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Analytical Solutions for Fourier and Non-Fourier Heat Conduction in Thermal Barrier Coating of Efficient Engine

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Keywords: thermal barrier coatings, efficient diesel engine, non-Fourier heat conduction, wave speed

ABSTRACT

In the present work, the Fourier, the C-V and the dual-phase-lags (DPL) heat conduction models are applied to study heat transfer in thermal barrier coatings (TBC) of a four-stroke cycle efficient diesel engine. An expanded separation of variables method is used to solve the heat conduction equations of the three models. Two typical cases ((a). the TBC subjected to a thermal shock. (b). the TBC subjected to continuous but unstable heating) are investigated. Wave phenomena in thermal propagation process were found in the C-V model and the DPL model. Several parameters effect on thermal wave speed is discussed. It is found that the working process of efficient diesel engine creates a temperature fluctuation at bottom of TBC. Several parameters effect on fluctuation range is discussed. The analytical method can be used for solving general heat transfer problems with transient boundary conditions.

INTRODUCTION

In aerospace and automobile industries, thermal barrier coatings (TBC) is always used to protect critical engineering components, such as turbine blades, combustion chambers and burner liners [Wang et al., 2016]. In addition to protecting TBC also improves running efficiency and prolongs life of engine [Zhang et al., 2015]. TBC plays an important role in heat conduction in efficient engine. Accurate prediction of heat transfer in TBC is useful for designing and manufacturing of efficient engine [Golosnoy et al., 2009]. Meanwhile along with the development of thermal spraying, thickness of the TBC is becoming smaller and smaller. Massive experiments demonstrate that heat conduction in thin film insulating structures such as thermal barrier coatings does not follow the Fourier-low [Frankel et al., 1987; Taitel, 1972]. Scientists and engineers are still looking for the best model for heat transfer in thin film.

In general, there are three kind of model for heat transfer in thin film, the Fourier model, the C-V model and the dual-phase-lags (DPL) model. The most widely used among them is the Fourier model:

$$q(\vec{r},t) = -k\nabla T(\vec{r},t) \tag{1}$$

It assumes an infinitely fast propagation of thermal signal. It means that any local temperature disturbance causes an instantaneous perturbation in the temperature at each point in the medium. Although this assumption is reasonable for most common engineering situations, it fails in heat conduction in thin film. Some recent experimental results show the existence of thermal waves in heat transfer in thin film. That may be caused by the finite speed of thermal signal [Liu et al., 2012].

In order to associate a finite heat propagation velocity, [Cattaneo, 1958 and Vernotte, 1958] modified the Fourier model. That is often referred to the C-V model.

$$q(\vec{r},t+\tau_q) = -k\nabla T(\vec{r},t) \tag{2}$$

It assumes that the temperature gradient and the heat flux occur at the different times. The relaxation time τ_q represents the time needed to establish the heat flux when a temperature gradient is suddenly imposed [Lin, 2013]. Several articles have been contributed to the study of heat transfer in thin film with C-V model [Kumar and Srivastava, 2015 and Afrin et al., 2011]. Numerical methods such as Finite element method (FEM), finite difference method (FDM), Boltzmann method are often used to study the

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C-V model [Akwaboa et al., 2013].

Considering the temperature gradient might also exists relaxation time, [Tzou, 1995] modified C-V model to DPL mode.

$$q(\vec{r},t+\tau_q)=-k\nabla T(\vec{r},t+\tau_T) \tag{3}$$

where τ_q represents relaxation time of heat flux, τ_T represents relaxation time of temperature gradient. If $\tau_q>\tau_T$, it means that the local heat flux vector results in the temperature gradient at the same location but an early time, the heat flow is often called gradient-precedence type. If $\tau_q<\tau_T$, it means that the temperature gradient results in the heat flux at an early time, the heat conduction is flux-precedence type [Kumar et al., 2015]. The DPL model is widely used in high intense and rapid heat and mass transfer in micro-scale structure. Many scholars present a lot of methods for solving the DPL model. For example [Xu et al., 2008] solved the DPL model by using the finite difference method (FDM) method. [Ramadan, 2009] employed hybrid scheme based on the Laplace transform to solve the DPL model. Yu-Ching Yang and Wen-Li Chen studied heat transfer in TBC of gas turbine by using a numerical method involving the hybrid application of the Laplace transform and control volume methods [Yang et al., 2015]. Ling Li and Ling Zhou developed an Expanded Lattice Boltzmann Method to solve the DPL model [Li et al., 2016]. From the above it can be seen that few literatures give the analytical solution for the DPL model.

This paper presents an Expanded Separation of Variables Method to solve the Fourier model, C-V model and DPL model. Analytical solutions of three models will be obtained. The heat transfer in TBC of a four-stroke cycle efficient diesel engine will be studied by using the analytic method. Several parameters effect on temperature variation and distribution will be discussed. The phenomenon of thermal wave propagation and reflection in TBC with C-V and DPL model will be present.

THE GOVERNNENT EQUATION AND BOUNDARY CONDITIONS

The structure of TBC of a four-stroke cycle efficient diesel engine can be show as figure 1.

