

Abbe Positioning Error Modeling and Compensation of CNC Machine Tools Based on Instantaneous Rotation Center

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Keywords : Computerized Numerical Control machine tool, guide rail system, Abbe positioning error, instantaneous rotation center, precise compensation.

ABSTRACT

In order to reduce the uncertainty of the imprecise Abbe offset calculation to the positioning error compensation of Computerized Numerical Control machine tools, the concept of instantaneous rotation center was proposed in this paper. The precise calculation model of Abbe offset and the compensation model of Abbe positioning error of the Computerized Numerical Control worktable was built based on instantaneous rotation center. An experimental system for determining the instantaneous rotation center was established by using a laser interferometer and speed sensors. The experiments for determining the instantaneous rotation center of the Computerized Numerical Control worktable with different X direction moving speed were carried out. The experimental results show that the instantaneous rotation center positions of the worktable change under different positions and speeds. The positioning error compensation experiments with the new compensation model based on the instantaneous rotation center and the traditional model were carried out, respectively. Under the speeds of 180 mm/min, 240 mm/min and 300 mm/min, the position errors of the Computerized Numerical Control worktable compensated by the instantaneous rotation center method are $0.81\ \mu\text{m}$, $1.12\ \mu\text{m}$ and $1.18\ \mu\text{m}$ less than errors compensated by using the traditional method respectively, the positioning errors compensation method based on the instantaneous rotation center can achieve a higher Abbe distance calculation accuracy and better Abbe positioning error compensation of the Computerized Numerical Control worktable.

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INTRODUCTION

The CNC (Computerized Numerical Control) machine worktable in each direction is supported by two guide rails. The positioning coordinate of the machine table is measured by the ball screw with encoder or grating. The axes of the workpiece are not coincident with the measuring system of the CNC machine tool due to machine structure limitation. So, the installation of the guide system disobeys the Abbe principle. Thus, according to the movement error of yaw, pitch and roll angle of the guide system, the Abbe positioning error caused by Abbe error of the machine table occurs, which directly affected the positioning and manufacturing accuracy of the machine tool. In order to compensate for the error of the machine tool and other mechanical equipment, many scholars have conducted numerous studies. Ferreira (1986) proposed an analytical model for the geometric error prediction of machine tools by using rigid body kinematics. The model constructed the relationship between the error vector of one machining point in the spatial machine space of machine tools and the dimensional and form errors of machine tool movement components. Okafor (2000) developed a mathematical model for the volumetric errors in the multi-axis machine tool by using rigid body kinematics, small angle approximation of the errors, and homogeneous coordinate method. Koenig, R. et al. (2007) analyzed the Abbe error definition, and realized real-time determination of Abbe error by using angle sensor measurement, etc. The correction method of Abbe error was put forward based on the measurement data analysis. Hongwei Liu et al. (2018) established the Abbe error model according to the Abbe principle and the detail analysis of the Abbe error, and compensated the machine geometry error by using software error compensation method. Ming-Fei CHEN et al. (2007) studied the traditional positioning error compensation method of two-dimensional worktable which realized the machine tool positioning error prediction and positioning accuracy correction. Gaoyan Zhong et al.

(2015) proposed a positional geometric error modeling method for five-axis CNC machine tools. The positioning error model of five-axis CNC machine tools was established and the positioning error was compensated by using recursive software-based error compensation method, which improved the positioning accuracy of the machine tool. Fan (2010) designed a real-time Abbe error detector based on the displacement and angle measurement principle of laser, which could measure the Abbe error in real time while the machine tool moves along the single axis. Raksiri (2002) developed a kinematic and geometric mathematical error model of a machine tool by using homogenous transformation matrices based on the analysis of the angle and linearity error, which could realize all errors compensation within the working space of the machine tool.

