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Design and Stress Analysis of a Rigid Involute Profiled Harmonic Reducer with Different Profile Modifications

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ABSTRACT

In this study, theoretically designed single-stage harmonic gear mechanisms with involute tooth profile structures, comprising rigid internal and external gears, were designed and modified according to stress values (gear root bending stress and Hertz contact stress). Two methods were used to achieve harmonic motion: the first involved cutting from the tooth head, and the second involved profile shifting within the tooth structure. Additionally, the effects of four different pressure angles (15°, 20°, 25°, and 30°) on both methods were analyzed. Gear calculations were performed using the KISSsoft® program. As a result, a gear pair capable of providing harmonic movement and achieving a high conversion rate in a single stage was designed and produced using a 3D printer. The study concluded that profile shifting was the most critical optimization method for ensuring harmonic movement without tooth interference and reducing stress values. This study aims to develop an alternative gear design for harmonic gearboxes produced in flexible gear systems or cycloid gear structures, where the tooth structure can be manufactured with conventional cutting tools and profile modifications.

INTRODUCTION

In many industries, gears are the most popular power transmission systems. They can bear large loads in comparison to conventional power trans-mission methods. Today, with the developments in technology, more power and higher speeds are required. This is possible with optimization studies in gear design.

The gear's capacity to carry loads increases with decreasing tooth tension. Gears in involute profile structure have been studied for a long time. The first comprehensive source on the subject is the work of Buckingam (1949). In the study by Colbourne, the geometric equations of gear wheels with various involute profile structures are described (Colbourne, 1987).

In the study by Litvin et al., gear theory, design, manufacturing techniques, and gear analysis with the finite method are explained in detail (Litvin & Fuentes, 2004). Radzevich et al. discussed practical methods of designing and manufacturing gears for production facilities of different volumes (Radzevich, 2016). In his study, Kapelevich used the direct gear de-sign method to generate the gear design independently of the cutting tool design. The geometric limits of the method are shown in the diagrams (Kapelevich, 2000). In the study by Yang, he created a mathematical model of asymmetric helical gear wheels and examined the undercut analysis. In the continuation of his study, he investi-gated the effect of static transmission faults occur-ring in gear wheels on assembly lines (Yang, 2005). In his study, Fetvacı simulated the tool trajectory in gears produced with pinion type cutting tools for internal and external gears and analyzed the gear geometry (Fetvacı, 2018).

There are many different analytical, experi-mental, and numerical methods, such as DIN, ISO, and AGMA, used in stress calculations on gear wheels (Shigley, 2011). In the study conducted by Jammi, the AGMA standard and finite element methods were used to calculate the stresses on the tooth bottom and gear side surfaces, and the contact ratios were also investigated. He found a difference of 10–15% be-tween the AGMA standard and finite element analysis (Jammi, 2013). In the study by Çavdar and his team, they used a new method for asymmetric gear profiles for their gears to carry more load. This method was developed as software, and the results were compared with the standards and verified by finite element analysis (Çavdar et al., 2005). Stresses at the tooth bottom are also caused by the cutting tool tip radius. Stress can be reduced by increasing the tip radius. Flodin et al. optimized

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the cutting tool tip radius to reduce the stress at the tooth bottom in gear wheels made of powder metal. Finite element analyses were performed on gears produced with standard cutting tools, full-round cutting tools, and optimized cutting tools, and the results were compared. In the study, a reduction of 18% in static stresses and 22% in dynamic stresses was found in the gear produced with the optimized cutting tool compared to the standard gear (Flodin & Andersson, 2013). Gear tooth geometry of external and internal involute gears can be described by the same equations, except the following rule: The number of teeth is given positive in external gears and negative in internal gears. However, studies show that different stress values occur in internal and external gear systems (von Eiff et al., 1990). The most common types of damage in gear applications are fractures at the base of the teeth or wear on the side surfaces. For this reason, the strength control of gears is examined from two aspects. One is the tooth root strength and the other is the side surface strength. When sizing gear wheels, they are required to provide these two strength values with sufficient safety. It is checked whether the stress values obtained in all strength calculations of the gear wheels are lower than the safety stress values of the material (Barış, 2013) Contact fatigue is an important failure mode of the gear (Li & Liu, 2018). In this mechanism, wear occurs because of deformations caused by high surface pressure between tooth surfaces gripping each other (Güllü & Yilmaz, 2017).

