中國機械工程學刊第四十一卷第四期第 441~448 頁(民國一百零九年)
Journal of the Chinese Society of Mechanical Engineers, Vol.41, No.4, pp 441~448 (2020)

An Evaluation Method of Gear Pitch Deviations Based on the Consideration of Installation Errors

Jie Tang*, Jian-Jie Wei** and Zhao-Yao Shi***

Keywords：gear metrology, pitch deviation, eccentricity, coordinate metrology.

ABSTRACT

To lower the influence of installation errors on gear pitch measurement, an evaluation method of pitch deviations is introduced, considering installation eccentricity error and installation wobbling error. The pitch measurement points are firstly transformed from the machine coordinate system into the workpiece coordinate system. Then gear functional center is fitted according to least squares equation established with the pitch measurement points and base circle center, and solved by Newton's method. After that, pitch deviations are evaluated according to ISO1328-1:2013. Experiments are carried out on a gear measurement center under four groups of different installation errors. The value of the variation among total cumulative pitch deviations of the product gear obtained by this method under different installation conditions is 0.39μm, whereas the variation obtained by the gear measurement center is 28.66μm. The method can be used to evaluate pitch deviations in the pitch measurement under certain installation errors

INTRODUCTION

Gears are a core component in mechanical transmission. The gear quality directly affects the performance, safety and life span of equipment. Gear accuracy measurement and evaluation is the main way to ensure the gear quality (Goch, 2002). Gear pitch deviations, as one of the gear elemental deviations, are mainly caused by the uneven distribution of gear teeth along the circumference (Shi, et al., 2003; Haertig et al., 2013).

The international gear precision standard (ISO1328-1:2013) gives the definition and evaluation method of pitch deviations, and inspection practice for which can be refered to the inspection code (ISO/TR10064-1: 2017).

Generally, gear pitch deviations can be measured on gear measurement center (GMC), coordinate measuring machines (CMM), etc. When gear pitch deviations are measured, installation errors often occur because of the misalignment between the rotation axis of the instrument and the center axis of the measured gear. The existence of the installation error affects the evaluation result of the total cumulative pitch deviation and even cause the wrong grade judgment of gear accuracy (Fujio et al., 1993; Sato et al., 2010). With the coordinate data obtained by the gear measuring center, the pitch deviations of the curve-face gear pair are obtained (Lin et al., 2016). The tolerance mathematical model for spur gear is obtained based on the mathematical models of pitch deviation and eccentricity deviation is obtained (Huang et al., 2017).

The highly accurate pitch calibration methods applicable to CMM or gear measuring machines were developed (Kniel et al., 2009), the calibration methods are based on the closure technique which allows the separation of the systematic errors of the measurement device and the errors of the gear. A method of evaluating pitch measurement accuracy based on a self-calibration technique or a comparative measurement technique using a sphere artifact is introduced (Kondo et al., 2012). Two gears' pitch deviations were measured by adopting the closure technique and multi-step method, in which the systematic errors can also be eliminated and it can

Paper Received October, 2019. Revised February, 2020. Accepted February, 2020. Author for Correspondence: Jie Tang

** Associate Professor, Beijing Engineering Research Center of Precision Measurement Technology and Instruments (Beijing University of Technology), Beijing 100124*

***Graduate student, Beijing Engineering Research Center of Precision Measurement Technology and Instruments (Beijing University of Technology), Beijing 100124*

**** Professor, Beijing Engineering Research Center of Precision Measurement Technology and Instruments (Beijing University of Technology), Beijing 100124*

also be used for highly accurate measurements of pitch deviations (Lou et al., 2012).

Pitch deviation measurements considering installation errors are extensively explored. A compensation method for installation errors is proposed through modifying the gear theoretical position parameters obtained by the ray-tracing method on a laser interferometer (Fang et al., 2014). Alignment angles are calculated with the eccentricity value obtained by measuring the upper and lower centers of the GMM and alignment angle errors can be compensated by modifying the measurement data (Liu et al., 2016). The previous evaluation and compensation method is analyzed through the full tooth profile measurement data based on opposite-direction dual scanning method (Xu et al., 2015). The gear base circle radius and base circle center are obtained by applying the Gauss-Newton method to solve the nonlinear equation established with tooth profile measurement points (Tang et al., 2009).

Generally, installation errors in gear measurement are divided into installation eccentricity error and installation wobbling error. The evaluation method of gear profile deviations considering installation errors (EMI-profile) is introduced to measure gear profile deviations without precise adjustment of installation errors (Tang et al., 2019). In gear pitch measurement, most scholars only considered one of the two installation errors. In this study, an evaluation method of gear pitch deviations based on the consideration of installation errors (EMI-Pitch) is proposed based on the consideration of installation errors and then verified in gear measurement experiments.

