中國機械工程學刊第四十卷第四期第 423~429 頁(民國一百零八年)
Journal of the Chinese Society of Mechanical Engineers, Vol.40, No.4, pp 423~429 (2019)

Bifurcation Analysis of a Quarter Car Vehicle System

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Keywords: quasi-periodic, nonlinear hysteretic damping, frequency response, bifurcation

ABSTRACT

A systematic analysis of the effects of non-linear, hysteretic, tire damping and stiffness forces, on the nonlinear, dynamic response of a two degrees of freedom, quarter-car suspension system. The dynamics of the system were evaluated through analysis of dynamic trajectories, power spectra, Poincare maps, bifurcation maps and frequency responses, respectively. The quasi-periodic solution for the twisting of the natural frequency was detected, and the coexistence of quasi-periodic and periodic solutions, due to the fold bifurcation, was also observed. Two resonances were found in the frequency response. The first resonance is the main resonance due to the lower natural frequency, and the second resonance is a subharmonic resonance of the higher natural frequency. The tire damping, combined with excitation amplitude and frequency, strongly impacted the dynamic characteristics. The results presented in this study provide a better understanding of the operating conditions under which undesirable dynamic motion takes place in a quarter-car system, and would therefore serve as a useful source of reference for engineers interested in designing and controlling such systems to extend the expected lives of automobiles.

INTRODUCTION

The dynamic characteristics of a vehicle's suspension system are among the most important and basic performance measurements of automobiles(Sharp et al.,1987). reviewed the information relating to the design of automobile suspension systems for ride, comfort and control of wheel load variations, for frequencies below body structure resonances. Many types of semi-active, electrorheological (ER) or magnetorheological (MR)dampers have been proposed for vibration attenuation of various dynamic systems, including vehicle suspensions. It has been demonstrated through experimentation, that unwanted vibrations of application systems can be effectively controlled by employing ER or MR dampers with appropriate control systems(Choi et al.,2001;Kwok et al.,2006;Spencer et al.,1997;Wereley et al.,1998;Sims et al.,2000;Andrzejewski et al.,2005) .

Many studies have been performed to understand the dynamic characteristics of vehicular suspension systems, especially nonlinear dynamic behaviors with load excitation. (Zhu et al.,2004) assume the suspension spring to be a nonlinear spring and nonlinear damper, and the chaotic response may appear as the vehicle moves over a bumpy road. Chaotic vibration of a quarter-vehicle model is investigated numerically in Ref.(Jiang et al.,2008). (Boada et al.,2005) present a neural controller for the control of a semi-active suspension system, in which the suspension system and tire are modelled as a linearity. (Zhou et al.,2016) have investigated the influence of nonlinear damping on the dynamic characteristics by the incremental harmonic balance method and Newmark method. Next, (Silveira et al.,2017) adopted the asymmetrical viscous damping to study the dynamic behaviour of vehavior of vehicle suspension system.

(Ahmadian et al.,2007) extended the passive and active suspensions work of (Chalasani et al.,1986;1986) to semi-active suspensions, using an hybrid control system. A frequency response analysis showed that, for both the passive and the hybrid configurations, the heave motion of the full vehicle model was very similar to the motion of the quarter car model used in the companion article. This indicates that the conclusions made with the quarter car model approximation for periodic inputs, are still valid for a typical full vehicle suspension system.

(Litak et al.,2008;2008) investigated possible chaotic motion in a nonlinear vehicle suspension system which was subjected to multi-frequency excitation from a road surface. They also investigated

Paper Received August, 2017. Revised March, 2018. Accepted July, 2018. Author for Correspondence: Lin-Jun Tong

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global, homoclinic, bifurcation and transition to chaos, in the case of a single-degree-of-freedom (SDOF), quarter-car model, excited kinematically by a road surface profile. (Litak et al.,2009) analyzed the motion of an SDOF quarter car model when the excitation consisted of an harmonic term with a small random noise component. Note that nonlinear hysteretic suspension damping and stiffness are adopted in their analyses.