Recently, the finite element method, which is a frequently used method in research, has been increasingly preferred in studies. Although these studies are usually purely numerically oriented, they are also frequently used in the verification of analytical or experimental methods. Hasl et al. used the finite element method to calculate the bending stresses, tooth contact deformations, and contact ratios of gear wheels made of different materials (Hasl C. et al., 2017). Marimuthu et al. investigated various parameters affecting the load-carrying capacity of asymmetric gears with a low contact ratio using the finite element method (Marimuthu & Muthuveerappan, 2016). In the study by Obsieger, he stated that the existing standards are insufficient for stress analysis of nonstandard gears and proposed the graphical method. His method is based on cutting tool and gear geometry and expressed with equations (Obsieger, 1980).

In this study, the gear designs of a rigid harmonic gearbox with an involute profile structure were studied. Based on two different ratios commonly used in gear systems (19 and 29), the tooth numbers of the gears of the harmonic gearbox were selected as Z1:58-Z2:60 and Z1:57-Z2:60. The pressure angles of each gear were examined at four different angles: 15°, 20°, 25°, and 30°. Firstly, the gears were designed without profile shifting by cutting only the tops of the teeth and then, gear designs were made using the profile shifting method and analyzed by the gear root bending stress and Hertz contact stress according to ISO 6336 standards. In both cases, the results were compared, and optimum gear mechanisms were proposed.

MATERIAL AND METHODS

The mechanism of the harmonic gearbox examined in this study is given in Figure 1. Compliance with harmonic motion was obtained by examining the tooth interferences for all models designed in the KISSSoft® programs and by cutting the models from the tooth heads and performing profile shifting until there was no interference. For verification, all drawn models were theoretically run with 2D and 3D simulation programs. As part of a third verification, a selected gear design was produced using a 3D printer and the movement was tested. The gearbox basically consists of a fixed internal gear, an external gear moving on an eccentric input shaft, and an output shaft, which is output through the external gear. The ratio calculation is given in Equation 1.

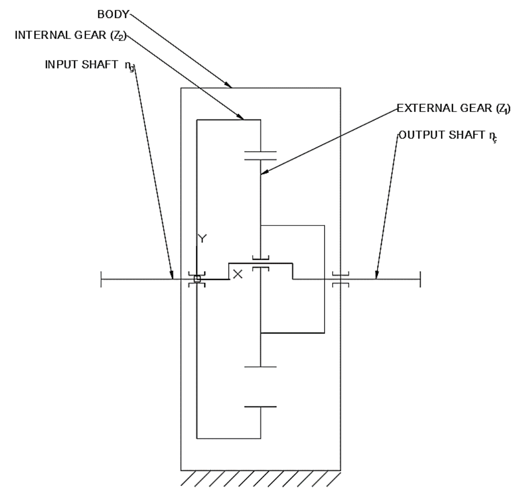

Figure 1. Designed Harmonic Reducer Mechanism

$$i_{1,2} = \frac{Z_1}{Z_2 - Z_1} \quad (1)$$

Gear Geometry Creation in the KISSsoft® Program

In this section, figures of the tooth geometries investigated in this study are given. The gears without profile shifting are given in Figure 2 (a) and (b) profile shifted gears are given in (c) and (d).

The value adopted for the harmonic gearbox is the tag value of a commercial electric motor with input speed n=1400 rpm and power rating P=3 kW. The numerical data of the gear pairs designed in this study are given in Table 1. Accordingly, the first group was analysed without profile shifting. Since profile shifting was not applied, the cutting process was applied to the upper diameters of the teeth.

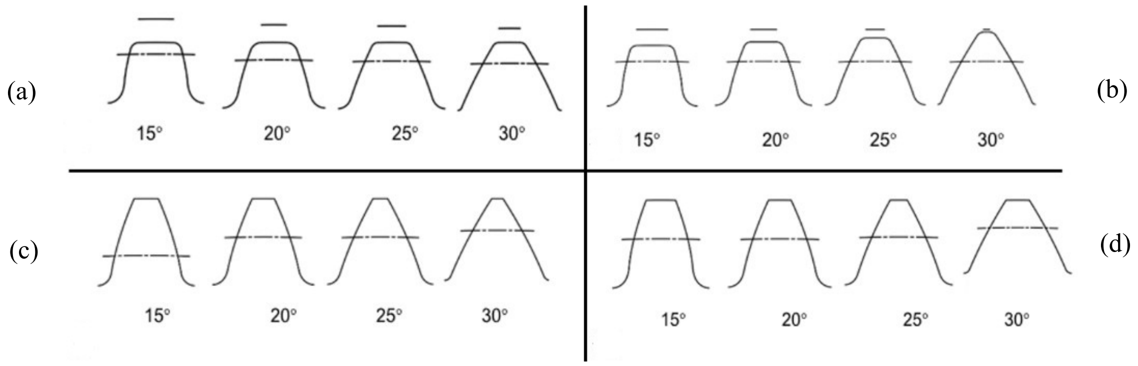

Figure 2. (a). Z1=58 pinion gear with pressure angles of 15°, 20°, 25°, and 30° in without profile shifting;
(b). Z1=57 pinion gear with pressure angles of 15°, 20°, 25°, and 30° in without profile shifting;
(c). Z1=58 pinion gear with pressure angles of 15°, 20°, 25°, and 30° in profile shifting;
(d). Z1=57 pinion gear with pressure angles of 15°, 20°, 25°, and 30° in profile shifting.