EVALUATION METHOD OF EMI-PITCH

The evaluation method of EMI-Pitch involves three main steps: establishing the installation error model, gear functional center fitting and the evaluation of pitch deviations. EMI-Pitch method aims to be used to assess pitch deviations of the measurement points of the gear tooth flank without precise positioning.

The specific procedure of EMI-Pitch method is shown in Figure 1. Considering the influences of installation errors, the transformation relationship between the Workpiece Coordinate System (WCS) and the Machine Coordinate System (MCS) was established according to the vector of the upper transverse plane and the gear functional center fitted with pitch measuring points in order to re-construct the measurement points under WCS. Pitch deviations were evaluated according to ISO1328-1:2013 in WCS.

Installation Error Model

During the gear pitch measurement, a probe was used to measure the points around the gear reference circle in the middle plane of face width. Fig. 2 shows the schematic diagram for pitch measurements.

In Fig.2, $O-XYZ$ is MCS, $O'-X'Y'Z'$ is WCS, $O_1-X_1Y_1Z_1$ is the Fitting Coordinate System (FCS), O' is the functional center of the workpiece, Z' is geometric center axis of the workpiece, O_1 is the intersection point of the rotary axis in MCS and the middle plane of facewidth.

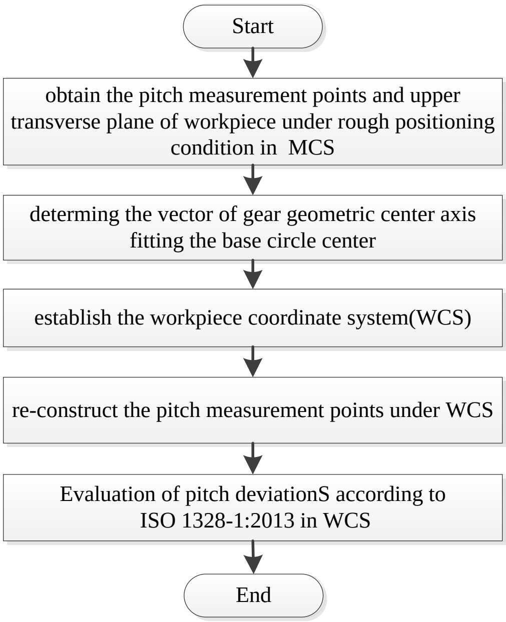


Fig.1. Procedure of EMI-Pitch method.

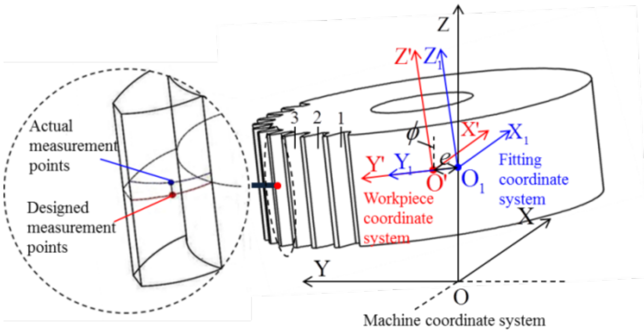


Fig.2. Schematic diagram of pitch measurements.

In gear pitch measurements, the gear functional center axis may not coincide with the rotary axis of measuring instrument, thus leading to the installation errors including installation eccentricity error and installation wobbling error. Installation errors cause the misalignment between actual measurement points and designed measurement points (see Fig. 2).

The installation error model aims to transform the measurement points for pitch deviations from the Machine Coordinate System into the Workpiece Coordinate System after obtaining the installation error for eliminating its influence. The installation error model can be described as:

$$\begin{bmatrix} X' \\ Y' \\ Z' \\ 1 \end{bmatrix} = B \cdot \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix} = B \cdot \left(M_Y \cdot M_X \cdot A \cdot \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \right), \qquad (1)$$

where A is the translation matrix of the Machine Coordinate System (MCS) relative to WCS, as expressed in Eq.(2); M_X is the rotation matrix of the MCS rotating around the X-axis, as expressed in Eq. (3); M_Y is the rotation matrix of the MCS rotating around the Y-axis, as expressed in Eq. (4); B is translation matrix of the Fitting Coordinate System, obtained from the functional center coordinates $O'(e_{x1}, e_{y1}, 0)$ of the gear in the Fitting Coordinate System, as expressed in Eq. (5); (X', Y', Z') is pitch measurement points in WCS; (X_1, Y_1, Z_1) is pitch measurement points in FCS; (X, Y, Z) is pitch measurement points containing installation errors in MCS.