In the traditional Abbe error compensation methods of the worktable mentioned above, the Abbe distance (Abbe offset) is calculated by the X, Y, and Z coordinate difference between the workpiece processing point and the measuring system of the machine tool. And the working table is assumed to rotate in the center. However, due to the influence of different geometric errors between the two machine guide rail systems, the actual rotation center of the worktable is time-varying during the movement of the worktable. So the Abbe offset size calculated by using the traditional method is not consistent with the actual size, which influenced the Abbe error compensation effect of the worktable. In order to compensate the positioning errors caused by the Abbe error of the machine guide systems precisely, the concept of instantaneous rotation center of the machine table is presented, which can describe the time-varying rotation center of the machine table precisely. The definition and determination methods of instantaneous rotation center are studied. The precise calculating model of Abbe offset is built based on instantaneous rotation center, and the precise compensation model of Abbe positioning error of the machine guide systems is built. The theoretical research results of instantaneous rotation center are verified by experiments, and two compensation effects of Abbe positioning error are compared by using the traditional and new Abbe offset calculation methods.

THE ABBE POSITIONING ERROR CHARACTERISTIC ANALYSIS OF THE CNC MACHINE WORKTABLE AND DETERMINATION METHOD OF INSTANTANEOUS ROTATION CENTER

Abbe error characteristics analysis of the CNC machine worktable

The CNC machine worktable is commonly supported by two guide rails, which is shown in Fig. 1. The different machining and assembly errors of the machine tool guide rails resulted in the different geometrical errors including the linear error, the straightness error and small angle error (yaw angle error, pitch angle error and roll angle error (2013)). These errors make the CNC machine worktable rotate around the X, Y and Z axes. As shown in Fig. 1, the point is not located on the measuring line of the coordinate measuring system (X and Y axes grating measuring system or ball screw), which disobeys the Abbe principle and causes the Abbe offset in the X, Y, and Z directions. Under the combined action of Abbe offset and angular motion error, Abbe positioning errors in the X, Y, and Z directions of the CNC machine worktable are generated.

As shown in Fig. 2, the Abbe positioning error caused by the yaw angle error is called the X-direction yaw Abbe positioning error of the guide systems. In the traditional Abbe error compensation method(Xiang , 2016; Zhang ,2017), the coordinate system XOY is established by using the grating reading head O as the origin of the coordinate system, and the yaw angle Abbe offset is the line length of BO between the processing point B and the grating system in the Y direction. Abbe positioning error caused by yaw angular motion is the line length of BB' . However, according to the actual geometrical error influence of the guide systems, when the worktable moves along in the X direction in the XOY plane, the XOY plane actually rotates around the point O_p caused by the yaw angle error. The point O_p is defined as the instantaneous rotation center. So the coordinate system $X'O_pY'$ is established by using the instantaneous rotation center O_p as the origin of the coordinate system. The actual processing point B is shifted to the point B_p under the influence of the yaw angle motion error α . So the actual yaw Abbe offset is the line length of BO_p , and the actual yaw Abbe positioning error is the line length of BB_p . Therefore, the traditional Abbe offset calculation method does not match the actual movement characteristic of the CNC machine worktable. The Abbe offset size can only be precisely calculated by using the method based on instantaneous rotation center, which has been confirmed and applied on Abbe error compensation of high-precision nanometer measuring machines (2016).

Determination method of instantaneous rotation center

From an analysis of the Abbe error, the instantaneous rotation center is defined as the point O_p on the worktable where the rotation speed is

zero. According to the theory of rigid body plane motion, when the velocity direction and size of any two points are given, their vertical lines are made through the two points, respectively, and the intersection of the vertical lines is the velocity instantaneous center (1988). For example, in order to determine the coordinates of the yaw instantaneous rotation center O_p , an XOY coordinate system is established on the worktable. The origin of coordinate system is set at the grating ruler reading head O , the coordinates of the two points M and N are set as $M(x_M, y_M)$, $N(x_N, y_N)$ respectively shown in Fig. 3.