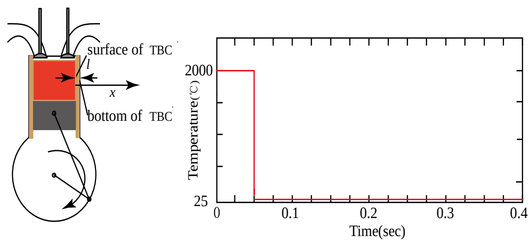

Fig.1. The structure of TBC and the thermal shock on it's surface

The TBC is sprayed onto the cylinder wall in older to improve thermal efficiency and service life of the engine. A first order approximation of Eq. (3) using Taylor series expansion is:

$$q+\tilde{\tau}_q\frac{\partial q}{\partial t}=-k\left[\frac{\partial T}{\partial x}+\tilde{\tau}_T\frac{\partial^2 T}{\partial t\partial x}\right] \tag{4}$$

The energy equation is:

$$\rho c\frac{\partial T}{\partial t}+\frac{\partial q}{\partial x}=0 \tag{5}$$

Substituting equation (4) into equations (3), one can obtain:

$$\rho c\tilde{\tau}_q\frac{\partial^2 T}{\partial t^2}=k\frac{\partial^2 T}{\partial x^2}+\tilde{\tau}_T k\frac{\partial^3 T}{\partial t\partial x^2}-\rho c\frac{\partial T}{\partial t} \tag{6}$$

The boundary conditions are:

At $x=0$:

$$T(0,\ t)=F(t) \tag{7}$$

At $x=l$:

$$k\frac{\partial T(l,\ t)}{\partial x}=0 \tag{8}$$

The initial condition is:

$$T(x,\ 0)=T_a\ ,\quad \frac{\partial T(x,0)}{\partial t}=0 \tag{9}$$

Applies dimensionless method to Eq. (6) one can obtain:

$$\bar{\tau}_q\frac{\partial^2\theta}{\partial\tau^2}=\frac{\partial^2\theta}{\partial\xi^2}+\bar{\tau}_T\frac{\partial^3\theta}{\partial\tau\partial\xi^2}-\frac{\partial\theta}{\partial\tau} \tag{10}$$

where:

$$\bar{\tau}_q=\frac{k\tilde{\tau}_q}{\rho cL^2},\ \bar{\tau}_T=\frac{k\tilde{\tau}_T}{\rho cL^2},\ \theta=\frac{T-T_a}{T_a},\ \xi=\frac{x}{L},\ \tau=\frac{kt}{\rho cL^2}$$

The corresponding dimensionless boundary conditions are:

At $\xi=0$:

$$\theta(0,\ \tau)=\frac{F(t)-T_a}{T_a}=f_1(\tau) \tag{11}$$

At $\xi=1$:

$$\frac{\partial\theta(1,\tau)}{\partial\xi}=0 \tag{12}$$

The corresponding dimensionless initial condition is:

$$\theta(\xi,\ 0)=0\ ,\quad\frac{\partial\theta(\xi,\ 0)}{\partial\tau}=0 \tag{13}$$

ANALYTICAL SOLUTION

Change of variable: obviously Eq.(10-13) are the second-order differential equations with non-homogeneous boundary conditions. An Expanded Separation of Variables Method will be employed to solve Eq.(10-13).

$$\theta(\xi,\tau)=v(\xi,\tau)+\sum_{i=1}^{2}g_i(\xi)f_i(\tau) \tag{14}$$

where i=1,2, $g_i(\xi)$ is shifting function, $v(\xi,\tau)$ is transferred function substituting Eq.(14) into eq.(10), one can obtain:

$$\bar{\tau}_q\frac{\partial^2 v}{\partial\tau^2}=\frac{\partial^2 v}{\partial\xi^2}+\bar{\tau}_T\frac{\partial^3 v}{\partial\tau\partial\xi^2}-\frac{\partial v}{\partial\tau}+\bar{q}_{ts} \tag{15a}$$

where:

$$\bar{q}_{ts}=\sum_{i=1}^{2}\left[\frac{d^2 g_i}{d\xi^2}f_i(\tau)-\bar{\tau}_q\frac{d^2 f_i}{d\tau^2}\right] \tag{15b}$$

If the boundary conditions of $v(\xi,\tau)$ satisfy eq. (16-17)
At $\xi=0$:

$$v(0,\tau)=0 \tag{16}$$

At $\xi=1$:

$$\frac{\partial v(1,\tau)}{\partial \xi}=0 \tag{17}$$

The corresponding initial condition is:

$$v(\xi,\tau)=-\sum_{i=1}^{2} g_i(\xi)\, f_i(0)$$
$$\frac{\partial v(\xi,\tau)}{\partial \tau}=-\sum_{i=1}^{2} g_i(\xi)\frac{df_i(0)}{d\tau} \tag{18}$$