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -Z_h \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad (2)$$

$$M_X = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\gamma_x & -\sin\gamma_x & 0 \\ 0 & \sin\gamma_x & \cos\gamma_x & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad (3)$$

$$M_Y = \begin{bmatrix} \cos\gamma_y & 0 & \sin\gamma_y & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\gamma_y & 0 & \cos\gamma_y & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad (4)$$

$$B = \begin{bmatrix} 1 & 0 & 0 & -e_{x1} \\ 0 & 1 & 0 & -e_{y1} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad (5)$$

where $O'(e_{x1}, e_{y1}, 0)$ is gear functional center coordinates in Fitting Coordinate System; Z_h/mm is the height of the middle plane of facewidth in Machine Coordinate System (MCS); γ_x /rad and γ_y/rad are the angles of the MCS around the X-axis and Y-axis obtained from the direction vector $\tau = (v_x, v_y, 1)$ of the functional center axis of the gear in the MCS, as expressed in Eq. (6):

$$\begin{cases} \gamma_x = \arctan(v_y) \\ \gamma_y = -\arctan\left(\dfrac{v_x}{\sqrt{(v_y)^2 + 1}} \right) \end{cases}, \qquad (6)$$

where $\tau = (v_x, v_y, 1)$ is the direction vector of the gear functional center axis in Machine Coordinate System (MCS).

Installation errors of gear to be measured are described as installation wobbling error, which is represented by gear axis direction vector $\tau = (v_x, v_y, 1)$ in MCS, and installation eccentricity error, which is represented by gear functional center coordinates $O'(e_{x1}, e_{y1}, 0)$ in FCS, as shown in Fig. 2.

To transform pitch measurement points from MCS into Workpiece Coordinate System, the gear axis direction vector $\tau = (v_x, v_y, 1)$ and the gear functional center coordinates should be determined firstly. The values of installation wobbling error ϕ(rad) and installation eccentricity error e (μm) are respectively calculated with Eq. (7) and Eq. (8).

$$\phi = \mathrm{atan}\left(\sqrt{v_x^2 + v_y^2}\right), \qquad (7)$$

$$e = \sqrt{(e_{x1})^2 + (e_{y1})^2}, \qquad (8)$$

where ϕ (rad) is the value of installation wobbling error; e (μm) is the value of installation eccentricity error.

The key step of the EMI-Pitch method is to determine the values of installation errors, where $\tau = (v_x, v_y, 1)$ is obtained by fitting the points of the gear upper transverse plane and the coordinates $O'(e_{x1}, e_{y1}, 0)$ are obtained by fitting the gear functional center with the pitch measurement points.

Gear Functional Center Fitting

Fitting gear functional center is to determine the coordinates of gear center $O'(e_{x1}, e_{y1}, 0)$ in Fitting Coordinate System. To establish the FCS $O_1 - X_1Y_1Z_1$ shown in Fig. 2, the vector $\tau = (v_x, v_y, 1)$ needs to be fitted by the gear upper transverse plane.

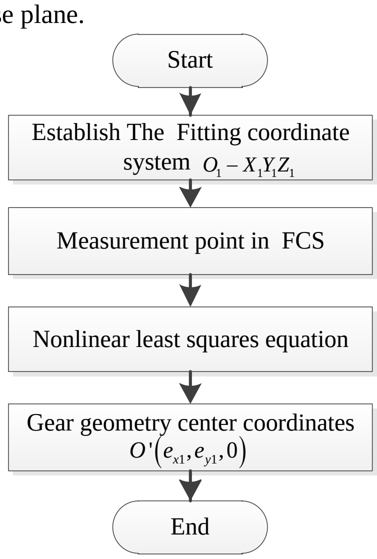


Fig. 3. Flow chart for fitting gear functional center.

Then the pitch measurement points in Fitting Coordinate System are obtained with Eq. (1) to eliminate the influence of installation wobbling error.

The gear functional center coordinates in Fitting Coordinate System are calculated by the least squares equation established by the geometric relationship between pitch measurement points and gear functional center, as shown in Fig. 3.

In the center fitting in Fitting Coordinate System, theoretically, the involute profile should be evenly distributed along the circumference and the gear base circle circumference angle of any two adjacent involutes is fixed (Fig. 4).

