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Fast Algorithm for the Inverse of the Real Gas Prandtl-Meyer Function

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Keywords： Supersonic nozzle. Prandtl Meyer function. Successive approximation method. bipartition algorithm. Stagnation temperature. Stagnation pressure. Real Gas. Berthelot state equation. Thermodynamic ratios. Gauss Legendre quadrature. Tolerance.

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ABSTRACT

The calculation of the flow parameters at the center of a throat expansion of a supersonic nozzle design in the real gas model is of practical interest in aerospace construction. The expansion center is characterized by the end of an infinite Mach lines which will be discretized by a finite number with continuous increase of Mach number and the *PM* value with a decrease of the temperature, density and the pressure. The flow parameters must be calculated by determining the inverse of the *PM*. This function depends on two variables which are the temperature and the density. The aim of this work is to develop a fast algorithm allowing to do the inverse of the *PM* in the context of real gas model by the determination of the temperature and the density corresponding to the given *PM* deviation which is itself depends on the flow deviation according to the shape of the supersonic nozzle. The Bernoulli equation must be added to construct two nonlinear algebraic equations with two coupled unknowns. The corresponding Mach number and pressure will be determined by analytic equations. The two equations are presented as integral of four complex functions, where the integration is made by the Gauss Legendre's quadrature. The numerical solving of system of equations is done by combining the successive approximation algorithm and the bipartition algorithm. The initial solution is chosen as the parameters corresponding to the previous Mach line to ensure the convergence and accelerate the numerical process. The comparison is made with previous algorithm.

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INTRODUCTION

In the aerospace industry, the problem of the supersonic nozzles design is a very important phase before moving on to construction. This phase requires a fairly large computation time by computer to obtain the results given the poor choice of the developed numerical algorithms. In some cases it is impossible to start to solve some problems because the development of algorithms is impossible because of difficulties of intermediate calculation or bad vision on the development or improvements of the algorithms or bad initial conditioning.

In (Salhi and Zebbiche, 2015), (Salhi, Zebbiche, 2016b), and (Salhi, 2017), a model based on the *RG* approach to determine the *P*0 effect on the thermophysical parameters of a supersonic flow in an unidirectional nozzle is developed, and in (Salhi, 2017), (Salhi, Zebbiche and Mehalem, 2016a), (Salhi, Zebbiche and Roudane, 2016b), an approach to the *PM* based always on the *RG* model to determine the P_0 effect on this function is presented. In these references, the approach is made by the *RG* Berthelot's state equation.

In (Salhi, 2017), a model of calculation based on the *MOC* taking into account the approach of *RG* to make the effect of *P*0 on the design of the supersonic 2D *MLN* giving a uniform and parallel flow at the exit section is presented. To reach the goal, the studies presented in (Salhi and Zebbiche, 2015), (Salhi, Zebbiche and Mehalem, 2016a, 2016b), (Salhi, 2017) and (Salhi, Zebbiche and Roudane, 217b) are used to develop the *MOC* model including the *PM*.

In (Salhi, 2017), (Salhi, Zebbiche and Mehalem, 2016a) and (Salhi, Zebbiche and Roudane, 2017), the problem encountered is the resolution of the system of non-linear equations with two unknowns based on the computation of 4 integrals of complex functions, especially for *PM* inversion, necessary at the throat nozzle expansion center to determine the flow parameters to allow the use of the *MOC*. The algorithm presented in (Salhi, 2017) requires a very high computation time with a discussed precision. So as solutions, the authors used a small number of mesh points to have only approximated results in a moderate time.

The aim of this work is to develop a fast and accurate algorithm allowing to do the inverse of the *PM* in the context of *RG* model by the determination of the temperature and the density corresponding to the given *PM* deviation which itself depends on the flow deviation according to the shape of the supersonic nozzle. The Bernoulli equation must be added to construct two nonlinear algebraic equations with two coupled unknowns *T* and *ρ*. The values of *M* and *P* are determined by analytic equations. The two nonlinear equations are presented by integral of four complex functions, where the integration is made numerically by the Gauss Legendre's formulae. The numerical solving of the system of equations is done by combining the successive approximation algorithm and the bipartition algorithm to accelerate the numerical process. The initial solution is chosen as the parameters corresponding to the previous Mach line. The application is made at the *MLN* expansion center and the *PN* lip to make design in a much reduced time with high precision.