The movement speed of the machine tool worktable in X direction is set as V_0 , and the speed V_{Mx} , V_{My} and V_{Nx} , V_{Ny} of point M , N are measured. So the coordinate $O_p(x, y)$ of the yaw instantaneous rotation center can be calculated by the following equation (1).

$$\begin{cases} y - y_M = -\frac{V_{Mx} - V_0}{V_{My}}(x - x_M) \\ y - y_N = -\frac{V_{Nx} - V_0}{V_{Ny}}(x - x_N) \end{cases} \quad (1)$$

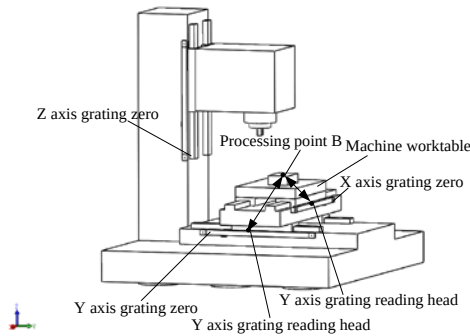


Fig. 1. CNC machine tool structure diagram.

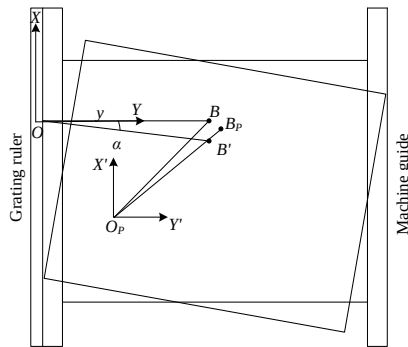


Fig. 2. Schematic diagram of Abbe offset calculation

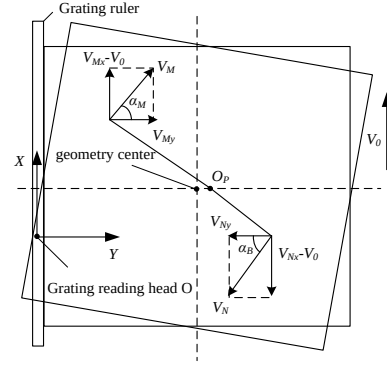


Fig. 3. Determination method of instantaneous movement center

ABBE POSITIONING ERROR COMPENSATION MODEL OF THE MACHINE WORKTABLE BASED ON INSTANTANEOUS ROTATION CENTER

It is known from Fig. 1 that the processed point B on the worktable is not located on the same axes line of the measurement system (XY axis grating measurement system), which does not obey the Abbe principle. So The Abbe positioning error δ of the CNC worktable is the product of rotation angle ε and Abbe offset L , where $\delta = L \tan \varepsilon$. When the machine table moving along the X direction, the instantaneous rotation center caused by the yaw angle error ε_z is defined as the yaw instantaneous rotation center and denoted as O_p . The simplified three-dimensional model of CNC worktable was built and shown in Fig. 4. The grating coordinate system $OXYZ$ is built, whose origin is set to grating reading head O . The position coordinate of yaw instantaneous center O_p is calculated and obtained by using the Eq. (1). The point B of the workpiece is chosen arbitrarily. When the worktable moves along the X direction, point B rotates around the instantaneous center O_p and reach the position of point B_p under the effect of the yaw angle error ε_z . So the Abbe positioning error of point B under the effect of the yaw angle error ε_z is the coordinate difference between point B_p and point B in the coordinate system $O_p X' Y' Z'$. If the three-dimensional coordinates of point B and B_p are set to $B(X_{B1}, Y_{B1}, Z_{B1})$ and $B_p(X_{BP}, Y_{BP}, Z_{BP})$ respectively, the three-dimensional coordinates of B_p can be calculated by the homogeneous coordinate transformation equation(2). So the actual Abbe positioning error δ_{PB} of point B under the action of yaw angle motion error is

calculated by the following equation (3).