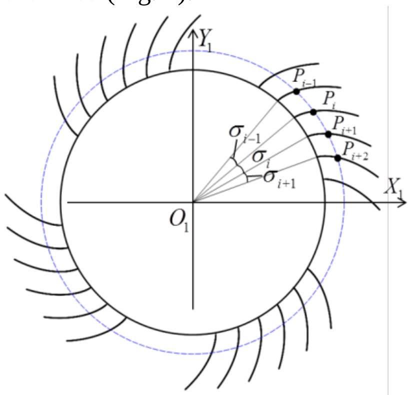


Fig. 4. Distribution of pitch measurement points.

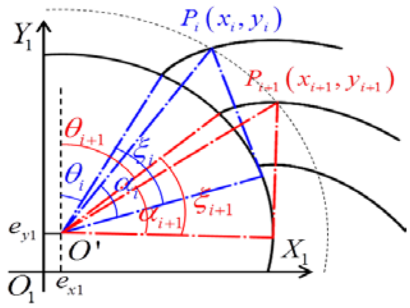


Fig. 5. Gear functional center fitting.

$$\sigma_{i-1} = \sigma_i = \sigma_{i+1} = \sigma_d = \frac{2\pi}{z}, \tag{9}$$

where z is the number of tooth; σ_d is the theoretical value of the gear base circle circumference angle of any two adjacent involutes.

Actually, the gear functional center deviates from its theoretical position due to the existence of installation errors, which introduce the inequal results in the base circle circumference angle of any two adjacent involutes, corresponding to the pitch measurement points $P_i(x_i, y_i)$ and $P_{i+1}(x_{i+1}, y_{i+1})$. To determine the functional center coordinates $O'(e_{x1}, e_{y1}, 0)$in Fitting Coordinate System, the least squares equation is established based on the geometric relationship between the pitch measurement points and the base circle center:

$$F(e_{x1}, e_{y1}) = min \sum_{i=1}^{n} (\Delta_i)^2, \tag{10}$$

where $\Delta_i = \sigma_i - \overline{\sigma_i}$, (11)

Δ_i/rad is the difference between the gear base circle circumference angle of any two adjacent involutes and its mean value in Fitting Coordinate System; *i=1, 2...n* (*n* is the number of σ_i); $(e_{x1}, e_{y1}, 0)$ is the base circle center coordinates of O'in FCS; $\overline{\sigma_i}$is the mean value of σ_i. If $\overline{\sigma_i} = \sigma_d$, the geometric relationship (shown in Fig. 5) can be expressed in Eq. (12):

$$\sigma_i = \theta_{i+1} - \theta_i \tag{12}$$

where

$$\begin{cases} \theta_i = arctan\frac{x_i - e_{x1}}{y_i - e_{y1}} - \xi_i + \alpha_i \\ \theta_{i+1} = arctan\frac{x_{i+1} - e_{x1}}{y_{i+1} - e_{y1}} - \xi_{i+1} + \alpha_{i+1} \end{cases}, \tag{13}$$

where ξ_i/rad and ξ_{i+1}/rad is the involute rolling angles corresponding to points $P_i(x_i, y_i)$ and $P_{i+1}(x_{i+1}, y_{i+1})$ according to the principle of involute (ISO21771:2007), as expressed in Eq. (13); α_i/rad and α_{i+1}/rad are respectively the pressure angles corresponding to points $P_i(x_i, y_i)$ and $P_{i+1}(x_{i+1}, y_{i+1})$, as expressed in Eq. (14).

$$\begin{cases} \xi_i = \frac{\sqrt{(x_i - e_{x1})^2 + (y_i - e_{y1})^2 - r_b{}^2}}{r_b} \\ \xi_{i+1} = \frac{\sqrt{(x_{i+1} - e_{x1})^2 + (y_{i+1} - e_{y1})^2 - r_b{}^2}}{r_b} \end{cases}, \tag{14}$$

where r_b (mm) is the base circle radius.

$$\begin{cases} \alpha_i = arccos\frac{r_b}{\sqrt{(x_i - e_{x1})^2 + (y_i - e_{y1})^2}} \\ \alpha_{i+1} = arccos\frac{r_b}{\sqrt{(x_{i+1} - e_{x1})^2 + (y_{i+1} - e_{y1})^2}} \end{cases}. \tag{15}$$

The functional center coordinate $O'(e_{x1}, e_{y1}, 0)$ in Fitting Coordinate System can be obtained by solving Eq. (10) with Newton's method.

