converter in this study

For this research, we used a particular simulation model of a two-body point absorber wave energy converter (Figure 13). A 20-meter-diameter float and a 6-meter-diameter spar make up this wave energy converter. The float weighs 727.01 metric tons and has a thickness of 5 meters. The spar/plate weighs 878.3 metric tons and is 38 meters tall. There is an 82-meter draft on this wave energy converter. Three anchors and three nylon mooring lines secure this wave energy converter to the ocean floor. Each set of mooring lines forms an obtuse angle in the plan view, measuring 120 degrees. The use of nylon, as opposed to the more conventional wire-rope, resulted in reduced total loads and higher compliance (stretching).

Simulations have been run using a piece of software called WEC-Sim, which is available for free download at <http://wec-sim.github.io/WEC-Sim/>. To be more specific, the extreme PTO (Power-Take-Off) force values of this wave energy converter have been determined for a selection of sea states that have been defined using a Pierson–Moskowitz spectrum. These values have been estimated for a period of fifty years. For the purpose of defining the aforementioned sea states along each of the two contours shown in Figure 11, five energy periods have been selected: 5.022 seconds, 6.48 seconds, 8.694 seconds, 12.182 seconds, and 20.119 seconds. The decision was made to select these energy periods such that they would encompass the peak values of the significant wave heights depicted in Figure 14. The selected sea states

are depicted in Figure 14 as "squares" made of solid black color, and a summary of these sea states may be found in Table 1. While the calculations were being done, irregular waves were generated based on the sea state parameters that were discussed earlier in order to calculate the wave excitation loads $\mathbf{P}_{wave}(t)$ in equation (13).

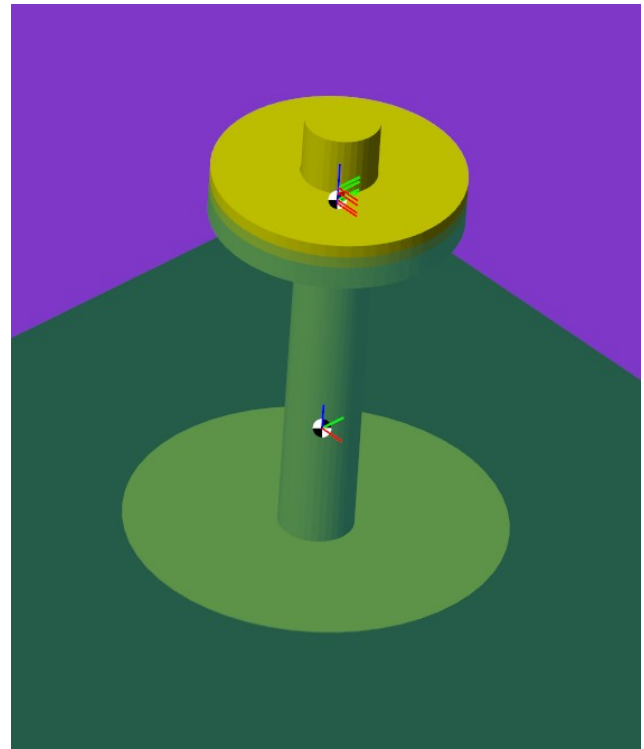


Fig. 13. The WEC-Sim simulation model of the chosen wave energy converter

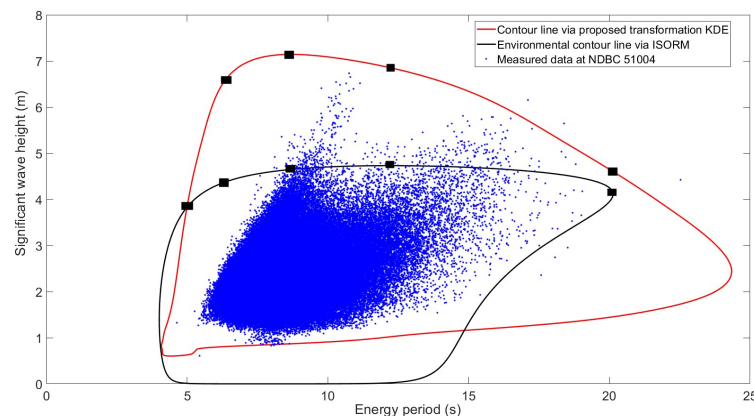


Fig. 14. The chosen sea states along the contours created by the traditional method and the proposed approach for NDBC 51004

Table 1 Values of significant wave heights from each

contour line method as shown in Figure 14

Energy period T_e, s	Spectral peak period T_p, s	Proposed Transformation KDE H_s, m	Rosenblatt ISORM H_s, m
5.022 s	5.87574 s	3.846 m	3.846 m
6.48 s	7.5816 s	6.687 m	4.401 m
8.694 s	10.172 s	7.143 m	4.634 m
12.182 s	14.25294 s	6.859 m	4.730 m
20.119 s	23.539 s	4.609 m	4.093 m