MATHEMATICAL FORMULATION

For our *RG* model, all state parameters can be defined by two state variables (Thompson, 1995), (Van Wylen, 1973) and (Annamalai, Ishwar and Milind, 2011), chosen by *T* and *ρ*. But for *HT* (Zebbiche and Youbi, 2007a, 2007b) and *PG* (Zebbiche, 2005) and (Emanuel and Argrow, 1988) models, all the state parameters depends only on one state variable. The expansion center *A* is the point of discontinuity, to see Fig. 1. At this point there is sudden increase of all the thermo-physical parameters. This discontinuity gives infinity of Mach line which is issued from point *A*. In each deviation *i* (*i*=2, 3, …, *N*) of a new Mach line, the *PM* takes the following value from the sonic Mach line,

$$\nu_i = (i-1)\,\Delta\theta \tag{1}$$

In relation (1), $\Delta\theta$ is given if the M_E is from the data, or calculated from θ^*.

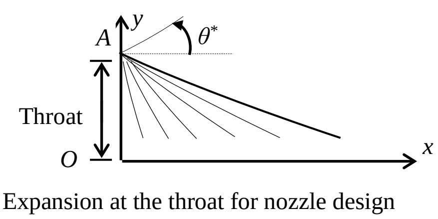


Fig. 1 Expansion at the throat for nozzle design

The problem consists in determining T_i, ρ_i, M_i and P_i corresponding to this deviation numbers *i*. Since *PM* depends on two variables *T* and *ρ* (Salhi, 2017), (Salhi, Zebbiche and Mehalem, 2016a) and (Salhi, Zebbiche and Roudane, 2017), we must solve the following two nonlinear algebraic equations simultaneously to obtain T_i and ρ_i. Equation (2) is obtained by equalization of the *PM* (Salhi, 2017), (Salhi, Zebbiche and Mehalem, 2016a) and (Salhi, Zebbiche and Roudane, 2017) with Eq. (1), and Eq. (3) is the Bernoulli equation (Salhi and Zebbiche 2015), (Salhi, Zebbiche and Mehalem, 2016a, 2016b), (Salhi, 2017), (Salhi, Zebbiche and Roudane, 2017). So,

$$f(T_i,\rho_i) = \nu(T_i,\rho_i) - \nu_i = 0 \tag{2}$$

$$\int_{T_i}^{T_{i-1}} E_1(t,\rho_i)\,dt + \int_{\rho_i}^{\rho_{i-1}} \left[\frac{1}{\rho} + E_2(T_i,t)\right] dt = 0 \tag{3}$$

Where

$$\nu(T_i,\rho_i) = \int_{T_i}^{T_*} \nu_1(t,\rho_i)\,dt + \int_{\rho_i}^{\rho_*} \nu_2(T_i,t)\,dt \tag{4}$$

And

$$\nu_1(T,\rho) = \frac{\sqrt{M^2(T,\rho)-1}}{V^2(T,\rho)}\,C_P(T,\rho) \tag{5}$$

$$\nu_2(T,\rho) = \frac{\sqrt{M^2(T,\rho)-1}}{V^2(T,\rho)}\,C_T(T,\rho) \tag{6}$$

$$E_1(T,\rho) = -\frac{C_P(T,\rho)}{S_V^2(T,\rho)},\quad E_2(T,\rho) = -\frac{C_T(T,\rho)}{S_V^2(T,\rho)} \tag{7}$$

$$M(T,\rho) = \frac{V(T,\rho)}{S_V(T,\rho)} \tag{8}$$

For the Berthelot state equation, the expressions of $V(T,\rho)$, $S_V(T,\rho)$, $C_T(T,\rho)$ and $C_P(T,\rho)$ are presented in (Salhi and Zebbiche, 2015), (Salhi, Zebbiche and Mehalem, 2016a, 2016b), (Salhi, 2017), (Salhi, Zebbiche and Roudane, 2017).