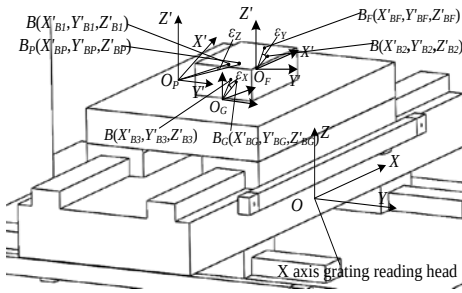


Fig. 4. The yaw instantaneous center of machine guide system

$$\begin{bmatrix} X'_{BP} \\ Y'_{BP} \\ Z'_{BP} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & -\varepsilon_Z & 0 & 0 \\ \varepsilon_Z & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X'_{B1} \\ Y'_{B1} \\ Z'_{B1} \\ 1 \end{bmatrix} \quad (2)$$

$$\begin{aligned} \delta_{PB} &= [X'_{BP} \ Y'_{BP} \ Z'_{BP}]^T - [X'_{B1} \ Y'_{B1} \ Z'_{B1}]^T \\ &= [-\varepsilon_Z \cdot Y'_{B1} \ \varepsilon_Z \cdot X'_{B1} \ 0]^T \end{aligned} \quad (3)$$

When the CNC worktable moves along X axes direction, there exist the Abbe positioning errors caused by the yaw angle error, pitch angle error and roll angle error at the same time. Like the calculation process of the yaw Abbe positioning error of the CNC worktable, the instantaneous rotation center caused by the pitch angle error ε_Y is defined as the pitch instantaneous center and denoted as O_F , the instantaneous rotation center caused by the roll angle error ε_X is defined as the roll instantaneous center and denoted as O_G . So the Abbe positioning error δ_{FB} of point B under the effect of the pitch angle error ε_Y is the coordinate difference between point $B_F(X'_{BF}, Y'_{BF}, Z'_{BF})$ and point $B(X'_{B2}, Y'_{B2}, Z'_{B2})$ in the coordinate system $O_F X'Y'Z'$, the Abbe positioning error δ_{GB} of point B under the effect of the roll angle error ε_X is the coordinate difference between point $B_G(X'_{BG}, Y'_{BG}, Z'_{BG})$ and point $B(X'_{B3}, Y'_{B3}, Z'_{B3})$ in the coordinate system $O_G X'Y'Z'$.

It can be deduced that the comprehensive Abbe positioning error of point B when the CNC machine tool worktable moves along X direction is calculated by the following equation (4).

$$\begin{aligned} \delta_B &= \delta_{PB} + \delta_{FB} + \delta_{GB} \\ &= [-\varepsilon_Z \cdot Y_{B1} + \varepsilon_Y \cdot Z_{B2} \ \varepsilon_Z \cdot X_{B1} - \varepsilon_X \cdot Z_{B3} \ -\varepsilon_Y \cdot X_{B2} + \varepsilon \cdot Y_{B3}]^T \end{aligned} \quad (4)$$

Similarly, the Abbe positioning error of point

when the CNC worktable moves in the Y direction can be calculated and analyzed according to the similar analysis process.

EXPERIMENTAL VERIFICATION OF THE ABBE ERROR COMPENSATION MODEL BASED ON INSTANTANEOUS ROTATION CENTER

Positioning and angle error measurement experiment of the XY machine worktable guide system

When the machine worktable moves at different speeds, the positioning, yaw angle, and pitch angle errors at the same position of the machine guide systems are different. In order to verify the above conclusion, the numerical control lathe is taken as the experimental object, and the machine tool motion speed is respectively set as 180 mm/min, 240 mm/min and 300 mm/min. The positioning error, yaw angle, and pitch angle errors of the X axes machine guide systems under different moving speeds were measured by using Renishaw XL-80 (Barman, 2010; Linares, 2014). The measuring range and sampling interval were selected as 200 mm and 5 mm respectively, and the experiments were repeated for three times. The error data were measured at 5 mm sampling interval. So, every error has 41 error data in the 200 mm measuring range. The average value of positioning error, yaw angle and pitch angle error were taken as the final measurement result, which was shown in Fig. 5. In the traditional Abbe error compensation method, the yaw and pitch Abbe offsets of measuring point B were 210 mm and 100 mm.