Using the two contour approaches that were stated earlier, WEC-Sim simulations have been carried out in order to determine the PTO heaving forces of the wave energy converter that was selected in the indicated sea states. During the process of performing WEC-Sim simulations, the significant wave height (H_s) and spectral peak period (T_p) pairings for each environmental contour approach have been utilized. Spectra of the Pierson–Moskowitz type have been derived from each $H_s - T_p$ pair in this instance. By utilizing a Simulink model and an input wave energy spectrum, the WEC-Sim software is able to simulate the dynamic responses of a wave energy converter. This is done in accordance with the ideas presented in Section 2.4. The wave energy converter dynamic responses, which came in the form of three-hour time series, were the outputs of the WEC-Sim simulations. In this particular instance, the PTO heaving forces were these responses. Because the PTO heave force is an essential component that must be taken into account during the design process of the wave energy converter and, more particularly, the PTO device, it was selected here. A three-hour extreme value was computed based on each WEC-Sim simulation time series, and the theories presented in Section 2.5 were used to analyze the data. The extreme PTO force values that were derived from the two contour approaches have been collected, and they are summarized in Table 2. This operation was repeated for each of the five energy periods that were selected earlier, and it was done for each of the two contours in Figure 14. Upon meticulous examination of the calculation findings in Table 2, one can ascertain that the anticipated extreme PTO force value derived from the proposed environmental contour technique is 3,362,900 N. This represents the 50-year design force value, as the environmental contour line methodology estimates long-term extremes through short-term analysis (see to Haver

and Winterstein (2009)). Table 2 indicates that the estimated 50-year PTO heaving force design value derived from the ISORM environmental contour approach is 2,207,200 N, significantly lower than the value predicted using the suggested environmental contour method. The elevated long-term design force value anticipated from the proposed environmental contour method will result in the development of a safer and more dependable wave energy converter PTO device. A greater force (3362900N) constitutes a more hazardous circumstance. If the wave energy converter PTO gadget can withstand this greater force, it can certainly endure a lesser force (2207200 N). However, if the wave energy converter PTO device is engineered to withstand a lesser force (2207200 N), it would certainly sustain damage if subjected to more hazardous conditions (equivalent to a greater force) during its future operations in a real-world context. Therefore, the suggested environmental contour approach is advised for forecasting the long-term extreme dynamic response values essential for the safe and effective design of a wave energy converter and its PTO mechanism.

Table 2. Most probable maximum PTO force values based on the two contour methods

Spectral peak period T_p, s	Most probable maximum PTO force values based on the proposed transformation KDE contour method (N)	Most probable maximum PTO force values based on the Rosenblatt ISORM contour method (N)
5.87574 s	1.8108e+06 N	1.8108e+06 N
7.5816 s	3.3629e+06 N	2.2072e+06 N
10.172 s	3.2701e+06 N	2.1239e+06 N
14.25294 s	2.1323e+06 N	1.6238e+06 N
23.539 s	8.8575e+05 N	7.0175e+05 N

During the course of our research, we came up with an additional calculation example in order to provide comparisons of the anticipated PTO force values for the 50-year design between the conventional KDE and the new KDE approaches. There are five different sea states that have been established along each of the two contours that are shown in Figure 12 (the green contour and the

magenta contour): 8.51 seconds, 10.75 seconds, 12.32 seconds, 14.66 seconds, and 15.67 seconds. These five energy periods have been chosen in order to generate these variations. The decision was made to pick these energy periods in such a way that they encompass the peak values of the significant wave heights shown in Figure 12.

Table 3 Values of selected H_s and the corresponding T_e from each contour line method as shown in Figure 12

Energy period T_e , s	Spectral peak period T_p , s	Proposed Transformation KDE H_s , m	Ordinary KDE H_s , m	Rosenblatt ISORM H_s , m
8.51 s	9.9567 s	9.23 m	7.15 m	7.15 m
10.75 s	12.5775 s	13.74 m	10.40 m	8.49 m
12.32 s	14.4144 s	15.06 m	12.11 m	9.15 m
14.66 s	17.1522 s	13.43 m	12.95 m	10.18 m
15.67 s	18.3339 s	10.61 m	4.51 m	10.61 m

Using WEC-Sim simulations, we were able to determine the PTO heaving forces of the wave energy converter that was chosen for the selected sea conditions. These results are presented in Table 3. For the purpose of carrying out WEC-Sim simulations, the significant wave height (H_s) and spectral peak period (T_p) pairs can be found in Table 3. These pairings have been applied for each environmental contour method. By utilizing a Simulink model and an input Pierson–Moskowitz wave energy spectrum, the WEC-Sim program is able to mimic the dynamic responses of the wave energy converter. This is accomplished by the utilization of the software. Dynamic responses of the wave energy converter were produced by WEC-Sim models in the form of three-hour time series. These outputs were created by the models. Specifically, the PTO heaving forces were the ones accountable for these responses in these particular instances. An extreme value forecast was made using the ideas that were discussed in Section 2.5. This prediction was based on each three-hour-long time series that was generated by the WEC-Sim simulation. A repetition of this method was carried out for each of the five selected energy periods that were discussed earlier along each of the two KDE contours shown in Figure 12. All of the extreme PTO force values that were determined from the two KDE contour methods were obtained as a consequence of this, and a summary of these values