The pressure can be calculated by the following Berthelot state equation (Salhi and Zebbiche, 2015), (Salhi, Zebbiche and Mehalem, 2016a, 2016b), (Salhi, 2017), (Salhi, Zebbiche and Roudane, 2017), (Thompson, 1995), (Van Wylen, 1973) and (Annamalai, Ishwar and Milind, 2011),

$$P(T,\rho) = \frac{\rho R T}{(1-b\rho)} - a\,\frac{\rho^2}{T} \tag{9}$$

The integrals in (2) and (3) are evaluated by the Gauss Legendre formulae of order 20 (Raltson and Rabinowitz, 1985).

For air, we have γ_{PG}=1.402, R=287.1029 J/(kg K), a=117.2666 Pa m^6, b=1.07334×10^{-3} m^3 and α=3056.0 K (Salhi and Zebbiche, 2015), (Salhi, Zebbiche and Mehalem, 2016a, 2016b), (Salhi, 2017), (Salhi, Zebbiche and Roudane, 2017), (Thompson, 1995), (Van Wylen, 1973) and (Annamalai, Ishwar and Milind, 2011).

ALGORITHM FOR EXPANSION CENTER

After mathematical calculations and adaptation of the successive approximations algorithm, Eq. (3) takes the form (10) at the iteration K+1,

$$\frac{\rho_i^{K+1}}{\rho_{i-1}}=\mathrm{Exp}\left(\int_{T_i^K}^{T_{i-1}} E_1(t,\rho_i^K)\,dt+\int_{\rho_i^K}^{\rho_{i-1}} E_2(T_i^K,t)\,dt\right) \tag{10}$$

Then, at the iteration (K+1), the relation (2) takes the following form,

$$f(T_i^{K+1})=\nu(T_i^{K+1},\rho_i^{K+1})-\nu_i=0 \tag{11}$$

This gives a nonlinear equation to one unknown T_i^{K+1}. The solution lies in the following interval (12) regardless of the value of K, which is determined by the use of the dichotomy algorithm (Raltson and Rabinowitz, 1985),

$$T_E \le T_i^{K+1} \le T_{i-1} \tag{12}$$

In Eq. (11), the value of ρ_i^{K+1} is given by (10). At iteration K=0, the solution at the point i is taken equal to the values of the previous Mach line corresponding to i-1, that is to say,

$$\rho_i^{K=0}=\rho_{i-1}\quad,\quad T_i^{K=0}=T_{i-1} \tag{13}$$

To determine the solution T_i^{K+1} according to the interval (12), the minimum N_T number can be calculated by the following relation (14) (Raltson and Rabinowitz, 1985):

$$N_T=1.4426\,\mathrm{Ln}\left(\frac{T_D-T_G}{\varepsilon}\right)+1 \tag{14}$$

Where:

$$T_D=T_{i-1}\qquad,\qquad T_G=T_E \tag{15}$$

The maximum value of N_T for the most unfavorable case is that, if $\varepsilon=10^{-8}$, in the supersonic regime $M_E \le 5.00$, cannot exceed 30 according to (14).

If this algorithm is used to solve an arbitrary point that is not related to the design problem of a supersonic nozzles, we can take an arbitrary initial value $\rho_i^{K=0}<\rho_*$ and $T_D=T_*$ and T_G=0.0. Here the iteration numbers N_T and N_ρ will be increased or decreased compared to our consideration.

The final solution at point i, after K iterations, will be obtained if condition (16) is satisfied

$$Max\left[\left|T_i^{K+1}-T_i^K\right|,\left|\rho_i^{K+1}-\rho_i^K\right|\right]<\varepsilon \tag{16}$$

The values of M_i and P_i are determined by the relations (8) and (9).

Concerning the angle θ_i, it is equal for example to ν_i for the *MLN* [9, 12], and equal to ν_E-ν_i for the *PN* (Zebbiche and Youbi, 2007b) and (Zebbiche, 2005).

The problem discussed is the improvement of the calculation time and the obtained precision in the design of the *MLN* and *PN*.

RESULTS AND COMMENTS

In the presentation of the results, the initial solution of T and ρ, at the iteration K=0, are chosen by $T_i^{K=0}=T_*$ and $\rho_i^{K=0}=\rho_*$; The values of T_* and ρ_* depends on T_0 and P_0.