(Naik et al.,2009) examined chaotic motion in a vehicle suspension system with hysteretic nonlinearity excited by a road profile. The quarter-car model with hysteretic nonlinear damping was also reduced into an SDOF system. (WeiWei et al.,2011) investigated the probability of vehicle suspension systems having occurred chaos and disturbed state character of closed-trajectories near the center, through the Melnikov function and Kolmogorov-Arnold-Moser (KAM) theory. (Siewe,2010) investigated the resonance, stability and period-doubling bifurcation of a quarter-car model, excited by the road surface profile, with non-symmetric potential, using the method of multiple-time scales.

(Palkovics et al.,1992) found that periodically exciting the system resulted in a chaotic behavior. In the more realistic case of a quarter car model with the introduction of a tire model extension, the model is able to describe the vibration of the system when the wheel-hop occurs. (Li et al.,2004) presented the investigation of possible chaotic motion in a vehicle suspension system with hysteretic nonlinearity, which was subjected to multi-frequency excitation from the road surface. (Dai et al.,2004) investigated the motion and stability of a four-wheel-steering vehicle, with the nonlinearity of the lateral tire forces, and the effects of the steering mechanism were considered as well. A new nonlinear model was developed for the steady state motion of the vehicle. The stability of a typical steady state motion generated by the model was also presented with the application of the bifurcation method.

(Brezas et al.,2015) proposed a time-domain optimal control approach to optimize ride and handing behaviour simualtaneously in an experimental situation. (Xie et al.,2015) developed a dual-mass piezoelectric bar harvester for energy harvesting from ambient vibrations of a vehicle suspension system subjected to roughness of road surface.

(Borowiec et al.,2010) experimentally studied the effects on a suspension system of three types of road surfaces: (a) asphalt, (b) sett, and (c) railway cross. They revealed that the effects of tires were of less importance, as the pneumatic tire structure can be treated as a linear system, with the vibration amplitude response is proportional to the excitation force. Moreover, (Soliman et al.,2001) investigated the effect of the suspension spring stiffness on vehicle dynamics, and the effect of using active suspension elements to obtain an improved ride. Their research indicated that increasing the tire damping leads to a decrease in the International Organization for Standardization (ISO) weighted body acceleration and the dynamic tire load. However, the changes in the values of tire damping parameters have a small effect on the ride comfort of the vehicle performance in a vertical direction. The tire damping had a very small effect of only 1-4 % on vehicle ride comfort, which allowed it to be disregarded. When the tire stiffness parameter is increased, the systems have higher wheel resonance peaks in the dynamic tire load response. The predicted values of the suspension working space and vertical acceleration were 8-10 % lower than the actual measurements. The peak resonance measured for body acceleration and suspension working space, at a specific frequency, was similar to that obtained theoretically.

In our previous work(Tong et al.,2013), a new hysteresis contact force model is proposed to model the contact characteristics of tire and road surface. The main motivation of the present paper is to study the effects of tire damping on the nonlinear characteristics of the two degrees of freedom, quarter-car system twist, by the two natural frequencies. The quarter-car model of (Samandari et al.,2010) was used and the quarter-car system parameters in Ref's.(Litak et al.,2008; Samandari et al.,2010) were adopted in order to compare the results numerically. The quasi-periodic solution, twisted by the two natural frequencies, the jump of frequency response and the multiple coexisting solutions were detected.

THE QUARTER-CAR MODEL

In this paper, we considered the two degrees of freedom quarter car as shown in Fig.1