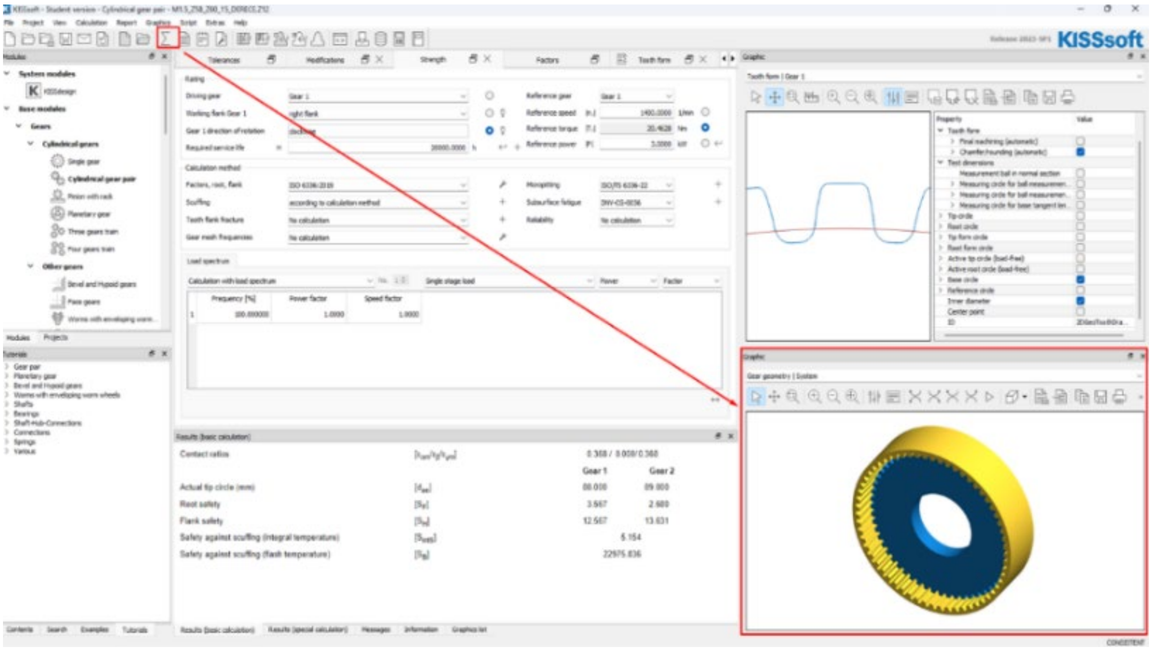

Figure 3. Z1=58 - Z2=60 creation of the gear pair without profile shifting in KISSsoft® program

Table 1. Harmonic Reducer Design Information

Group No	Model	Pressure Angle (°)	Internal Gear (Z_2)	External Gear (Z_1)	Profile shift coefficient		Tip alternation of gear (mm)		Eccentricity between gears distance (mm)	Ratio (i)
					(Z_1)	(Z_2)	(Z_1)	(Z_2)		
Group 1	Model 1	15	60	58	0	0	-1	-1	1.5	29
	Model 2	20			0	0	-0.75	-0.75	1.5	
	Model 3	25			0	0	-0.70	-0.70	1.5	
	Model 4	30			0	0	-0.60	-0.60	1.5	
	Model 5	15	60	57	0	0	-0.75	-0.75	2.25	19
	Model 6	20			0	0	-0.58	-0.58	2.25	
	Model 7	25			0	0	-0.40	-0.40	2.25	
	Model 8	30			0	0	-0.1250	-0.1250	2.25	
Group 2	Model 9	15	60	58	0.528	-1.2	0	0	1.99	29
	Model 10	20			0	-0.5578	0	0	2	
	Model 11	25			0	-0.4324	0	0	1.955	
	Model 12	30			-0.093	-0.2073	0	0	1.86	
	Model 13	15	60	57	0	-0.5515	0	0	2.71	19
	Model 14	20			0	-0.3923	0	0	2.6555	
	Model 15	25			0	-0.2471	0	0	2.55	
	Model 16	30			0.05	-0.05	0	0	2.25	

In the analyses, 18CrNiMo7-6 was defined as cementation steel for the gears. The mechanical properties of the material are given in Table 2.