The boundary condition must satisfy eq. (19-20)
At $\xi=0$:

$$\sum_{i=1}^{2} g_i(0)\, f_i(\tau)=f_1(\tau) \tag{19}$$

At $\xi=1$:

$$\frac{d\sum_{i=1}^{2} g_i(1)\, f_i(\tau)}{d\xi}=0 \tag{20}$$

Assuming:

$$g_1(\xi)=1 \tag{21}$$
$$g_2(\xi)=0 \tag{22}$$

Obviously eq. (21-22) satisfy eq. (19-20).
Solution of the transformed variable: the characteristic equation of Eq. (15) is

$$\overline{\tau}_q\frac{\partial^2 v}{\partial \tau^2}=\frac{\partial^2 v}{\partial \xi^2} \tag{23}$$

Assuming:

$$v(\xi,\tau)=Y(\xi)G(\tau) \tag{24}$$

Substituting Eq.(24) into Eq.(23), one can obtain Eq.(25-26)

$$\frac{d^2Y}{d\xi^2}+\lambda^2 Y=0 \tag{25}$$
$$\frac{d^2G}{d\tau^2}+\frac{\lambda^2}{\overline{\tau}_q}G=0 \tag{26}$$

Substituting Eq.(22) into Eq(14-15), one can obtain:

$$Y(0)=0 \tag{27}$$
$$\frac{dY(1)}{d\xi}=0 \tag{28}$$

Obviously Eq (25,27,28) are second-order differential equations. Solutions of Eq.(25) can be expressed as:

$$Y(x)=\begin{cases} Ae^{-\lambda\xi}+Be^{\lambda\xi},\lambda<0 \\ A+B\xi,\lambda=0 \\ A\cos\lambda\xi+B\sin\lambda\xi,\lambda>0 \end{cases} \tag{29}$$

Substituting the boundary conditions:
If $\lambda<0$, substituting $\xi=0$，$\xi=1$ into Eq,(27), one can obtain:

$$A+B=0 \tag{30}$$
$$\left(-\lambda Ae^{-\lambda}+B\lambda e^{\lambda}\right)=0 \tag{31}$$

According to Eq,(30,31), one can obtain: A=0，B=0
Obviously if $\lambda=0$, one can obtain A=0，B=0
If $\lambda>0$ substituting $\xi=0$ ，$\xi=1$ into Eq.(27) .

$$A\cos 0+B\sin 0=0 \tag{32}$$
$$-A\lambda\sin\lambda+B\lambda\cos\lambda=0 \tag{33}$$

According to Eq,(32,33), one can obtain:

$$B\lambda\cos\lambda=0 \tag{34}$$

So:

$$\cos\lambda=0 \tag{35}$$

So:

$$\lambda=\frac{2n-1}{2}\pi(n=1,2,3......) \tag{36}$$

So:

$$Y_n(\xi)=\sin\lambda_n\xi,\quad \lambda_n=\frac{2n-1}{2}\pi(n=1,2,3....) \tag{37}$$

Obviously Eq.(37) satisfy Eq.(38)

$$\int_0^1 Y_m Y_n d\xi=\begin{cases} 0 \quad \text{for} \quad m\neq n \\ \delta_n=\frac{1}{2} \quad \text{for} \quad m=n \end{cases} \tag{38}$$

So, $Y_n(\xi)$ satisfy the orthogonal condition. The mode superposition method can be used to derive the solution of the transformed. Assuming:

$$v(\xi,\tau)=\sum_{n=1}^{\infty} Y_n(\xi)B(\tau) \tag{39}$$

Substituting Eq. (39) into Eq. (15) and multiplying it by $Y_n(\xi)$, then integrating it from 0 to 1, one can obtain:

$$\overline{\tau}_q\frac{d^2B_m}{d\tau^2}+(\overline{\tau}_T\lambda_m^2+1)\frac{dB_m}{d\tau}+(\lambda_m^2)=\overline{q}_m \tag{40a}$$

where:

$$\overline{q}_m=\frac{1}{\delta_m}\int_0^1 \overline{q}_{ts}Y_m d\xi \tag{40b}$$

Further, the general solution of Eq. (40) is can be expressed as:

$$B_m(\tau)=V_{m1}(\tau)B_m(0)+V_{m2}(\tau)\frac{dB_m(0)}{d\tau} + \int_0^{\tau} V_{m2}(\tau-\chi)\overline{q}_m(\chi)d\chi \tag{41}$$

The fundamental solutions $\{V_{m1},\ V_{m2}\}$ of Eq. (40) can be expressed as:

Case1: If $\left(\overline{\tau}_T\lambda_m^2+1\right)^2-4\overline{\tau}_q\lambda_m^2>0$:

$$V_{m1}=\frac{1}{(\chi_{11}-\chi_{21})}\left[-\chi_{21}e^{-\chi_{11}\tau}+\chi_{11}e^{-\chi_{21}\tau}\right]$$
$$V_{m2}=\frac{1}{(\chi_{11}-\chi_{21})}\left[-e^{-\chi_{11}\tau}+e^{-\chi_{21}\tau}\right] \tag{42a}$$

Where:

$$\chi_{11}=\left[\frac{\left(\bar{\tau}_T\lambda_m^2+1\right)-\sqrt{\left(\bar{\tau}_T\lambda_m^2+1\right)^2-4\bar{\tau}_q\lambda_m^2}}{2\bar{\tau}_q}\right],\quad \chi_{21}=\left[\frac{\left(\bar{\tau}_T\lambda_m^2+1\right)+\sqrt{\left(\bar{\tau}_T\lambda_m^2+1\right)^2-4\bar{\tau}_q\lambda_m^2}}{2\bar{\tau}_q}\right]. \tag{42b}$$