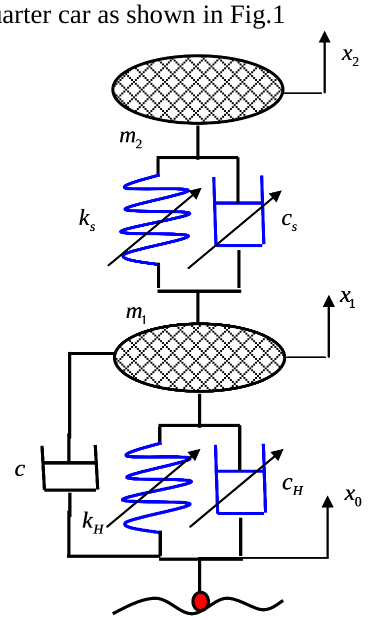

Fig. 1 Quarter-car model

The nonlinear hysteretic damping and stiffness forces are adopted in the mathematical model. The relationship between the tire force and the displacement in the tire is expressed as:

$$F_{us} = k_H^1 \left(x_1 - x_0\right) + k_H^3 \left(x_1 - x_0\right)^3 \qquad (1)$$

where k_H^1 and k_H^3 are the nonlinear stiffness coefficients, x_1 is the displacement of unsprung mass and x_0 is the road excitation, which is assumed to be a sinusoidal function as:

$$x_0 = A \sin 2\pi\omega t \qquad (2)$$

where A is the amplitude of the road surface roughness, $\omega = v_0/\lambda$, v_0 is the vehicle speed and λ is the wave length of the harmonic wave.

For modeling the suspension system, a softening stiffness was adopted. The stiffness force of the suspension system was then expressed as(Naik et al.,2009) :

$$F_{k_s} = k_s^1 \left(x_2 - x_1\right) - k_s^3 \left(x_2 - x_1\right)^3, k_1^3 > 0 \quad (3)$$

where k_s^1 and k_s^3 are the stiffness coefficient and x_2 is the displacement of sprung mass. A magnetorheological damper is generally used in the vehicle suspension, so the hysteretic behavior of the damper can be modeled as(Naik et al.,2009) :

$$F_{c_s} = -c_s^1 \frac{d\left(x_2 - x_1\right)}{dt} + c_s^3 \left[\frac{d\left(x_2 - x_1\right)}{dt}\right]^3 \qquad (4)$$

where c_s^1 and c_s^3 are the damping coefficients. The damping of the tire is assumed to be viscous as:

$$F_{c_H} = c_H \frac{d\left(x_1 - x_0\right)}{dt} \qquad (5)$$

Recalling the vehicle model shown in Fig.1, the equation of quarter-car system can be represented

$$m_2 \frac{d^2 x_2}{dt^2} = -F_{k_s} - m_2 g - F_{c_s} \qquad (6)$$

$$m_1 \frac{d^2 x_1}{dt^2} = F_{k_s} + F_{c_s} - m_1 g - F_{k_H} - F_{c_H} \qquad ($$

Substitution of Eq. (1-5) into Eqs. (6) and (7), obtains:

$$\begin{aligned} & m_2\ddot{x}_2 - c_s^1\left(\dot{x}_2 - \dot{x}_1\right) + k_s^1\left(x_2 - x_1\right) - k_s^3\left(x_2 - x_1\right)^3 + c_s^3\left(\dot{x}_2 - \dot{x}_1\right)^3 = -m_2 g \\ & m_1\ddot{x}_1 + c_H\left(\dot{x}_1 - \dot{x}_0\right) + c_s^1\left(\dot{x}_2 - \dot{x}_1\right) + k_H^1\left(x_1 - x_0\right) - k_s^1\left(x_2 - x_1\right) \\ & \qquad + k_H^3\left(x_1 - x_0\right)^3 + k_s^3\left(x_2 - x_1\right)^3 - c_s^3\left(\dot{x}_2 - \dot{x}_1\right)^3 = -m_1 g \end{aligned} \qquad (8)$$

NUMERICAL ANALYSIS

The main motivation of the work presented here, is to investigate the effect of tire damping on the nonlinear bifurcation characteristics of the quarter-car model. The suspension system parameters of the quarter-car model were chosen to be similar to those used in Ref's ((Litak et al.,2007;2008; Tong et al.,2013), in order to compare the results to the single degree of freedom model, as listed in Table 1.