(a) Measurement device

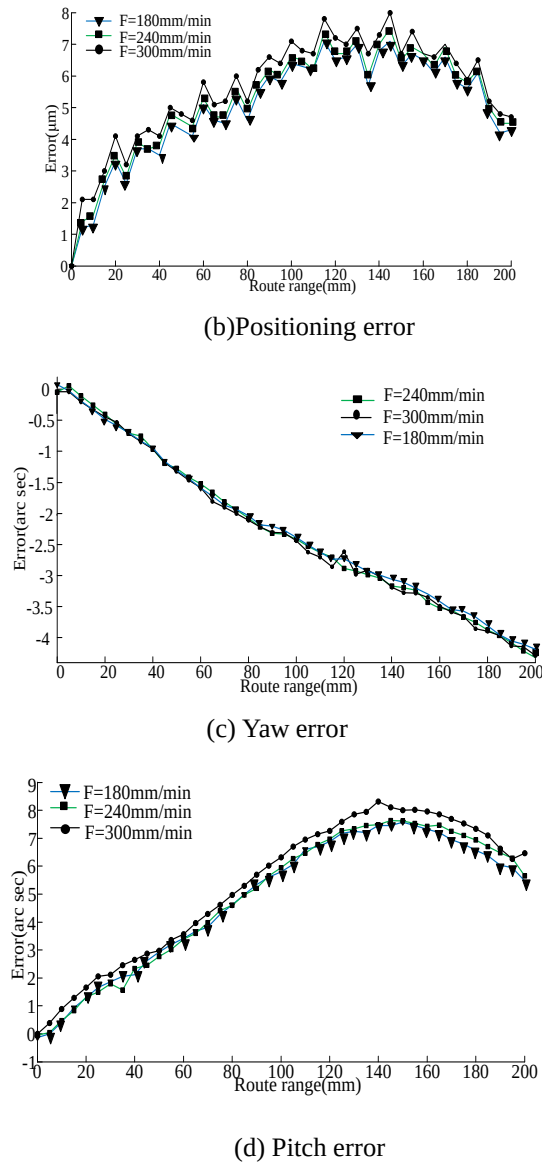


Fig. 5. Positioning, yaw angle, pitch angle error curve of X-direction machine guide system and measurement device

Yaw instantaneous rotation center determination experiments of the XY Machine worktable moving along the X direction

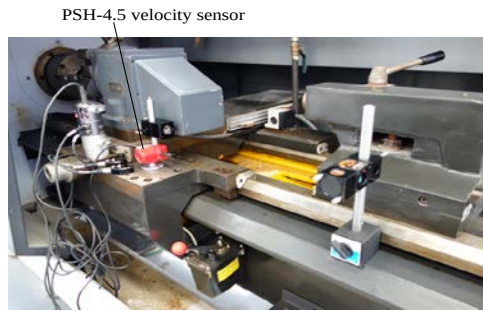
In order to determine the position of the yaw instantaneous rotation center, it is necessary to measure the speed at the M and N points. According to the analysis in the previous section, a coordinate system needs to be established to measure the coordinates of M and N points. In order to ensure that the measured machine tool guide system errors and the measured Abbe offset are under the same experimental conditions, a laser interferometer and a PSH-4.5 horizontal velocity sensor were used to build measurement device shown in Fig. 8 (a) (b). The laser interferometer is used to measure the

positioning error, yaw angle, pitch angle and other angle errors in the process of machine tool table movement. The speed sensor measures the speed data in two-dimensional direction. According to the measured two-dimensional velocity data, the coordinates of the yaw instantaneous rotation center are calculated, and the yaw Abbe offset of is calculated. The yaw Abbe positioning error is calculated by substituting the results into equation (2). The positioning error of the machine tool table is compensated by the calculated yaw Abbe positioning error, and then the compensation result is compared with the original positioning error measured by the laser interferometer. The compensation effect can be observed by the comparison result.