Table 2: Mechanical Properties of Material (https://www.kisssoft.com, 2024)

Surface Hardness	HRC 61
Infinite life endurance for tooth bottom stress σ_{FLimit} (MPa)	430
Fatigue strength for Hertz pressure σ_{HLimit} (MPa)	1500
Young's Modulus (MPa)	206000
Poisson Ratio	0.3
Tensile Strength (MPa)	1200
Yield Strength (MPa)	850

Stress Analysis of Gears

International and industry standards such as ISO 6336, AGMA 2101-D04, DIN 3990, and ANSI/AGMA 2001-D04 are often used to analyze gear stresses. These standards are used to define important criteria for the design, strength, and performance of gears. The gear forces acting on the gears are shown in Figure 7. Within the scope of the study, the gears were analyzed according to the ISO 6336 standard. In the study, gear root bending stresses and Hertz contact stresses were calculated.

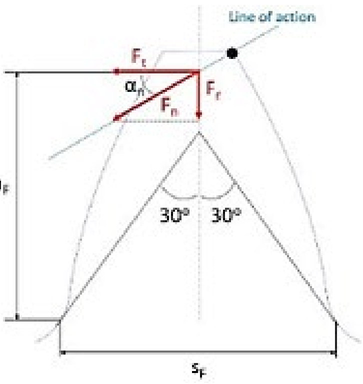

Figure 3. The gear forces acting on the gears

Calculations According to Gear Root Bending Stress

In the study, gear root bending stresses and Hertz contact stresses were calculated by the KISSsoft® AG Program. KISSsoft® is a modular calculation program for the design, optimization and verification of machine elements according to international standards (https://www.kisssoft.com, 2024). It was used the ISO 6336 standard at gear calculations. Following equations were used in the program.

$$\sigma_{F0} = \frac{F_t}{b.m}.Y_F.Y_S.Y_\beta \tag{2}$$

$$\sigma_F = \sigma_{F0}.K_A.K_V.K_{F\beta}.K_{F\alpha} \tag{3}$$

In the equations 2 and 3, F_t is the tangential force, b is the tooth width, and m is the module value. Y_F is the form factor, Y_S is the stress correction factor, Y_β is the helix angle factor, the factor K_A, called application factor, is used to take into account the effect of external overloading. It depends on the application. The dynamic factor K_V considers the internal dynamic loads. The factors $K_{F\beta}$ and $K_{F\alpha}$ are used to model the uneven load distribution in the contacts along the face width and in the transverse direction and is the permissible bending stress. Maximum gear root bending stresses were calculated according to Eq. 3.

Hertz contact stress Calculations

The equations used in the Hertz contact stress calculations according to ISO 6336:2019 are given in Eq. 4 and 5.

$$\sigma_{H0} = Z_H.Z_E.Z_\varepsilon.Z_\beta.\sqrt{\frac{F_t}{d_1.b}.\frac{u+1}{u}} \tag{4}$$

$$\sigma_H = Z_B.\sigma_{H0}.\sqrt{K_A.K_V.K_{H\beta}.K_{H\alpha}} \tag{5}$$

In the equations, is the nominal contact stress, Z_H is the zone factor, Z_E is the elasticity factor, Z_ε is the contact ratio factor, Z_β is the helix angle factor, d_1 is the pitch diameter, u is the gear ratio, σ_H is the Hertz contact stress, Z_B is the single pair tooth contact factor, $K_{H\beta}$ is the face load factor and $K_{H\alpha}$ is the transverse load factor. The calculation results are given below.

RESULTS

Results of The Stress Calculations

Stress analysis results of all studied designs are given in Table 3. In the tables, σ_F is the tooth root stress, σ_{Max} is the tooth root stress at the point with the highest stress, σ_{Min} is the tooth root stress at the place with the lowest stress, σ_{F0} is the nominal tooth root stress from, σ_{H0} is the nominal contact stress, and σ_H is the contact stress, σ_{Max} and σ_{Min} values, were found by calculating the von mises equivalent stress with KISSoft® program.

When the data in Table 3 is examined, root bending stress σ_F in models without profile shifting at 15°, 20° and 25° pressure angles, it was found that the stress values decreased as the angle increased. However, it increased at 30° pressure angle. In the models with profile shifting, it was found that the tooth root bending stress values decreased as the pressure angle increased. In the analysis results, it was seen that the results were lower than the yield strength of the material. When the Hertz contact stresses were analyzed, it was observed that the contact stresses decreased as the pressure angle increased in the models without profile shifting. In the models with profile shifting, it was determined that the contact stresses in-creased as the pressure angle increased. When the analysis results of all models are examined, it is seen that σ_{FLimit} and σ_{HLimit} values were in the safe line. However, the change in stress values with gear modifications is remarkable. Another important point that can be said for stress analyses is that KISSSoft® is an important tool that allows all stress values to be obtained quickly for each design change. Studies in the literature show that, results close to the correct result are given with deviations as low as 8% maximum (Irsel, 2021).