Table 1 presents the values of the thermophysical parameters (T/T_0, ρ/ρ_0, M and P/P_0) corresponding to the values of ν with $\varepsilon=10^{-5}$ when T_0=2000K and P_0=50 bar. The values are determined by the developed program by inverting the *PM* coupled with the Bernouli equation.

Table 1 : Supersonic parameters correspond to *PM* values for T_0=2000 K, P_0=50 bar and $\varepsilon=10^{-6}$.

ν(°)	T/T_0	ρ/ρ_0	M	P/P_0
1	0.83305	0.55137	1.07348	0.45740
5	0.79176	0.46801	1.22816	0.36872
10	0.74924	0.39206	1.38313	0.29209
20	0.67043	0.27549	1.66955	0.18346
30	0.59367	0.18855	1.95964	0.11109
40	0.51739	0.12407	2.27198	0.06367
50	0.44152	0.07774	2.62245	0.03403
55	0.40388	0.06023	2.81815	0.02411
60	0.36657	0.04591	3.03204	0.01668
65	0.32972	0.03436	3.26880	0.01122
70	0.29355	0.02519	3.53467	0.00732
75	0.25830	0.01803	3.83786	0.00461
80	0.22430	0.01256	4.18900	0.00279
85	0.19191	0.00847	4.60143	0.00161
90	0.16148	0.00550	5.09192	0.00088
95	0.13330	0.00341	5.68270	0.00045
100	0.10755	0.00200	6.40625	0.00021

Tables 2 presents the necessary iteration umbers N_T and N_ρ as a function of ε. We note the influence of all parameters on N_T and N_ρ, especially ε and ν. We have observed that the influence of T_0 and P_0 on N_T and N_ρ is quite slight with N_T=20 and N_ρ=4 for $\varepsilon=10^{-5}$.

If we apply this algorithm on the design process of a supersonic nozzle, the iteration numbers *NT* and *Nρ* are slightly small by those found in table 2, since the initial solution will be taken close to the found solution according to the proposal (13), which gives a very advantageous result to further accelerate the numerical calculation process.

According to (14), the number *NT* depends on the search interval of the solution Ti^{K+1} by the choice of *TD* and *TG* and especially the desired accuracy ε.

For comparison, the difference between the results presented in the table 1 and those in (Salhi, 2017) can reach 29% with the values in table 1 are lower than those in Ref. [3]. For example, from (Salhi, 2017), if ν=60°, we have M=4.21502, with an error ε=28% approximately. Consequently, the other parameters, and, according to (Salhi, 2017), are equal to $T/T0$=0.46852, $\rho/\rho0$=0.06971 and $P/P0$=0.02433 giving respectively errors of 22%, 34% and 31%.

Table 2: Numbers of iterations corresponded to desired precision for ν_i=60°, T_0=2000 K and P_0=50 bar.

ε	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}	10^{-8}
N_T	7	10	14	17	20	24	27	30
N_ρ	3	4	4	4	5	5	5	6

COMPARISON WITH ALGORITHM OF (SALHI, 2017)

The first comparison on the integrals of (3) and (4) is that (Salhi, 2017) used Simpson's formula with 100 points for applications, while our algorithm uses Gauss Legendre quadrature of order 20 with its high power and precision. We know that despite 100 points are used, the precision given by (Salhi, 2017) is still very small compared to the precision given by our squaring. Therefore, the execution time of our algorithm is lowered 5 times in this context compared to that of (Salhi, 2017).

The second comparison on the technique used to determine (T_i, ρ_i) as a solution to the expansion center *A*, the Ref. (Salhi, 2017) searches for the root by the scanning of the intervals [T_E, T_*] and [ρ_E, ρ_*] on a number of points equal to 100×100 for each Mach line *i* (*i*=2, 3, ..., *N*). Then it determines the root on the basis that the values given by Eqs. (2) and (3) are the smallest possible in absolute values. Since there is a physical solution for each deviation ν_i, the solution will be accepted with a discussion of accuracy. The authors of (Salhi, 2017) have used this technique to avoid the computation of the partial derivatives of the functions under the integral sign presented in (2) and (3), since all the algorithms use the Jacobean formulated on the computation of these derivatives. It should be noted that the computation of the partial derivative with respect to T has a major difficulty compared to the partial derivative with respect to ρ. The execution time is very high since their algorithm calculates 4×100×100=40000 integrals by the Simpson formula for each Mach line *i* (*i*=2, 3, …, *N*). For information, our algorithm makes a number of 4×N_T×N_ρ integrations by the Gauss Legendre formulae of order 20 for each value of ν_i.