The reflector mirror of laser interferometer and speed sensor were fixed at M and N on the machine tool. The sampling time of PSH-4.5 horizontal velocity sensor was set to 20 s, the sampling frequency of PSH-4.5 horizontal velocity sensor was set to 1 KHz, the moving speed of the machine tool was set to 180 mm/min, 240 mm/min, 300 mm/min respectively through the upper computer software, and the speed values V_{Mx} , V_{My} , V_{Nx} , V_{Ny} at positions M and N were collected. The measurement results of V_{Mx} , V_{My} , V_{Nx} , V_{Ny} at the moving speed of 240 mm/min of the machine tool are shown in Fig. 8 (c) (d) (e) (f). The position coordinates of the yaw instantaneous rotation center at different speeds can be determined by using equation (1), which was shown in Fig. 9 (a). The yaw instantaneous rotation center coordinate positions at the speed of 240 mm/min was shown in Fig. 9 (b). The machine table geometric center in the figure represents the center of the original guide system worktable, which is the default Abbe distance calculation reference point in the traditional Abbe error compensation method. We can see that the yaw instantaneous rotation center coordinate at the same position under different moving speeds of the CNC worktable are different. The yaw instantaneous rotation center coordinate at the different position under the same moving speeds of the CNC worktable are also different.



(a) Measurement device



(b) Measurement device

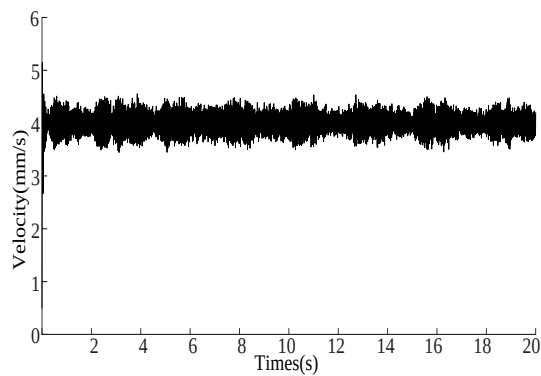
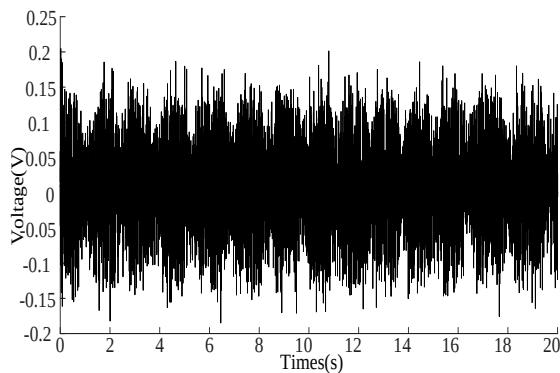
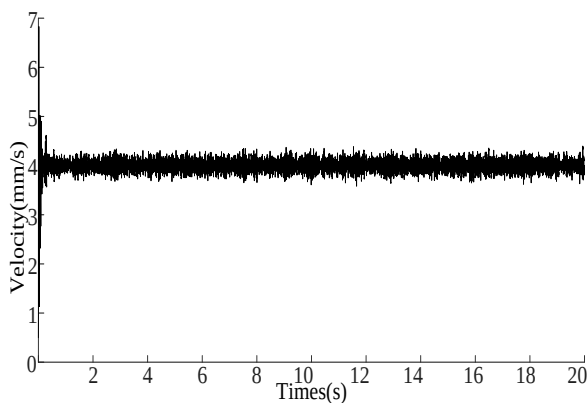
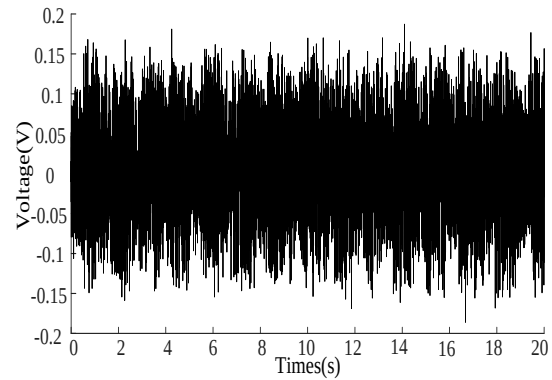
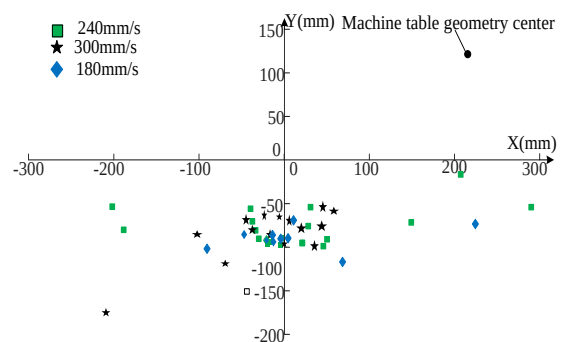
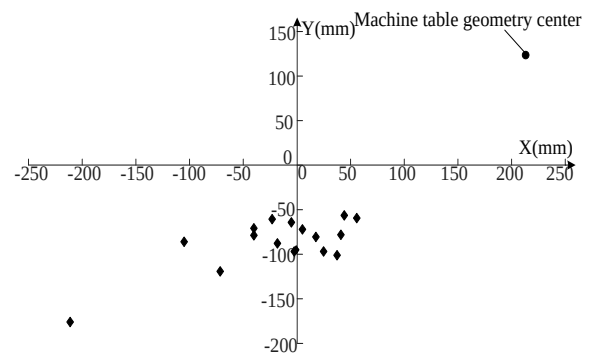
(c) V_{Mx} under 240 mm/min speed(d) V_{My} under 240 mm/min speed(e) V_{Nx} under 240 mm/min speed(f) V_{Ny} under 240 mm/min speed