Table 1 System parameters of the quarter-car model

Parameters	Symbols	Units	Values
Sprung mass	m_2	Kg	250
Unsprung mass	m_1	Kg	40
Sprung mass spring coefficient	k_s^1	N/m	200,000
	k_s^3	N/m3	300,000
Sprung mass damping coefficient	c_s^1	Ns/m	250
	c_s^3	Ns3/m3	25
Unsprung mass spring coefficient	k_H^1	N/m	800,000
	k_H^3	N/m3	160,000
Unsprung mass damping coefficient	c_H	Ns/m	400

Before the numerical analysis is made, the approximate natural frequencies of the corresponding two degrees of freedom system (8) were calculated, when disregarding the nonlinear component, as:

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \mathbf{K}\mathbf{X} = 0 \qquad (9)$$

Where

$$\mathbf{M} = \begin{bmatrix} m_2 & 0 \\ 0 & m_1 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} -c_s^1 & c_s^1 \\ c_s^1 & c_H - c_s^1 \end{bmatrix}, \mathbf{K} = \begin{bmatrix} k_s^1 & -k_s^1 \\ -k_s^1 & k_H^1 + k_s^1 \end{bmatrix}$$

The two approximate natural frequencies are $4.0132\,\mathbf{Hz}$ and $25.2471\,\mathbf{Hz}$ and the frequency response is shown Fig. 2.

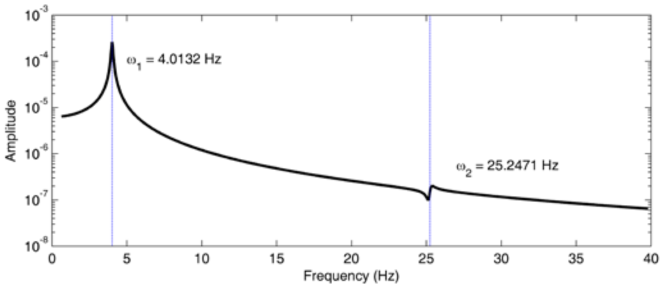

Fig. 2 Frequency response of system (9)

Recalling the nonlinear suspension system of quarter-car model, the dynamic response of the system is studied numerically utilizing the Runge-Kutta algorithm. To study the nonlinear dynamic characteristics of the vehicle under road excitation, the bifurcation map, poincare map, frequency spectrum, and frequency response curve were used to analyze the effects of system damping and road excitation. It should be noted that the final steady-state response at a particular input parameter is used as the initial condition for the next parameter in the numerical calculation. One defines the forward strategy when th parameter is increasing, whereas it is backward. Th frequency response curve, as equivalent root-mear square (rms) is defined as:

$$A^{rms} = \sqrt{\sum_{i=1}^{N} A_i^2} \qquad (10)$$

Here A_i is the r**th** harmonic amplitude of $\mathbf{x}(t)$

Effect of excitation frequency

Fig. 3 shows the forward (a) and backward (b) bifurcation map for the suspension mass according to the parameters given in Table 1, assuming that the roa excitation amplitude is $f = 0.2m$, and th excitation frequency varies from 2 to 16 Hz. As see from these figures, the system underwent quas periodic motion when road excitation frequency wa lower than $\omega \approx 4.95\mathbf{Hz}$ in the forward bifurcatio map, and/or $\omega \approx 3.51\mathbf{Hz}$ in the backwar bifurcation map. On the other hand, multipl coexisting stable solutions are possible in the range c 3.51 to 4.95 Hz. For example, the two kinds c multiple, coexisting stable solutions were detected a $\omega = 4.00\mathbf{Hz}$ as shown in Fig. 4 and Fig. 5

In Fig. 4, the phase portrait and corresponding spectra of the coexisting quasi-periodic solution are illustrated when the initial values were at -0.2298, 6.0892, 0.0176 and -4.1816. The poincare map is a closed curve, as shown in Fig. 4 (a) by the red cycle. The road excitation excited the two natural modals of the quarter-car model, and the system response was quasi-periodic for the ratios ω_1/ω_2 of the two natural frequencies, which were irrational. The peaks at the two natural frequencies were obviously detected. When the initial value settings were -0.4737, -2.2901, -0.0728 and 5.1517, the period-1 solution as observed as shown in Fig. 5 However, the two coexisting stable solutions were dominated by the lower natural frequency of ω_1.