Considering the influence of gear functional eccentricity (the distance between the gear functional center to its ideal center), the circumferential angles corresponding to the adjacent tooth profiles of actual gears σ_i are different. So the non-linear least square equation can be established by using multiple groups of measuring points along the circumference, and the average value of the circumference angle corresponding to multiple groups of adjacent tooth profiles are taken as the reference value, as shown in Fig. 4.This can eliminate the influence of installation eccentricity in the evaluation of pitch deviations, and the fitting results are expected to be closer to the gear functional center.

Evaluation of Pitch Deviations

The pitch measurement points in Workpiece Coordinate System are obtained with the installation error model in EMI-pitch (Fig. 6).

The evaluation result of pitch deviations according to ISO1328-1:2013 is shown in Fig. 7.

Based on the involute principle, the rolling angle ξ_i (rad) of pitch measurement point (x_i, y_i) can be calculated as

$$\xi_i = \sqrt{x_i^2 + y_i^2 - r_b^2}/r_b. \tag{16}$$

The coordinate (X_i, Y_i) of pitch point on the design profile corresponding to the rolling angle is obtained as

$$\begin{cases} X_i = r_b\left(cos(\xi_i) + \xi_i \, sin(\xi_i)\right) \\ Y_i = r_b\left(sin(\xi_i) - \xi_i \, cos(\xi_i)\right) \end{cases}, \tag{17}$$

where r_b (mm) is base circle radius; ξ_i (rad) is the rolling angle of involute.

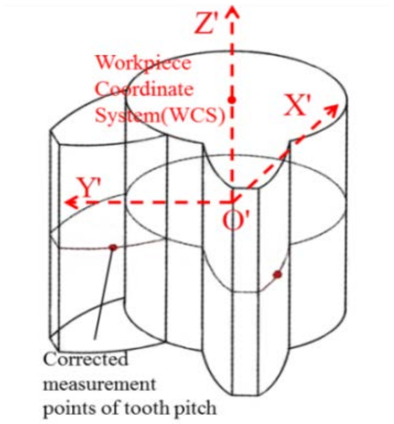


Fig. 6. Gear pitch measurement points in WCS.

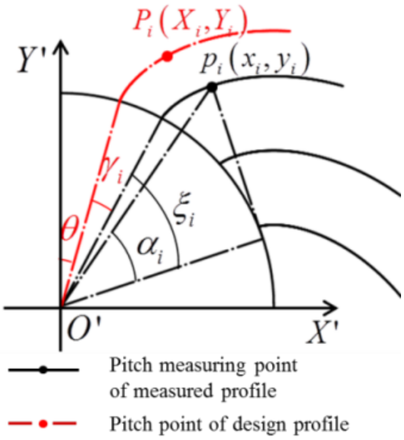


Fig. 7. Evaluation of pitch deviations.

The design profile is rotated around the origin of WCS, so that the design profile passes through the corresponding pitch measurement points in succession. The rotation angle γ_i (rad) passing through each measurement point of the design profile can be calculated as

$$\gamma_i = \begin{cases} arccos\left(\frac{X_i x_i - Y_i y_i}{(X_i)^2 + (Y_i)^2}\right) & \gamma_i \in (0, \pi) \\ 2\pi - arccos\left(\frac{X_i x_i - Y_i y_i}{(X_i)^2 + (Y_i)^2}\right) & \gamma_i \in (\pi, 2\pi) \end{cases}. \tag{18}$$

The value of individual cumulative pitch deviation of each pitch measurement point is obtained as

$$F_{pi} = r.\left((\gamma_i - \gamma_1) - \frac{2\pi}{z}(i-1)\right). \tag{19}$$

The total cumulative pitch deviation is:

$$F_p = max(F_{\mathrm{pi}}) - min(F_{\mathrm{pi}}). \tag{20}$$

The individual single pitch deviation is:

$$f_{\mathrm{pi}} = F_{pk} - F_{pk-1}. \tag{21}$$

The single pitch deviation is:

$$f_p = \max|f_{\mathrm{pi}}| = max(F_{pk} - F_{pk-1}), \tag{22}$$

where r is the reference circle radius; $r = \frac{mz}{2}$; m is module; z is tooth number; $k=1,2\ldots\ldots z$.

MEASUREMENT EXPERIMENTS

Basic Experimental Parameters

Gear measurement experiments are carried out to verify the EMI-Pitch evaluation method. The product gear is measured on a GMC. The product gear is an involute cylindrical spur gear with the Grade-3 accuracy (ISO 1328-1:1995). The parameters of the product are shown in Table 1.

The gear installation error is introduced by inserting a feeler gauge between the gear and an expandable arbor. The thickness values of the feeler gauges used are respectively 0 mm, 0.01 mm, 0.03 mm and 0.05 mm and the insertion depth of the feeler gauge from the upper transverse plane is about 8 mm, as shown in Fig. 8. Totally four groups of experiments were carried out.