Table 3. Results of The Stress Calculations

	σ_F (MPa)	σ_{Max} (MPa)	σ_{Min} (MPa)	$\sigma_{F0\ ISO\ 6336}$ (MPa)	σ_{H0} (MPa)	σ_H (MPa)
Model 1 (15°)	207.59	109.06	-124.14	111.38	75.97	106.81
Model 2 (20°)	195.03	96.84	-118	102.42	67.01	95.68
Model 3 (25°)	187.70	176.36	-151.65	96.19	61.38	88.97
Model 4 (30°)	333.67	130.99	-181.15	166.98	57.73	84.96
Model 5 (15°)	231.71	121.81	-140.18	125.93	94.68	132.56
Model 6 (20°)	217.47	108.84	-131.84	115.61	83.51	118.70
Model 7 (25°)	207.43	101.37	-130.9	107.71	76.36	110.26
Model 8 (30°)	342.76	137.01	-189.24	171.57	69.64	103.24
Model 9 (15°)	261.36	119.09	-142.48	128.22	36.41	54.52
Model 10 (20°)	241.36	99.84	-121.69	105.94	35.73	56.87
Model 11 (25°)	207.21	90.6	-117.38	95.11	37.46	58.36
Model 12 (30°)	202.46	102.68	-143.21	99.73	40.42	60.70
Model 13 (15°)	216.40	54.08	-61.31	76.89	46.01	81.77
Model 14 (20°)	251.37	92.6	-112	96.47	49.31	84.30
Model 15 (25°)	205.82	85.79	-112.15	87.78	53.77	87.11
Model 16 (30°)	214.65	156.09	-275.57	90.81	71.03	104.58

Results of the FEM Analysis

KISSSoft® program can also perform finite element analyzes in a modular manner. In this study, tooth root stress values were also evaluated by finite element analysis. The results are seen in Figure 4-7. The 2D finite element analysis results of the model of the Z58-Z60 gear pair made within the scope of the study without profile shift are given in Figure 4.

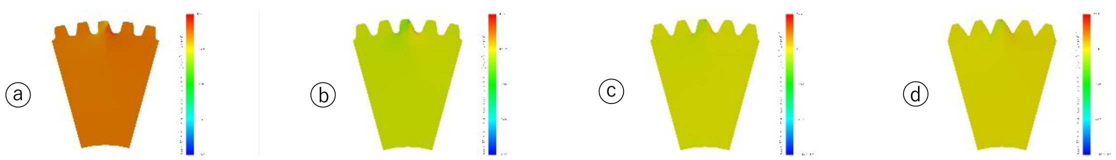

Figure 4. Models without profile shifting. a) Visual Results Model of 1 Z58-Z60 15°; b) Visual Results of Model 2 Z58-Z60 20°; c) Visual Results of Model 3 Z58-Z60 25°; d) Visual Results of Model 4 Z58-Z60 30°.

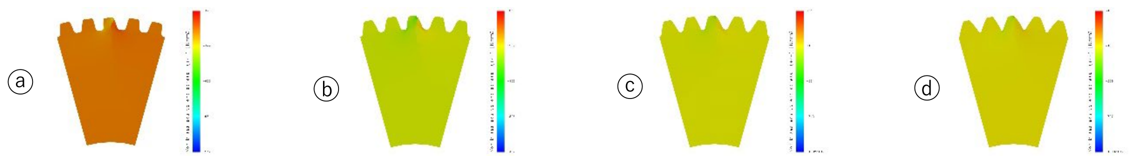

Figure 5. Models without profile shifting. a) Visual Results Model of 1 Z57-Z60 15°; b) Visual Results of Model 2 Z57-Z60 20°; c) Visual Results of Model 3 Z57-Z60 25°; d) Visual Results of Model 4 Z57-Z60 30°.

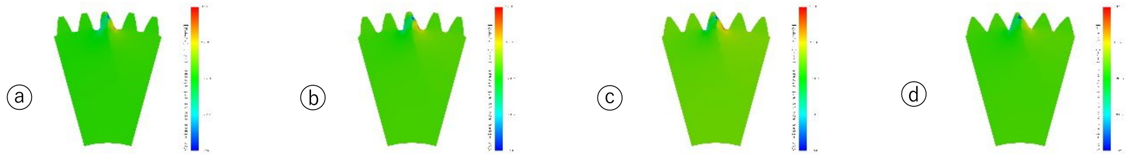

Figure 6. Models with profile shifting. a) Visual Results Model of 1 Z58-Z60 15°; b) Visual Results of Model 2 Z58-Z60 20°; c) Visual Results of Model 3 Z58-Z60 25°; d) Visual Results of Model 4 Z58-Z60 30°.