Moreover, as the excitation frequency increases, the period-1 solution will diverge into a quasi-periodic and chaotic solution in the range of 10-14 Hz. Fig. 6 shows the frequency responses, where (a) is equivalent root-mean-square x_2^{rms} and (b) is the mean values x_2^{mean} of the suspension mass. Two resonance regions were observed at approximately $\omega \approx \omega_1$ and $\omega \approx \omega_2/2$. The first resonance is the main resonance due to the lower natural frequency, and the second resonance is a subharmonic resonance of the higher natural frequency. The multiple coexisting stable solution appears in the frequency response as a jump phenomena as shown in Fig. 6.

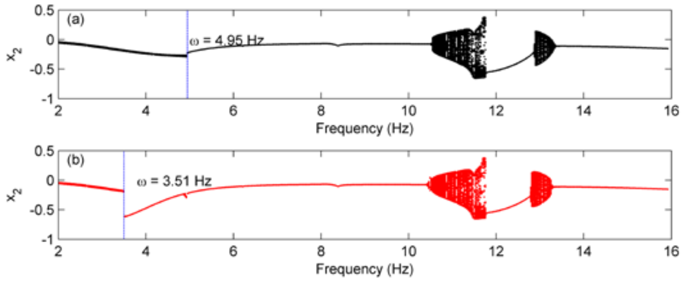

Fig. 3 Bifurcation map with respect to excitation frequencies for $f = 0.2m$, $c_h = 100\ Ns/m$.
(a) Forward bifurcation (b) Backward bifurcation.

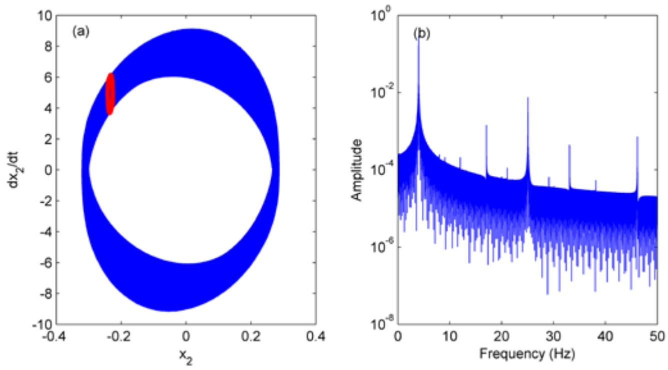

Fig. 4 Coexisting stable solution
Quasi-periodic solution for $\omega = 4\mathbf{Hz}$, (a) is the phase portrait (red cycle is the Poincare map), (b) is the frequency spectrum when initial values are -0.2298, 6.0892, 0.0176 and -4.1816.

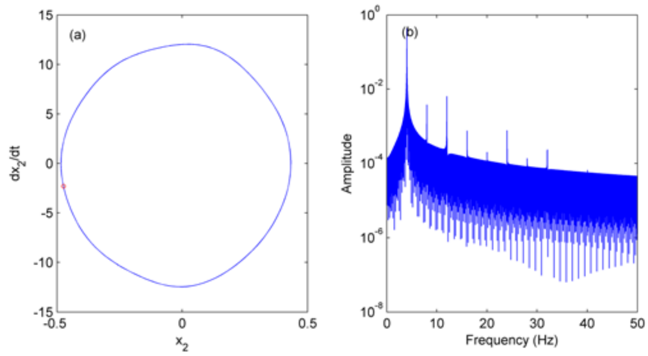

Fig. 5 Coexisting stable solution
Period-1 solution for $\omega = 4\mathbf{Hz}$, (a) is the phase portrait (red cycle is the Poincare map), (b) is the frequency spectrum when initial values are -0.4737, -2.2901, -0.0728 and 5.1517.