Case2：If $\left(\bar{\tau}_T\lambda_m^2+1\right)^2-4\bar{\tau}_q\left(\lambda_m^2\right)<0$

$$V_{m1}=e^{-\chi_{12}\tau}\left[\cos\chi_{22}\tau+\frac{\chi_{12}}{\chi_{22}}\sin\chi_{22}\tau\right],\quad V_{m2}=\frac{1}{\chi_{22}}e^{-\chi_{12}\tau}\sin\chi_{22}\tau \tag{43a}$$

$$\chi_{12}=\frac{\left(\bar{\tau}_T\lambda_m^2+1\right)}{2\bar{\tau}_q},\ \chi_{22}=\sqrt{\frac{\lambda_m^2}{\bar{\tau}_q}-\left[\frac{\left(\bar{\tau}_T\lambda_m^2+1\right)}{2\bar{\tau}_q}\right]^2} \tag{43b}$$

The corresponding dimensionless initial condition is:

$$B(0)=\frac{1}{\delta_m}\int_0^1-\sum_{i=1}^{2}g_i(\xi)f_i(0)d\xi \tag{44a}$$

$$\frac{dB(0)}{d\tau}=\frac{1}{\delta_m}\int_0^1\frac{-d\sum_{i=1}^{2}g_i(\xi)f_i(0)}{\tau}Y(\xi)d\xi \tag{44b}$$

C-V model and Fourier model:

If $\bar{\tau}_T=0$, the DPL model reduce to the C-V model. The solution of the C-V model can be expressed as the same as the DPL model. If $\bar{\tau}_T=\bar{\tau}_q=0$ the DPL model reduce to the Fourier model. Eq.(40a) reduce to Eq.(45)

$$\frac{dB_m}{d\tau}+\lambda_m^2B_m=\bar{q}_m, \tag{45a}$$

$$\bar{q}_m=\frac{1}{\delta_m}\int_0^1\bar{q}_{ts}Y_m d\xi \tag{45b}$$

The solution of Eq.(45) can be expressed as:

$$B_m(\tau)=e^{-\lambda_m^2\tau}B_m(0)+e^{-\lambda_m^2\tau}\int_0^\tau e^{\lambda_m^2\tau}\bar{q}_m d\tau \tag{46}$$

NUMERICAL RESULT AND DISCUSSION

The TBC subjected to a thermal shock: as shown in fig.1. in older to study the thermal wave propagation process, a constant temperature is applied to the TBC surface. The constant temperature is used to emulate the thermal shock in power stroke. The thickness of the TBC is l=0.5mm. The density of the TBC is $\rho=5210(kg/m^3)$; The thermal conductivity of the TBC is $k=1.02(W/mK)$; The specific heat of TBCs is $c=502(J/KgK)$; The initial condition is $T_a=25\,^{O}C$.

Figure 2 shows the temperature variation of TBC due to the thermal shock with Fourier models. It is found that the temperature increase first and then decrease with the time. The smaller the value of x the faster the growth and reduction rate of temperature is. The temperature rises to 320℃ at bottom of TBC. It indicates that the thermal insulation effect of the TBC is good.

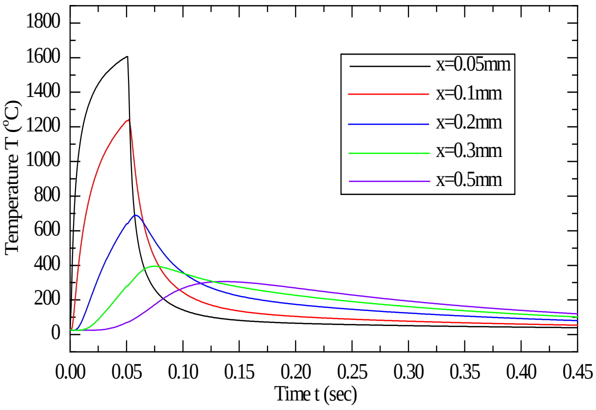

Fig.2. Temperature variation of TBC due to the thermal shock with Fourier models

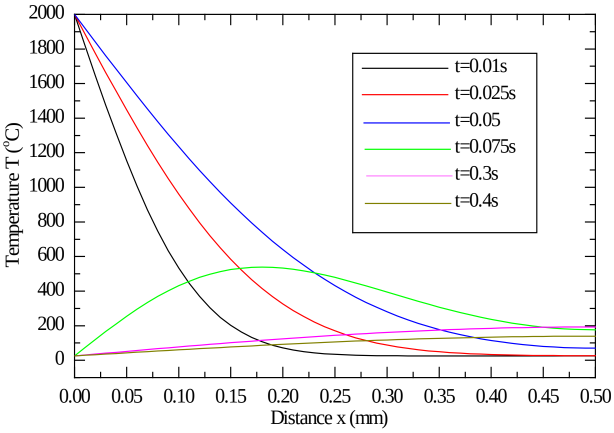

Fig.3. Temperature distribution in TBC due to the thermal shock with Fourier models

Figure 3 shows the temperature distribution in TBC due to the thermal shock with Fourier models. It is found that if $t\le 0.5$ the temperature decreases with the distance. The higher the value of t the slower the decrease of the temperature is. If $t>0.5$, because of cooling of nature air ,the temperature of TBCs surface fall to 25 ℃ . At $0<x\le 0.5$mm the temperature decreases with the time t.