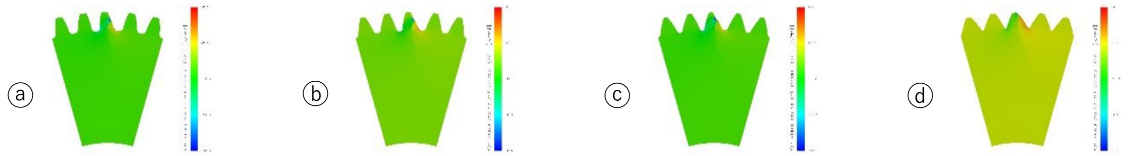

Figure 7. Models with profile shifting. a) Visual Results Model of 1 Z57-Z60 15°; b) Visual Results of Model 2 Z57-Z60 20°; c) Visual Results of Model 3 Z57-Z60 25°; d) Visual Results of Model 4 Z57-Z60 30°.

In this study, the material of the gear wheels was selected as 18CrNiMo7-6. The yield strength of this material is 850 MPa and tensile strength is 1200 MPa (Table 2). When the finite element analysis results of all models were examined, it was seen that all designs remained within safe limits. However, it is seen that the stress values vary significantly according to design differences. In addition, in order to easily compare the numerical data obtained from the analysis, maximum tooth root bending stress values and hertz stress values are given in Figure 8-9.

Comparasion of the Stress Values

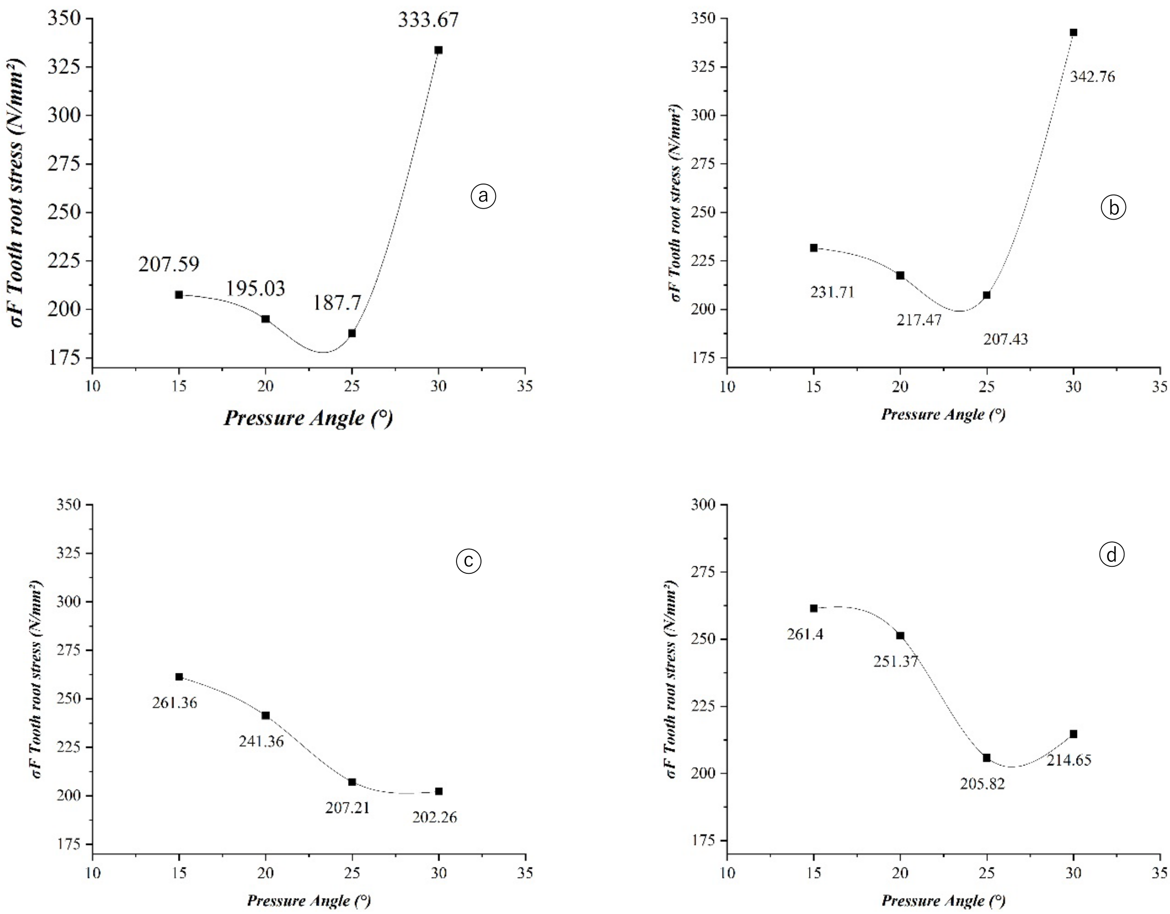

Figure 8. Variation of σ_F Stress with Pressure Angle. a) Z58-Z60 models without profile shifted; b) Z57-Z60 models without profile shifted; c) Z58-Z60 models profile shifted; d) Z57-Z60 models profile shifted.