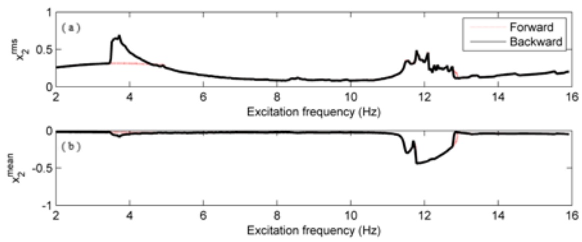

Fig. 6 Frequency response with respect to

excitation frequency in Hz for: $f = 0.2m$, $c_h = 100\, Ns/m$.(a) is the equivalent root-mean-square and (b) is the mean values.

Effect of excitation amplitude

To illustrate the effect of the excitation amplitude on the system dynamics, numerical simulations were performed as a previous subsection. Fig. 7 shows the frequency response for different excitation amplitudes, of $f = 0.1m$, $f = 0.2m$ and $f = 0.3m$, and with the excitation frequency varying from 2 to 16 Hz. The forward and backward bifurcations are abbreviated as F and B in the legends, respectively. The frequency response curves show that the excitation amplitude will excite the main resonance around 4 Hz and the rms values x_2^{rms} increase linearly with increasing amplitude excitation. Moreover, it is obvious that for low levels of road excitation amplitude, such as $f = 0.1m$, the jump phenomena is not detected, in all the other excitation frequencies considered in the range. The peak appears at a subharmonic resonance range of the higher natural frequency. To draw comparisons, in Ref.(Tong et al.,2013), the critical road profile amplitude (0.31m) for the suspension system was defined when the frequency response was sensitive to the initial condition. However, in the present work, the critical road amplitude may be 0.2m based on that definition. The conflicting conclusion can be understood by considering the following three points. First, the nonlinear hysteretic damping and stiffness force model, adopted in the suspension model, may lead to fold bifurcation, where one stable solution will break into two stable solutions and one unstable solution, which cannot be detected based on the previous numerical analysis algorithm. Mathematically, the fold bifurcation is the reason for the multiple, coexistin stable solutions in the suspension system, or sensitivity to the initial values. Second, in Ref.(Li et al.,2008), Litak explained the multiple coexisti solutions for the discontinuity, signaling jur between the resonant and non-resonant vibratio which may be different from the results presented our work. Third, the optimization of the suspensi system must take into account the nonlin bifurcation characteristics. When the excitati amplitude was $f = 0.3m$, the displacement of the sprung mass increased rapidly, and the system loses stability near subharmonic resonance.

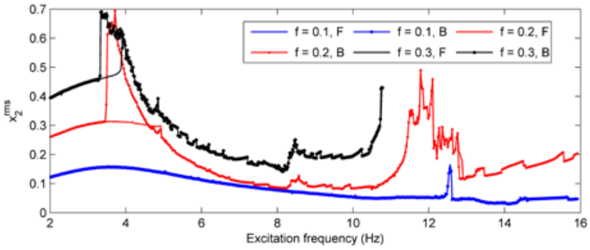

Fig. 7 Frequency response for different excitation amplitudes of f . The forward and backward bifurcations are denoted as F and B in the legend, respectively.

Effect of tire damping

Fig. 8 shows the frequency responses for different tire damping when $f = 0.2m$. The frequency response curves show that, in the subharmonic resonance region, the tire damping influence was slight, but the damper did not absorb the vibration excited by the road roughness in the main resonance region. It is obvious that the vibration amplitude increases as the tire damping increases, and the jump phenomena is detected just as the tire damping reaches $c_h = 100\, Ns/m$, in the main resonance range. However, when the excitation amplitude decreases to $f = 0.1m$, the system appears to behave according to different rules, as seen by comparing to the case when $f = 0.2m$, as shown in Fig. 9. When neglecting tire damping, or $c_h = 0\, Ns/m$, the frequency response is influenced linearly and decreases with decreasing road excitation amplitude. When $c_h = 100\, Ns/m$ and $c_h = 200\, Ns/m$, the damping increases the vibration amplitude in the main resonance range, but one can obtain the desired results of reducing vibration in the subharmonic range.