Table 1. Parameters of product gear.

Parameters	Values
Module m/mm	3.5
Number of teeth z	36
Facewidth b/mm	25
Modification coefficient x	0
Center hole diameter φ/mm	40
Pressure angle α/°	20
Helix angle β/°	0

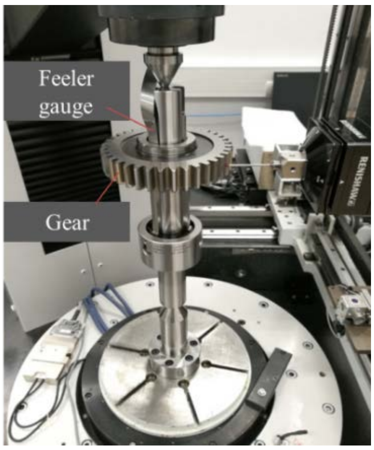


Fig. 8. Measurement experiment.

The direction vector τ of the product gear is the normal vector of the fitting plane and obtained with 25 points homogeneously measured on the upper transverse plane of the gear on the GMC by the least squares plane fitting. The gear functional center O' is calculated after the pitch measurement points in MCS are transformed into the coordinates FCS according to the gear functional center fitting method. The tooth number of the product gear selected in gear functional center fitting are Z_5, Z_6, Z_7, Z_{14}, Z_{15}, Z_{16}, Z_{23}, Z_{24}, Z_{25}, Z_{32}, Z_{33}, and Z_{34}, as shown in Fig. 9

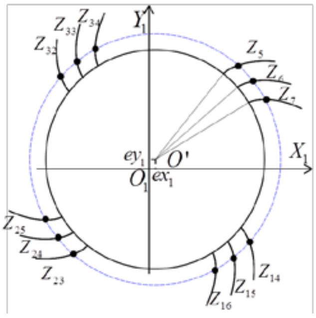


Fig. 9 Tooth number selected in gear functional center fitting

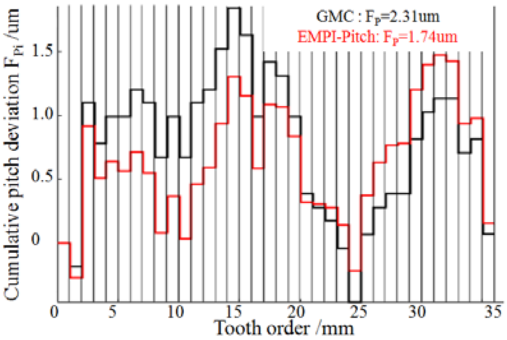
Table 2. Four groups of installation errors in the gear measurement experiments.

Group order	Thickness of feeler gauges /mm	Functional center axis direction vector τ /μm	Gear geometry center O' /μm	Installation wobbling φ/rad	Installation eccentricity e/μm
Group 1	0	(0,0,1000)	(-0.11,-0.47,0)	0	0.48
Group 2	0.01	(-0.19,0.05,1000)	(-3.27,2.10,0)	0.0002	4.27
Group 3	0.03	(-0.79,0.11,1000)	(-7.34,2.35,0)	0.0008	7.71
Group 4	0.05	(-1.15,0.35,1000)	(-12.19,7.27,0)	0.0012	14.19

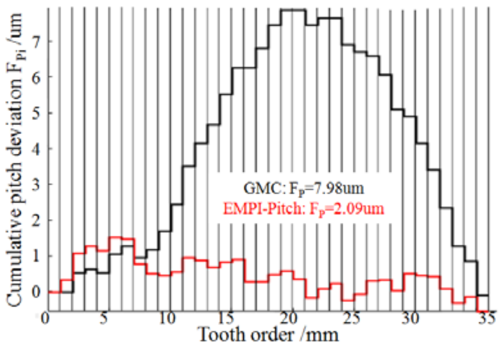
Determination of Installation Errors

Four groups of installation errors are listed in Table 2. The values of installation wobbling error φ (rad) and installation eccentricity error e (μm) are determined with the vector τ and the center O', as expressed in Eq. (1) and Eq. (2).