Fig. 6 Measuring device and measurement result of yaw instantaneous rotation center



(a) The instantaneous rotation centers position under different moving speeds



(b) The instantaneous rotation center position under 240 mm/min moving speed

Fig. 7. The coordinate position of the yaw instantaneous rotation center

Abbe Positioning Error Compensation Results Comparison

In order to compare the error compensation effect between traditional error compensation method and the error compensation method based on the instantaneous rotation center on the Abbe positioning error of the CNC worktable, the corresponding

calculations were performed. According to the analysis in the previous section, the yaw offset of the machine tool at different moving speeds can be determined. The actual Abbe positioning error δ_{PB} of point B caused by the yaw angle motion error was calculated by using equation (3) and the measured yaw angle error data in, which was used to compensate the worktable positioning error measured at different speeds. As shown in Fig. 10, the positioning error compensated by using the traditional method is recorded as the compensated positioning error 1, and the positioning error compensated by using the instantaneous motion center compensation method is recorded as the compensated positioning error 2.

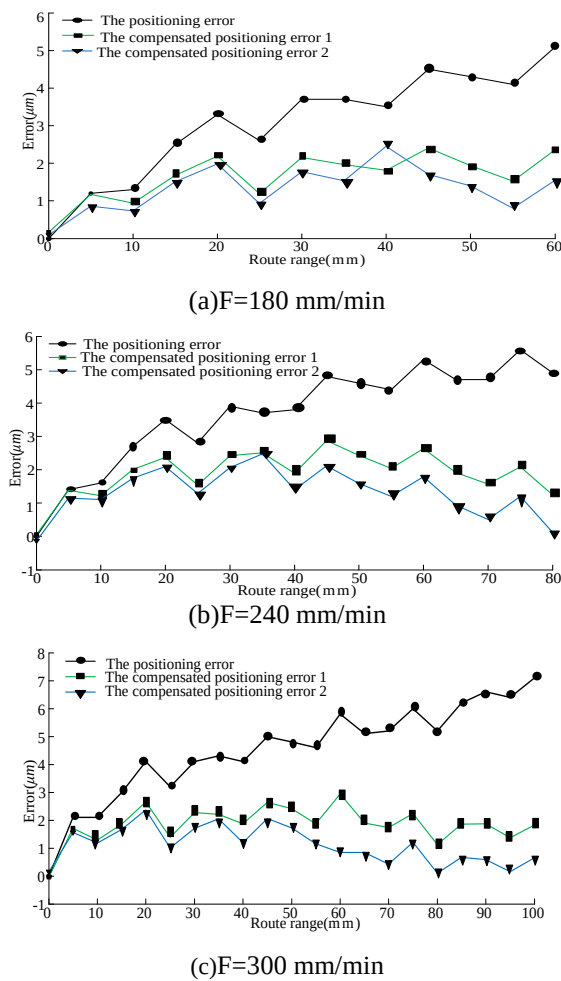


Fig. 8 Positioning error compensation results of the worktable under different moving speeds