Figure 4 shows the effect of τ_q on temperature variation in different location with C-V model. As can be seen from the fig.4., wave phenomena exist in thermal propagation process. According to fig.4. the wave speed can be presented in Table1. It is found that if the value of τ_q remains unchanged, the thermal wave travels at the same constant velocity. The smaller the value of τ_q the greater the speed of thermal wave propagation is. The wave speed can be calculated according to fig.4.as fellow:

$$C=x/t_r \tag{47}$$

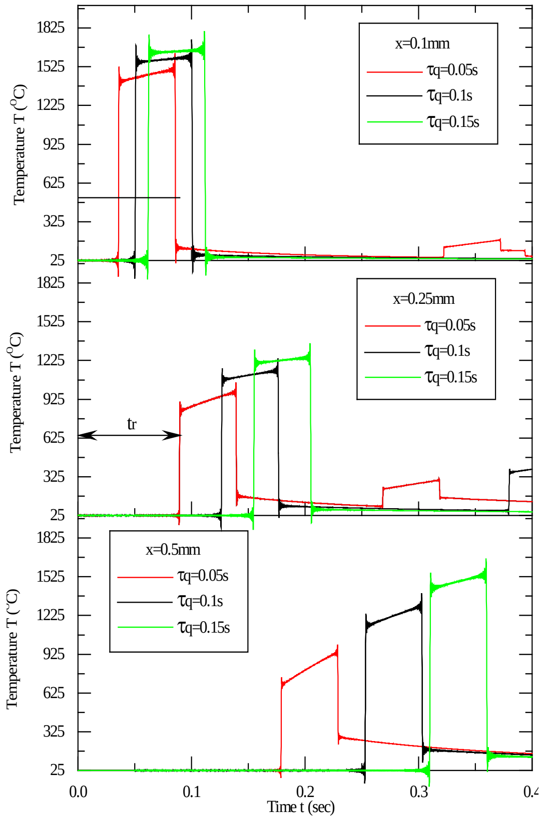

Fig. 4. The effect of τ_q on temperature variation in different location with C-V model

Table 1. Wave velocity at different positions with difference τ_q

τ_q (s)	X(mm)	t_r (s)	C(mm/s)
0.05	0.1	0.036	2.78
0.05	0.25	0.09	2.78
0.05	0.5	0.18	2.78
0.1	0.1	0.051	1.97
0.1	0.25	0.127	1.97
0.1	0.5	0.253	1.97
0.15	0.1	0.062	1.61
0.15	0.25	0.155	1.61
0.15	0.5	0.31	1.61

Figure 5 shows the effect of τ_q on temperature distribution in TBC with C-V model. The thermal propagate in the form of wave. Transmitted and reflect wave can be observed. If $\tau_q = 0.05\text{s}$ the thermal wave speed can be calculated as follow:

$$C = l_1/(\Delta t) = (0.278-0.139)/(0.15-0.1) = 2.78\text{mm}/\text{s} \quad (48)$$

$$C = (l_2 + l_3)/(\Delta t) = (0.5-0.278+0.5-0.444)/(0.25-0.15) = 2.78\text{mm}/\text{s} \quad (49)$$

By using the same method, we can obtain: if $\tau_q = 0.1\text{s}$ $C = 1.97\text{mm}/\text{s}$, $\tau_q = 0.15\text{s}$ $C = 1.61\text{mm}/\text{s}$ .It is found that wave speed calculated from fig.5. is the same as fig.4..

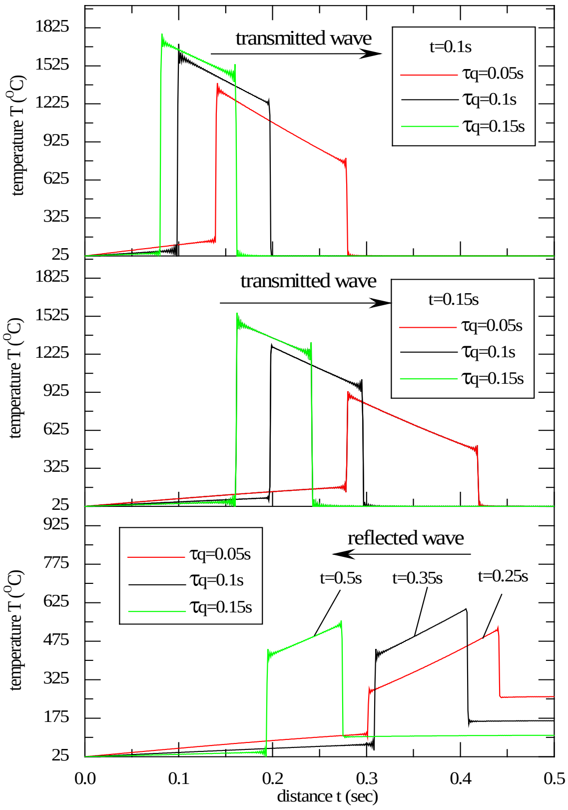

Fig. 5. Effect of τ_q on temperature distribution in TBC with C-V model

Figure 6 shows the effect of τ_T on temperature variation in different location with DPL model. Figure 7 shows effect of τ_T on temperature distribution in TBC with DPL model. We can also observe wave phenomenon in fig.6. and fig.7. During the propagation and reflection stages, the thermal wave decreases constantly.