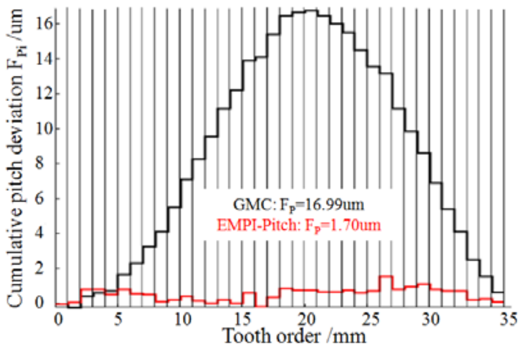
Measurement Results Considering Installa-tion Error



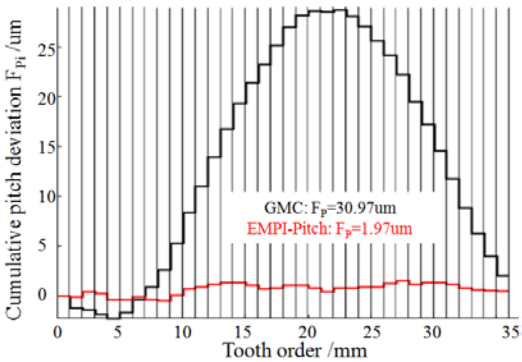
(a) Group 1



(b) Group 2



(c) Group 3



(d) Group 4

Fig. 10. Comparison of the evaluation results obtained with EMI-Pitch and GMC.

Four groups of pitch measurement data were obtained on the GMC. The data are re-evaluated with the EMI-Pitch method according to ISO1328-1:2013.

The comparison results of the four groups of total cumulative pitch deviations (F_p) respectively obtained with the GMC and EMI-Pitch method are shown in Fig. 10(a)- Fig. 10(d) and the evaluation results are listed in Table 3.

It can be found from Fig. 10(a)-Fig. 10(d) that installation errors have a significant effect on the evaluation of pitch deviations and that the effect increases with the increase in the installation error. The maximum difference among the total cumulative pitch deviations of the same product gear evaluated by GMC under different installation conditions was 28.66 μm. Under the same measurement data of pitch measurement points, the maximum difference among the total cumulative pitch deviations of the same product gear evaluated by EMI-Pitch method under different installation conditions was 0.39 μm (Table 3).

Table 3. F_p obtained with EMI-Pitch and GMC.

Group No.	F_p/μm GMC	F_p/μm EMI-pitch
Group 1	2.31	1.74
Group 2	7.98	2.09
Group 3	16.99	1.70
Group 4	30.97	1.97
max(Δ)	28.66	0.39

Notes: max(Δ) indicates the value of the variation among the four groups of the evaluation results.

It can be found that the EMI-Pitch can significantly eliminate the influence of installation errors and yield a more consistent evaluation result. The EMI-Pitch method can be applied in the evaluation of gear pitch deviations under certain installation errors.

CONCLUSIONS

In gear pitch measurements, installation errors including installation eccentricity error and installation wobbling error cause the misalignment between actual measurement points and designed measurement points, thus influencing the evaluation results of pitch deviations.

To eliminate the influences of installation errors, the evaluation method of pitch deviations based on the consideration of installation errors (EMI-Pitch) is introduced. The installation error model is used to transform the pitch measurement points from the Machine Coordinate System (MCS) into the Workpiece Coordinate System (WCS) based on the values of installation errors. The coordinates of gear functional center are determined by the least squares equation established based on the geometric relationship between the pitch measurement points and the base circle center. Newton's method is used to solve the least squares equation. Then the pitch deviations are evaluated through the pitch measurement points reconstructed in WCS according to ISO1328-1:2013.

The gear measurement experiments were carried out. The product gear measured on a GMC is an involute cylindrical spur gear with the Grade-3 accuracy (ISO1328-1:1995). The pitch measurement data under four groups of different installation errors were obtained on the GMC. The measurement data were re-evaluated with the EMI-Pitch method. The value of the variation among the total cumulative pitch deviations of the product gear evaluated by EMI-Pitch method under different installation conditions is 0.39 μm, whereas the variation obtained by the GMC is 28.66 μm.

The EMI-Pitch can eliminate the influences of installation errors and obtain more consistent evaluation results. It can be applied to process the measurement points of the gear tooth flank for pitch deviations under consideration of installation errors, especially for the gears with higher accuracy and the situation that the installation eccentricity error is significantly larger than the gear functional eccentricity caused by manufacturing.

ACKNOWLEDGEMENTS

This research is supported by National Key R&D Program of China (Grant No. 2017YFF0204804).

REFERENCES

Fang, S.P., Zhu, X.D., Yang, P.Ch., Cai, Q.W., Komori, M., and Kubo, A., "Analysis and Compensation Method for Installation Error in Measuring Gear Tooth Flank with Laser Interferometry," *Optical engineering* , Vol.53, No.8, pp.084111(2014).