From Refs (Salhi and Zebbiche, 2015), (Salhi, Zebbiche and Mehalem, 2016a, 2016b), (Salhi, 2017) and (Salhi, Zebbiche and Roudane, 2017), for each pair (T, ρ), we need respectively 18, 42, 23, 35 mathematical operations to obtain the values of $V(T, \rho)$, $S_V(T, \rho)$, $C_T(T, \rho)$ and $C_P(T, \rho)$ after removing the additional mathematical operations.

Note that the integrals in (2) and (3) are formulated by the functions $\nu_1(T, \rho)$, $\nu_2(T, \rho)$, $E_1(T, \rho)$ and $E_2(T, \rho)$, and according to (5), (6), (7) and (8), we note again that the 4 said functions are formulated by $V(T, \rho)$, $S_V(T, \rho)$, $C_T(T, \rho)$, $C_P(T, \rho)$ and $M(T, \rho)$. Then, as a numerical technique, to reduce the execution time as much as possible, it is recommended to avoid the calculation of the 4 integrals separately and to group the computation in two quadratures to avoid the repetition of the calculations of the terms form the functions $\nu_1(T, \rho)$, $\nu_2(T, \rho)$, $E_1(T, \rho)$ and $E_2(T, \rho)$ since the Gauss Legendre quadrature order and the bounds of the integration are the same respetively for $\nu_1(T, \rho)$, $E_1(T, \rho)$ and the same for $\nu_2(T, \rho)$, $E_2(T, \rho)$. Then we need only 8×20 additional mathematical operations to evaluate 2 integrals at the same time witch calculated 2 times without repetition, instead of repeating each time the calculations of $V(T, \rho)$, $S_V(T, \rho)$, $C_T(T, \rho)$, $C_P(T, \rho)$ in the calculation if the 4 integrals are evaluated separately. Then in total we need of (18+42+23+35+8)×20×2=2520×2 mathematicals operations to evaluate the 4 integrals for each iteration without repetition. Then the total number of mathematical operations, for each point of Mach line *i* (*i*=2, 3, …, *N*) is equal to 5040×N_T×N_ρ.

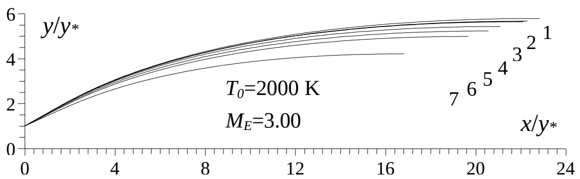
In (Salhi, 2017), the evaluation of the 4 integrals presented in (2) and (3) is done separately. For this, and given the skew of the Simpson formula, the number of mathematical operations performed for each iteration is equal to (18+42+23+35+8)×100×4 =50400. Compared with the number of our mathematical operation, the execution time is decreased 10 times on this path.

Two solutions can be chosen for the algorithm of (Salhi, 2017) is the use of a very powerful computer to hide this disadvantage, or to make calculations for large meshes only to have first solutions that do not contain a number of lines Mach enough. So given the obtained N_T and N_ρ according to table 2, we can say that our execution time is lowered in the unfavorable case to (100×100)×10/(30×6)=555 times without comparing the accuracy between the two algorithms.

For example, for N=100 Mach lines, if our calculation is done in 1 second, the algorithm of (Salhi, 2017) does the same calculation in about 9 minutes, without speaking about the precision obtained. So, our algorithm can be used successfully for meshes of very fine design.