That may be caused by damping effect of the thermal diffusivity. If the value of τ_q remains unchanged, the amplitude of the thermal wave decreases with increasing the value of τ_T. But with of the thermal wave increases with increasing the value of τ_T. The thermal wave with different value of τ_T travel together as shown in fig.6. and fig.7.So we can we can infer that the speed of the thermal wave with different value of τ_T is the same.

TBC subjected to continuous but unstable heating:

During 4-stroke engine working process, the temperature of TBC surface is unstable. Figure 8 shows the temperature variation at TBC surface in a working process. The corresponding boundary condition at the TBC surface can be presented as Eq.(50).

$$T(0,t)=\begin{cases}25 & 0\le t\le 0.05\text{s}\\ 25+500\sin\left[10\pi\left(t-0.05\right)\right] & 0.05<t\le 0.10\text{s}\\ 1800\cos\left[6\pi\left(t-0.1\right)\right] & 0.10<t\le 0.15\text{s}\\ 800 & 0.15<t\le 0.20\text{s}\end{cases} \tag{50}$$

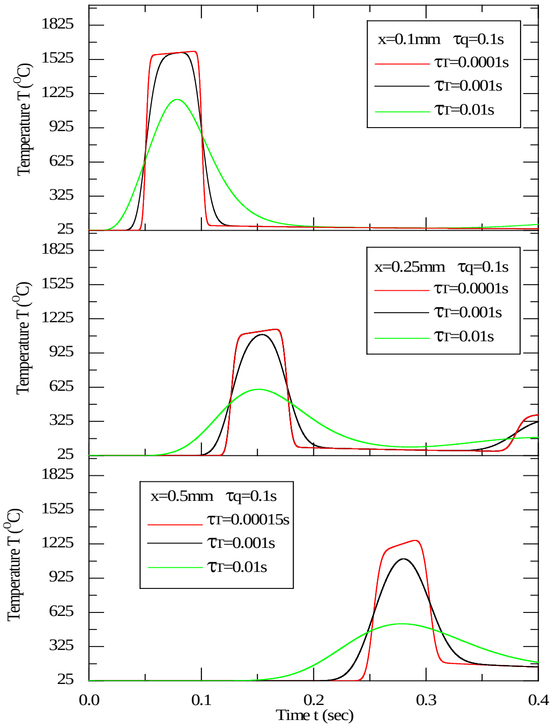

Fig.6. The effect of τ_T on temperature variation in different location with DPL model