can be found in Table 7. The predicted extreme PTO force value according to the suggested transformation KDE contour method is 3310700 N, as can be shown by perusing Table 4 for the computation results. Given that the environmental contour line technique can be used to calculate long-term extremes from short-term analyses (e.g., Haver and Winterstein (2009)), this effectively represents the value of the design force that will be generated fifty years from now. Table 4 also shows that the standard KDE contour approach gives an estimated design value of 2653200 N for the 50-year PTO heave force. In comparison to the value anticipated using the suggested transformation KDE contour approach, this number is substantially lower. An unsafe wave energy converter PTO device will undoubtedly be designed because to the low long-term design force value estimated utilizing the conventional KDE contour technique. Therefore, to guarantee a safe and effective design of a wave energy converter and its PTO mechanism, it is recommended to use the environmental contour approach to make predictions of the long-term extreme dynamic response values.

Table 4. Most probable maximum PTO force values based on the two KDE contour methods

Spectral peak period T_p , s	Most probable maximum PTO force values based on the proposed transformation KDE contour method (N)	Most probable maximum PTO force values based on the ordinary KDE contour method (N)
9.9567 s	3.3107e+06 N	2.6532e+06 N
12.5775 s	2.5461e+06 N	1.9327e+06 N
14.4144 s	2.0723e+06 N	1.6782e+06 N
17.1522 s	1.8409e+06 N	1.7923e+06 N
18.3339 s	1.6732 e+05 N	0.8711e+05 N

CONCLUDING REMARKS

In this research, we present a new KDE (Kernel Density Estimation) method to improve the accuracy of calculating the tails of the distribution of sea state parameters and to perform well in extrapolation, with the goal of overcoming the limitations of parametric approaches. The following

are the key conclusions from applying our suggested transformation KDE approach to forecast the probability distribution tails of four ocean wave datasets taken at four National Data Buoy Center Stations: By comparing the predicted outcomes with those from the parametric method, we can confirm that the suggested method is accurate. Two of the previously described wave datasets have been used in conjunction with this transformation KDE approach to derive precise 50-year environmental contour lines. Comparing the projected environmental contour lines from the IFORM and ISORM contour methods with the derived ones is an ongoing effort. The environmental contour lines that are derived and other contour lines that are obtained using a parametric contour approach are used to forecast the PTO force values for a point absorber Wave Energy Converter (WEC) over a 50-year design period. We can see that the WEC PTO system will be safer and more reliable if it is developed using the anticipated 50-year design PTO force value of 3362900 N, which is based on one of the proposed transformation KDE contours and whose accuracy has been thoroughly validated in this article.

Unexpected or contentious findings from this study include the fact that our suggested transformation KDE method outperforms current parametric methods for predicting extreme sea state parameter values. In other words, the use of the transformation KDE approach allows for the prediction of environmental contour lines of extreme sea states and more precise tails of the probability distributions of sea state parameters. As a result, WEC designs will be safer and more dependable since the expected force values for the long term will be based on the suggested KDE contour modification. We conclude that the transformation KDE contour approach should be used to forecast the extreme dynamic response values in the long run, which is crucial for the secure and effective design of wave energy converters and related PTO devices.

PLANNED FUTURE WORK

In our future studies we will explore broader applications of our new transformation KDE method to other fields such as offshore wind farm planning, oil rig safety, and climate risk modeling. Our paper states that IFORM and ISORM approaches underestimate extreme sea states, but it is extremely difficult to invent a statistical index to quantify these underestimations. In our future studies we will try to

invent a statistical index to quantify these underestimations.

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NOMENCLATURE

H_s	significant wave height (m)
T_e	energy period (s)
α	scale parameter of the 3-parameter Weibull distribution
β	shape parameter of the 3-parameter Weibull distribution
γ	location parameter of the 3-parameter Weibull distribution
$F_{H_s}(h)$	marginal probability distribution of the significant wave height

$F_{T_p|H_s}(t|h)$ conditional distribution of the spectral
peak period on H_s

$\Phi(\cdot)$ the standard normal distribution function.

$\mathbf{M}_{RB}$ wave energy converter rigid body inertia
matrix (kg).

$\mathbf{x}(t)$ wave energy converter positions (m).

$\mathbf{A}(\infty)$ wave energy converter infinite-frequency
added mass matrix (kg).

$\bar{\mathbf{K}}(t)$ kernel functions.

$\mathbf{P}_{wave}(t)$ wave excitation loads (N)

$\mathbf{P}_{visc}(t)$ viscous loads (N)

$\mathbf{P}_{ext}(t)$ external loads. (N)

$-\mathbf{P}_{hs}$ hydrostatic loads (N)

K_{PTO} stiffness of the power-take-off system (N/m)

C_{PTO} damping of the power-take-off system (Ns/m))

$\mathbf{x}_{rel}$ the relative motion (m)

$\dot{\mathbf{x}}_{rel}$ relative velocity (m/s)

g probability density function of a new sample
($Y_1, \dots, Y_n$)

$t(\square)$ the box-cox transformation function