It can be seen from the comparison results that the positioning error of the worktable compensated by the traditional method is decreased by 2.74 μm , 3.66 μm , 5.27 μm , whereas that compensated by the compensation method based on the instantaneous rotation center is decreased by 2.74 μm , 4.87 μm , 6.45 μm under the moving speeds of 180 mm/min, 240 mm/min and 300 mm/min. The positioning error compensated by using the new compensation method

is decreased by 0.81 μm , 1.12 μm , 1.18 μm more than that by using the traditional method. The experimental results show that the Abbe positioning error compensation method based on instantaneous rotation center can achieve more accurate compensation effects than the traditional positioning error compensation method.

CONCLUSIONS

(1) A concept and determination method for the instantaneous rotation center of the CNC worktable are proposed in this paper. The theoretical model and experimental results show that the positions of the CNC worktable instantaneous rotation centers in X, Y, Z direction are different change with different moving speeds.

(2) The movement characteristics of CNC worktable and the change rule of Abbe positioning error of machine tool guide systems are analyzed. The Abbe positioning error compensation model based on instantaneous rotation center is established. The positioning error compensation experiments with the new compensation model based on the instantaneous rotation center and the traditional model are carried out, respectively. The experimental results show that the Abbe positioning error compensation model based on the instantaneous rotation center can achieve higher Abbe offset calculation accuracy and more precise positioning error compensation. This method can be used to achieve high-precision positioning error compensation for CNC worktable.

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NOMENCLATURE

- O grating reading head
- B processing point
- BO the yaw angle Abbe offset
- BO_p actual yaw Abbe offset
- O_p the yaw instantaneous rotation center.
- O_F the pitch instantaneous center
- O_G the roll instantaneous center
- ε_Z yaw angle error
- ε_Y pitch angle error
- ε_X roll angle error
- V_0 The movement speed of the machine tool worktable in X direction
- $V_{Mx} V_{My} V_{Nx} V_{Ny}$ the speed of point M, N
- δ Abbe positioning error
- δ_{PB} actual Abbe positioning error of point B under the action of yaw angle motion error
- θ the roll angle of the sprung mass of vehicle
- ε_f the front roll steer coefficient
- ε_r the rear roll steer coefficient

基於瞬時旋轉中心的數控 機床阿貝定位誤差建模與 補償

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摘要

為了減少因為阿貝偏移量計算不精確對數控機床定位誤差產生的影響，本文提出了瞬時旋轉中

心的概念，建立了基於瞬時旋轉中心的阿貝偏移量精確計算模型和機床工作台阿貝定位誤差精確補償模型。利用激光干涉儀和速度傳感器建立了瞬時旋轉中心的確定實驗裝置，開展了不同運動速度下數控機床導軌系統 X 方向的瞬心確定實驗。實驗結果表明，在不同速度下和不同位置處工作台瞬時運動中心的位置時刻在改變。採用基於瞬時旋轉中心的新型補償模型和傳統模型分別進行了定位誤差的補償實驗。在 180 mm/min，240 mm/min，300 mm/min 的速度下，經過基於瞬時運動中心補償後的定位誤差比傳統方法補償後的定位誤差分別多減小了 $0.81\ \mu\text{m}$ ， $1.12\ \mu\text{m}$ ， $1.18\ \mu\text{m}$ 。相比較於傳統方法的補償結果，基於瞬時旋轉中心的補償方法能夠實現更高精度的阿貝偏移量的計算和機床導軌系統阿貝定位誤差的補償。