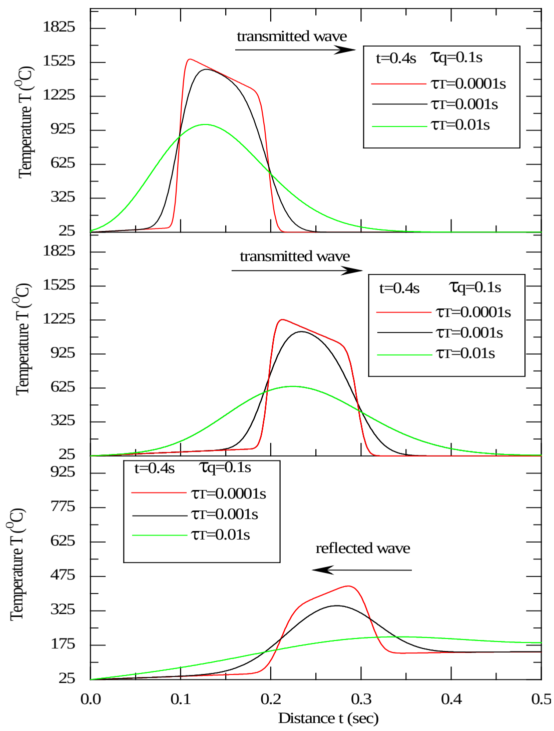

Fig.7. Effect of τ_T on temperature distribution in TBC with DPL model

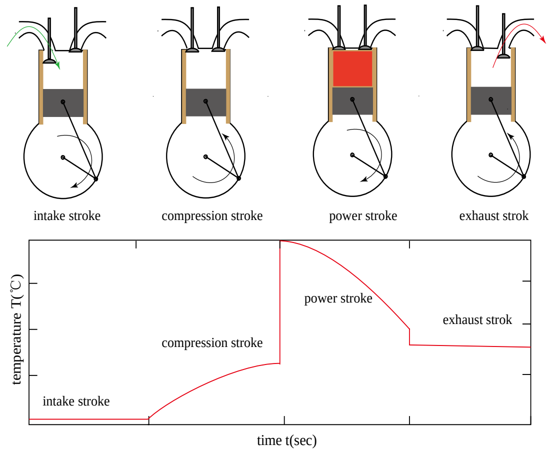

Fig.8. Temperature variation at TBC surface in a working process

Figure 9, figure 10, and figure 11 show the temperature variation in different location during 10 second period in the Fourier model, C-V model and DPL model, respectively. It is found that at the beginning of working process the average temperature will increase gradually and reach steady temperature fluctuation finally. In Fourier model the lager the value of x the smaller the range of temperature fluctuation is. This conclusion is also suitable for C-V and DPL model. In C-V model the lager the value of τ_q the lager range of temperature fluctuation is. In DPL model the lager the value of τ_q and τ_T the smaller the range of temperature fluctuation.

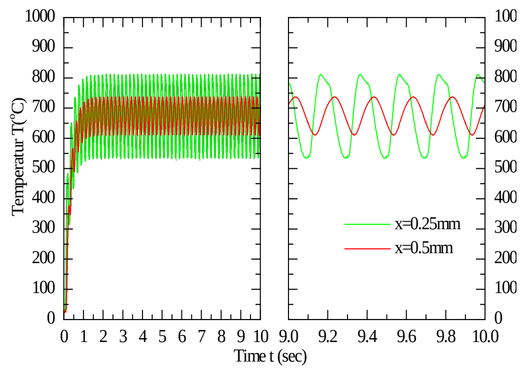

Fig.9. Temperature variation in different location during 10 second period in the Fourier mode

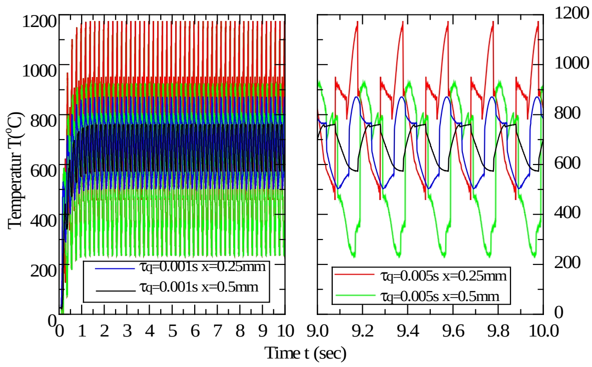

Fig.10. Temperature variation in different location during 10 second period in the C-V model

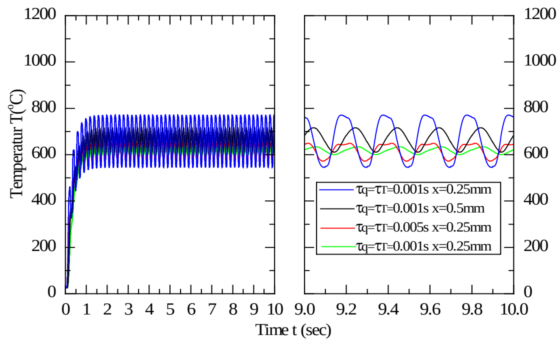

Fig.11. Temperature variation in different location during 10 second period in the DPL mode

CONCLUSIONS

In this work, the heat transfer in TBC is investigated. The analytical solutions of three models (the Fourier model, the C-V model and Dual-phase-lag model) are obtained. All of the conclusions are as follows:

(1) The results of the three models varied wildly.

(2) Wave phenomena exist in thermal propagation process in C-V model and DPL model.

(3) In C-V model, if the value of τ_q remains unchanged, the thermal wave travels at the same constant velocity. The smaller the value of τ_q the greater the speed of thermal wave propagation is.

(4) In DPL model, the amplitude of the thermal wave decreases with increasing the value of τ_T. The thermal wave speed has nothing to do with the value of τ_T.

(5) After running a few times, the temperature of TBC fluctuate within a certain range. The lager the value of x the smaller the range of temperature fluctuation is.

(6) In DPL model the lager the value of τ_q and τ_T the smaller the range of temperature fluctuation.

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REFERENCES

Afrin, N, Y.W Zhang, and J.K Chen. “Thermal Lagging in Living Biological Tissue Based on Nonequilibrium Heat Transfer between Tissue, Arterial and Venous Bloods”, International Journal of Heat and Mass Transfer Vol. 54, No. 11, pp.2419-2426, 2011.

Akwaboa, Stephen, Patrick Mensah, and Yusuf Dag. “Study of Heat Transfer in a Thermal Barrier Coatings System Using the Dual Phase Lag Model Based on Mean Value Finite Volume Method”, ASME 2013 International Mechanical Engineering Congress and Exposition, 2013.

Cattaneo, C. “A Form of Heat Conduction Equation Which Eliminates the Paradox of Instantaneous Propagation”, Compte Rendus Vol. 247, No. 4, pp.431-433, 1958.

Frankel, J. I., Brian Vick, and M. N. Özisik. “General Formulation and Analysis of Hyperbolic Heat Conduction in Composite Media”, International Journal of Heat & Mass Transfer Vol. 30, No. 7, pp.1293-1305, 1987.