Fujio, H., Kubo, A., Saitoh, S., Suzuki, M., Tochimoto, S., Hanaki, H., and Honda, T., "Laser Holographic Measurement of Tooth Flank Form of Cylindrical Involute Gear," *Journal of Mechanical Design*, Vol.116, pp.721-729(1993).

Goch, G., and Gunther, A., "Future Gear Metrology: Superfical Description and Inspection of Flanks," *VDI Berichte*, Vol. 2, No. 1665, pp.751-768 (2002).

Haertig, F., Lin, H., Kniel, K., and Shi, Zh.Y., "Laser Tracker Performance Quantification for the Measurement of Involute Profile and Helix Measurements," *Measurement*, Vol. 46, pp .2837-2844(2013).

Huang, K., Xu, R., and Chen, Q., “A Tolerance Mathematical Model Based on Fractal Theory for Tolerance Analysis of Gear,” *Journal of the Chinese Society of Mechanical Engineers*, Vol.38, No.5, pp.525~536(2017).

ISO, Cylindrical gears — ISO System of Flank Tolerance Classification — Part 1: Definitions and Allowable Values of Deviations Relevant to Flanks of Gear Teeth, ISO 1328-1: 2013.

ISO, Code of Inspection Practice — Part 1: Measurement of Cylindrical Gear Tooth Flanks, ISO/TR 10064-1: 2017

ISO, Gears—Cylindrical Involute Gears and Gear Pairs—Concepts and Geometry, ISO 21771: 2007.

Kniel, K., Hartig, F., Osawa, S., and Sato, O., “Two Hghly Accurate Methods for Pitch Calibration,” *Measurement Science and Technology*, Vol.20, No.115110, pp.1-11(2009).

Kondo, Y., Osawa, S., Sato, O., Komori, M., and Takatsuji, T., “Evaluation of Instruments for Pitch Measurement Using a Sphere Artifact,” *Precision engineering*, Vol.36, No.4, pp.604–11(2012).

Lin, Ch., Cao, X.J., Fan, Y., and Zeng, D., “Pitch Deviation Measurement and Analysis of Curve-face Gear,” *Measurement*, Vol.81, pp.95-101(2016).

Liu, Y.S., Fang, S.P., Wang, H.Y., Taguchi, T. and Takeda, R., “Compensation Method for the Alignment Angle Error in Pitch Deviation Measurement,” *Measurement science and technology*, Vol.27, No.055006(2016).

Lou, Zh.F., Ling, S.Y., He, H.Z., Wang, X.D., Ma, Y., Wang, L.D., “A Kind of Multi-step Method for Measuring Pitch Deviation of a Gear,” *Measurement Science and Technology*, Vol.23, No.115002, pp.1–7(2012).

Shi, Zh.Y., Fei, Y.T., and Xie, H.K., “100 Years of Gear Measurement Technology⁈ Review & Prospect,” *Engineering Science*, Vol. 5, No. 9, pp.13-17(2003) (in Chinese).

Sato, O., Osawa, S., Kondo, Y., Komori, M., and Takatsuji, T., “Calibration and Uncertainty Evaluation of Single Pitch Deviation by Multiple-Measurement Technique,” *Precision engineering*, Vol.34, pp.156-163(2010).

Tang, J., and Shi, Zh.Y., “Reverse Calculation of the Parameters of Involutes and Application,” *Journal of Beijing University of Technology*, Vol.35, No.1, pp.8-12(2009) (in Chinese).

Tang, J., Wei, J.J., and Shi, Zh.Y., “An Evaluation Method of Gear Profile Deviations Based on the Consideration of Installation Errors,” *Measurement*, Vol.146, pp.806-814(2019).

Xu, B., So, I., Yuki, S., and Gao, W., “Pitch Deviation Measurement of an Involute Spur Gear by a Rotary Profiling System,” *Precision engineering*, Vol.39, pp152-160(2015).

考慮安裝誤差的齒輪齒距偏差評定方法

湯潔 魏建傑 石照耀

北京市精密測控技術與儀器工程技術研究中心(北京工業大學)

摘要

考慮安裝偏心誤差和偏擺誤差的齒距偏差評定方法,是將齒距測量點從機器坐標系變換到工件坐標系,建立最小二乘法及牛頓法得到齒輪回轉中心,再由 ISO1328-1:2013 評定偏差。齒輪測量中心試驗表明,應用該方法同一齒輪 4 種安裝誤差條件下齒距累積總偏差數值相差 0.39μm,未用該方法數值相差 28.66μm。該方法可用於一定安裝誤差條件下齒輪齒距偏差測量。