When the graphs of Figure 8-a and 8-b were examined, it was seen that there was no regular decrease or increase between the pressure angle change and the stress values. The main reason for this was that the stress values emerge with the combination of this effect together with the pressure angle due to the reduction of the gear contact surfaces between the gear wheels due to the cutting of the teeth from their upper parts. As a result, the profile shifting method was also examined after the high stress values obtained as a result of the cutting method.

When the graphs of Figures 8-c and 8-d were analyzed, it was seen that although there was not a linear decrease between the change of the pressure angle and the stress values, the stress values decrease regularly in the Z:60-Z:58 gear pair. All stress results obtained were lower than the σ_{Flimit} value of 430 MPa. The main reason for this was that the gear contact surfaces have increased due to profile shifting and the stress values have decreased. However, it was observed that there was no linear decrease. The main conclusion drawn from these graphs was that there was no regular correlation between changing the pressure angle and stress values.

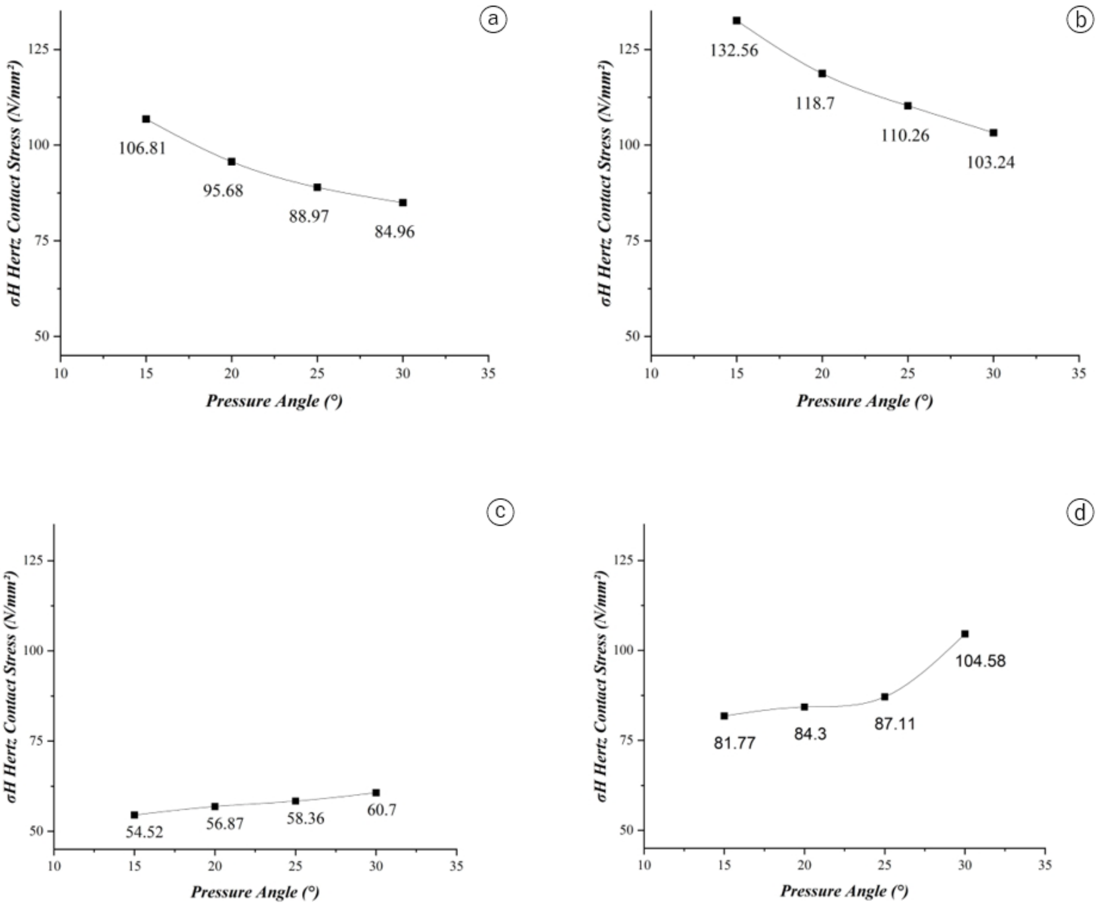

Figure 9. Variation of σ_H Contact Stress with Pressure Angle. a) Z58-Z60 models without profile shifted; b) Z57-Z60 models without profile shifted; c) Z58-Z60 models profile shifted; d) Z57-Z60 models profile shifted.