Golosnoy, I. O., A. Cipitria, and T. W. Clyne. “Heat Transfer through Plasma-Sprayed Thermal Barrier Coatings in Gas Turbines: A Review of Recent Work”, Journal of Thermal Spray Technology Vol. 18, No. 5-6, pp.809-821, 2009.

Kumar, P, D Kumar, and K.N Rai. “A Numerical Study on Dual-Phase-Lag Model of Bio-Heat Transfer During Hyperthermia Treatment”, Journal of Thermal Biology Vol. 49, pp.98-105, 2015.

Kumar, Sumit, and Atul Srivastava. “Thermal Analysis of Laser-Irradiated Tissue Phantoms Using Dual Phase Lag Model Coupled with Transient Radiative Transfer Equation”, International Journal of Heat and Mass Transfer Vol. 90, pp.466-479, 2015.

Li, Ling, Ling Zhou, and Mo Yang. “An Expanded Lattice Boltzmann Method for Dual Phase Lag Model”, International Journal of Heat And Mass Transfer Vol. 93, pp.834-838, 2016.

Lin, Shueei.Muh. “Analytical Solutions of Bio-Heat Conduction on Skin in Fourier and Non-Fourier Models”, Journal of mechanics in medicine and biology Vol. 13, No. 04, 1350063, 2013.

Liu, Kuo.Chi, Yao.Nan Wang, and Yuen.Shin Chen. “Investigation on the Bio-Heat Transfer with the Dual-Phase-Lag Effect”, International Journal of Thermal Sciences Vol. 58, pp.29-35, 2012.

Ramadan, K. “Semi-Analytical Solutions for the Dual Phase Lag Heat Conduction in Multilayered Media”, International Journal of Thermal Sciences Vol. 48, No. 1, pp.14-25, 2009.

Taitel, Yehuda. “On the Parabolic, Hyperbolic and Discrete Formulation of the Heat Conduction Equation”, International Journal of Heat and Mass Transfer Vol. 15, No. 2, pp.369-371, 1972.

Tzou, D. Y. “A Unified Field Approach for Heat Conduction from Macro- to Micro-Scales”, Journal of heat transfer Vol. 117, No. 1, pp.8-16, 1995.

Vernotte, Pierre. “Les Paradoxes De La Théorie

Continue De Léquation De La Chaleur", Comptes Rendus Hebdomadaires Des Seances De L Academie Des Sciences Vol. 246, No. 22, pp.3154-3155, 1958.

Wang, L., D. C. Li, J. S. Yang, F. Shao, X. H. Zhong, H. Y. Zhao, K. Yang, S. Y. Tao, and Y. Wang. "Modeling of Thermal Properties and Failure of Thermal Barrier Coatings with the Use of Finite Element Methods: A Review", Journal of The European Ceramic Society Vol. 36, No. 6, pp.1313-1331, 2016.

Xu, F, K.A Seffen, and T.J Lu. "Non-Fourier Analysis of Skin Biothermomechanics", International Journal of Heat and Mass Transfer Vol. 51, No. 9, pp.2237-2259, 2008.

Yang, Yu-Ching, Wen-Lih Chen, Tzu-Sheng Yeh, Win-Jin Chang, and Haw-Long Lee. "Numerical Study of Dual-Phase-Lag Heat Conduction in a Thermal Barrier Coating with a Hybrid Method", Journal of the Brazilian Society of Mechanical Sciences and Engineering Vol. 38, No. 1, pp.287-296, 2015.

Zhang, Xiaomin, Long Zhang, and Zhongxiang Chu. "Thermomechanical Coupling of Non-Fourier Heat Conduction with the C-V Model: Thermal Propagation in Coating Systems", Journal of Thermal Stresses Vol. 38, No. 10, pp.1104-1117, 2015.

NOMENCLATURE

c specific heat of TBC

k thermal conductivity of TBC

l thickness of TBC

t time variable

τ dimensionless time, $kt / \rho cl^2$

x position coordinate

ξ dimensionless position coordinate, x / l

τ_q phase lag for heat flux

$\overline{\tau}_q$ dimensionless phase lag for heat flux, $k\tau_q / \rho cl^2$

τ_T phase lag for temperature gradient

$\overline{\tau}_T$ dimensionless phase lag for temperature gradient, $k\tau_T / \rho cl^2$

T temperature of TBC

T_a the initial temperature of TBC

高效引擎熱障塗層傳裡葉和非傳裡葉熱傳導的解析解

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摘 要

本文利用傳裡葉模型，C-V 模型和雙向滯(DPL)模型研究了熱在某四衝程高效柴油機熱障塗層（TBC）中的傳導過程。採用一種拓展的分離變數法求解了三種模型的解析解。利用求得的解析解研究了兩種典型的情況：a). TBC 受到熱衝擊；b). TBC 受到持續不穩定加熱。發現在 C-V 模型和 DPL 模型中熱傳導具有波動現象。討論了幾個參數對熱波速度的影響。發現在高效柴油機工作工程中熱障塗層底部溫度呈波動狀態。討論了幾個參數對溫度波動範圍的影響。本文的解析方法也可運用與邊界條件不斷變化的普通傳熱問題。