Figure 2 show the effect of P_0 on the contour of the 2D *MLN*, giving M_E=3.00 at the exit when

T_0=2000 K. The design is done with a very large mesh point number using the algorithm presented in this work on the calculation of the flow parameters at the expansion center. Five values of P_0 were taken which are 1 bar, 5 bar, 10 bar 50 bar and 100 bar. In this figure, the contours for the same data for the *PG* (Emanuel and Argrow, 1988) and *HT* (Zebbiche and Youbi, 2007a) models were added for comparison purposes. The corresponding numerical design results are presented in in table 3. The influence of P_0 on the nozzle contour and on all the design parameters is clearly visible. In this case, the size of the nozzle given by our model *RG* is quite large compared to the size of the nozzles given by *PG* and *HT* models, whatever the value of P_0. This result is very advantageous making it possible to say that in order to have a complete expansion inside the nozzle according to the real gas flow behavior, a large space of the nozzle is required compared to that given by the *PG* and *HT* models.



Curve 1 : *RG* (P_0=1 bar) , Curve 2 : *RG* (P_0=5 bar)
Curve 3 : *RG* (P_0=10 bar) , Curve 4 : *RG* (P_0=50 bar)
Curve 5 : *RG* (P_0=100 bar) , Curve 6 : HT (T_0=2000 K)
Curve 7 : *PG* (γ=1.402)

Fig. 2 P_0 effect on the *RG* 2D *MLN* design

Table 3: Design values for the nozzles of Fig. 2

ν(°)	P_0 (bar)	L/y_*	C_{Mass}	C_F	y_E/y_*
1	1	22.307	45.398	0.353	5.673
2	5	22.190	45.166	0.351	5.579
3	10	22.102	44.986	0.350	5.571
4	50	21.387	43.533	0.342	5.569
5	100	21.085	42.911	0.339	5.359
6	*HT*	19.671	20.306	0.336	4.981
7	*PG*	16.818	17.291	0.296	4.211

CONCLUSIONS

This work enabled us to develop an algorithm making fast calculation of the reverse of *PM* for *RG* model, and determine all thermophysical parameters in the expansion center of the supersonic nozzle designed on *RG* model. The following conclusions are obtained:

1. The numbers N_T and N_ρ depends especialy on the desired precision ε.
2. The Gauss Legendre formulae of order 20 is used for the numerical evaluation of the presented 4 integrals with great precision.
3. The dichotomy algorithm is used for *T* calculation and the algorithm of successive approximations is suitable for the ρ calculation.
4. The *PM* depends on a single variable for the *HT* and *PG* models and on two variables for *RG* model, which affects the simplicity of calculation and the execution time between the three models.
5. Our algorithm can be used for very fine design meshes giving a great precision in a very reduced computation time.

Develop an algorithm to solve the two nonlinear algebraic equations to calculate T_2/T_1 and ρ_2/ρ_1 through the normal shock wave based on the *RG* model.

The second problem is to use this algorithm for the supersonic nozzle design of *RG* model.

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NOMENCLATURE

M	Mach number.
x, y	Coordinate of a point
a	Constant of intermolecular forces.
b	Constant of molecular size
α	Molecular vibration energy constant
t	Integration variable
v_1	First part for Prandtl Meyer function
v_2	Second part for Prandtl Meyer function
v	Prandtl Meyer function
E_1	First function for Bernoulli equation
E_2	Second function for Bernoulli equation
S_V	Sound velocity.
V	Flow velocity
θ	Flow angle deviation.
N_ρ	Number of iteration for solution ρ
N_T	Number of iteration for solution T
$\Delta\theta$	Step from the expansion center for θ.
P	Pressure.
T	Temperature.
R	Gas constant.
C_P	Specific heat to constant pressure.
C_T	Specific heat at constant temperature
C_F	Thrust coefficient of *MLN*
L	Length of the nozzle.
C_{Mass}	Non-dimensional nozzle mass.
y_E/y_*	Critical section ratio of *MLN*
γ	Specific heats ratio.
ρ	Density.
ε	Tolerance of calculation (desired precision).
f	Nonlinear equations
PG	Perfect gas model
HT	High Temperature model
RG	Real Gas model
PM	Prandtl Meyer function
MLN	Minimum Length Nozzle
MOC	Method Of Characteristics
PN	Plug Nozzle

Subscripts

0	Stagnation condition (combustion chamber).
*	Critical condition.
E	Exit section of the nozzle
i	Point.
K	For numerical iteration process
G, D	End of interval, see equations (14) and (15).