When the figures in Figures 9-a and 9-d are analyzed, it was seen that the hertz contact stresses are lower than the fatigue strength σ_{HLimit} value for hertz pressure. As seen in the graphs in Figures 9-a and 9-b, it was observed that the hertz contact stresses decreased as the pressure angle increased. In the graphs in Figures 9-c and 9-d, it was seen that the hertz contact stresses increased with the increase in the pressure angle. With the use of the profile shifting method, contact stresses were found to be even lower. It is recommended that the profile shift method be used in such gear systems.

When all pressure angle and profile shifting optimizations are taken together, it is seen that each changing profile structure affects the resulting stresses differently. It is seen that increasing the pressure angle up to 25° generally reduces the gear root bending stress values. And, it has also been observed that there are no major differences in terms of the 20° profile used in the standard. When looking at the Hertz contact stress values, no correlation was seen with the pressure angle. It should be considered that these two values should be taken into account in each profile modification.

In their study, Maiti and Roy stated that to avoid tooth tip interference in internal gear and external gear systems, the minimum tooth difference of the number of teeth of the internal gear and external gear pair should be 5 (Maiti & Roy, 1996). However, in this study, it has been seen from the analyzes that harmonic movement can be achieved even with the difference in the number of 3 and 2 teeth, thanks to the tooth modifications. After all these analyses, in order to check that the harmonic movement is achieved with the specified profile modifications in the rigid gears, an optimum design was selected from the designed gear mechanisms considering the minimum stress values obtained and the idea that the design was closest to the standards and was produced using PLA filament via a 3D printer. Figures 10 and 11 show the 3D printed and assembled version of the harmonic gearbox chosen in this study. The model was printed using the Z58 - Z60 gear pair with profile shift with a profile pressure angle of 20°. As a result of this study, it was determined that the minimum tooth difference between the inner gear and the outer gear pair could be at 2 (58-60).

Figure 10. 3D Printed image of Model 10 Z58-Z60 20° harmonic Reducer

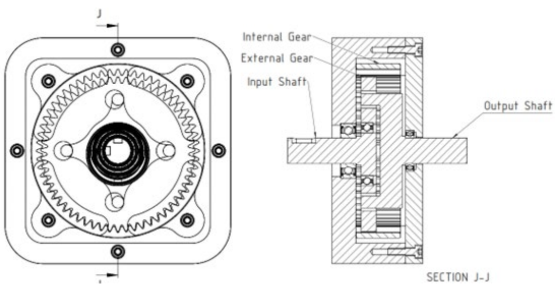

Figure 11. Technical drawing of Model 10 Z58-Z60 20° harmonic reducer

CONCLUSIONS

In this study, gear root bending stress and Hertz contact stress occurring in the gear at different press-sure angles in the rigid harmonic reducer gear pair with involute profile structure were investigated. In this context, 16 different models were studied for optimization data. In the first stage of the study, designs were made without profile shifting, only by cutting from the tops of the teeth and analyzes of these de-signs were performed. In the second part of the study, designs were made with profile shifting and analyzes of these designs were performed. The results of the study are summarized below.

Before the study, it was predicted that, especially the toot root stresses would decrease with increasing pressure angle, but when the results of the study were examined, it was seen that there was no linear increase or decrease. Also, it was observed that the root bending stress and Hertz contact stress values were higher in the models without profile shifted than in the models with profile shifted. Generally, low stress values were observed with profile shifting. In this study, it is selected to produce a Z58 - Z60 gear pair with a cycle ratio of 29 and a pressure angle of 20°. The main reason for this is the high gear ratio and easy production. Although any desired model can be easily produced using the 3D printer method, in practice machining method that is widely used in gear manufacturing, gear wheel pressure angles are generally produced at 20°. When the pressure angle is different from 20°, a special cutting tool is required to produce the gear wheel. As a result of the study, it was seen that harmonical movement can be achieved by cutting the top of the teeth and profile shifting. Besides this, it was understood while profile shifting makes the gear system suitable for harmonic movement, it also reduces tooth root and hertz stresses. As a result, it is thought that involute profile harmonic gear mechanisms providing high transformation ratios can be produced in a single stage by both using standard tools and various tooth modifications, and that studies on this subject are promising for the future. In fact, considering that in the future, through developments in 3D manufacturing, gears can be produced directly using additive manufacturing methods without using tools, it is also thought that all desired gear modifications can be provided directly using additive manufacturing methods.

Acknowledgments

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