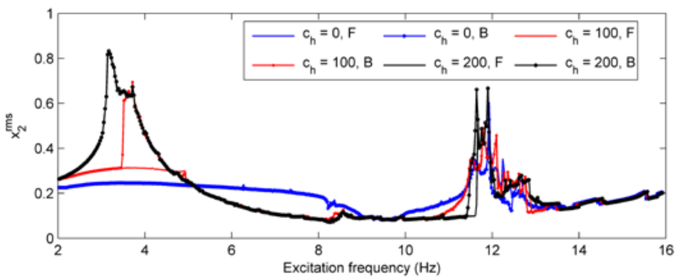

Fig. 8 Frequency response for different tire damping, and an excitation amplitude of $f = 0.2$.

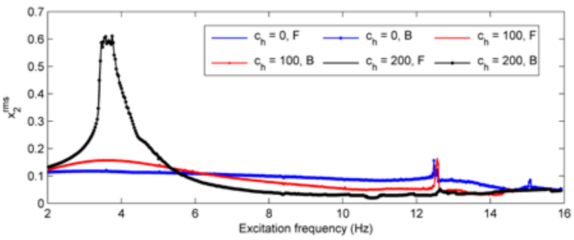

Fig. 9 Frequency response for different tire damping and an excitation amplitude of $f = 0.1$.

CONCLUSIONS

This study presented a numerical analysis of a two degrees of freedom, quarter-car suspension system subjected to nonlinear hysteretic damping and stiffness forces from tire damping. The dynamics of the system are evaluated by reference to its dynamic trajectories, power spectra, Poincare maps, bifurcation maps and frequency responses, respectively. The analysis has investigated the dynamic response of quarter-car suspension system as a function of road excitation amplitude, frequency and the tire damping coefficient. The numerical simulation results have shown that (i) the quasi-periodic solution for the twisting of the natural frequency was detected and the coexisting quasi-periodic and periodic stable solutions were also observed in the present model. (ii) Two resonances were found in the frequency response. The first resonance is the main resonance due to the lower natural frequency, and the second resonance is a subharmonic resonance of the higher natural frequency. The multiple coexisting stable solutions appear in the frequency response as jump phenomena. (iii) The tire damping strongly impacts the dynamic characteristics. The vibration amplitude increases with increased tire damping, and the jump phenomena is detected at median tire damping, in the main resonance range, with large excitation amplitudes. (iv) When the excitation amplitude is small, the damping increases the vibration amplitude in the main resonance range, but the desired results of reducing vibration in the subharmonic range were obtained.

Overall, the results presented in this study provide a detailed understanding of the effects of tire damping on the nonlinear dynamic response of a two degrees of freedom, quarter-car system. Specifically, the results enable suitable values for the damping coefficient, the stiffness of suspension, and tire damping, to be specified such that resonance, quasi-periodic motion and chaotic behavior can be avoided, reducing the amplitude of the vibration within the system and thus extending the system life.

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四分之一車輛動力學系統的分岔分析

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摘要

基於二自由度 1/4 車輛懸掛系統，系統的分析了輪胎非線性、遲滯效應、阻尼和剛度對車輛非線性動力學響應的影響。以動態跡線、功率頻譜、Poincare 映射圖、分岔圖和頻率响應為指标，分析了系統動力學性能，得到了扭轉固有頻率的准週期解，發現了由於鞍結分叉點導致的准週期解與週期性解的共存現象。通過頻率响應分析發現了兩處共振：第一處共振是由於固有頻率較低而導致的主共振；第二處共振為高固有頻率的次諧波共振。同時，發現輪胎阻尼、激勵的幅值與頻率對動力學性能影響極大。研究結果將有利於分析 1/4 車輛動力學系統在出現不良動態時的實際運行工況，可為同類系統設計人員延長汽車服役期提供有益的參攷。