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Flexible Job Shop Scheduling Problem with Parallel Batch Machines to Minimize Total Weighted Flowtime with Machine Eligibility Restrictions in Automobile Gear Manufacturing Industry

R. Murugesan*, M. Rajeshwari**, Praveena Kumara K.M*** and G. K. Jagatheswari****

Keywords: Flexible Shop Job Scheduling Problem with parallel batch machines; Positive selection based Modified Clonal Selection Algorithm; Gear Manufacturing.

ABSTRACT

In this article, we address a Flexible Shop Job Scheduling Problem with parallel machine batch processing for minimizing total weighted flowtime of the production of automotive gears manufacturing industry. Scheduling has specifically addressed the bottleneck functioning of the Pre-Heat treatment stage of the vehicle gear manufacturing process. Numerous real-world scenarios have been taken into account, including machine eligibility constraints, sequence-dependent setup delays, and unequal release times. Dynamic beginning conditions are taken into consideration using a mathematical model that has been suggested. It is determined that the scheduling problem is NP-hard and used the meta- heuristic algorithm that is a Modified Clonal Selection Algorithm by Positive Selection method (PSMCSA) have been offered as ways to approach the issue. The performance of the suggested algorithms is assessed based on planned computational experiments. The proposed heuristic algorithms may consistently produce solutions that are close to the optimal ones as calculated by statistics in an acceptable amount of time, according to extensive computational assessments.

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* *Department of Mathematics, REVA University, Bangalore, India. Email id: contactmurugu@gmail.com.*

** *Department of Mathematics, School of Engineering, Presidency University, Bangalore, India. praveenmsk82@gmail.com*

*** *Department of Mathematics, REVA University, Bangalore, India. rajeakila@gmail.com.*

**** *Department of Science and Humanities, PES University-EC Campus, Bangalore-560 100, India. jagatheswarigk@pes.edu*

INTRODUCTION

Effective production planning and scheduling are essential for a company to remain competitive in a world where dynamic events are happening more often and affecting the production system in more complex ways. It can be quite difficult to schedule dynamic events correctly in real time on a daily basis. For many years, research has recognised the value of effective scheduling [Yugma, C., et al. (2012)] and the proper dynamic optimisation of manufacturing systems is increasingly the focus of research [Kopanos, G.M., et al. (2010)]. Researchers' literature surveys [Zhang, G., et al. (2009)] have demonstrated that artificial intelligence techniques [Klotz, E., Newman, A.M. (2013)] are among the best options for addressing dynamic occurrences like machine failure, adjustments in processing times, receipt of new orders, etc. [Ham, M., et al. (2011)]. According to the survey [Park, J., et al. (2018)], the complexity of the multi objective optimisation of the Flexible Job Shop Scheduling Problem (FJSSP) is not addressed by utilising just one artificial intelligence method [Fattahi, P., et al. (2009)]. Thus, the findings of the researchers indicate that it is reasonable to build various hybrid approaches. Researchers improve each method's effectiveness and resilience through hybridization by combining its benefits and removing its drawbacks [Ham, M., et al. (2011)], [Murugesan, R., Sivasakthi Balan, K. (2012)]. The flexibility of the production system, which is made possible by the proper mathematical description, actually reflects the reality of the complexity of the FJSSP problem [Imanipour, N. (2006)]. Effective solutions are made possible by the quick adaption of the suggested algorithms [Fattahi, P., Fallahi, A. (2010)] and the dependable automatic processing of dynamic events and system changes [Park, J., et al. (2018)]. Despite some techniques' drawbacks, including priority rules, their widespread use in commercial in the modern world, utilisation is beneficial [Zhang, Q., et al. (2012)].

The adoption of an efficient supporting simulation method is essential in any scenario, especially in the era of Industry 4.0 and the capacity to produce digital twins that are affordable and time-efficient [Baykaso ˘glu, A., et al. (2020)]. An efficient decision algorithm can be connected to a real-world production problem or service process using the potential of a data driven simulation methods that gathers vast amounts of data in real time [Jemmali, M., et al. (2022)]. In the era of Industry 4.0, where commercial tools are no longer the only means of attaining satisfying operation, the relevance of optimised dynamic production systems is highlighted [Jemmali, M., et al. (2022)]. The optimisation of crucial production parameters [Park, J., et al. (2018)], [Zhang, Q., et al. (2012)] highlights the need for highly connectable and potent supporting simulation environments [Torabi, S.A., et al. (2005)] and goes well beyond the scope of candid simulation interfaces [Sahraeian, R., Namakshenas, M. (2015)].

The presented study work's primary goal is to illustrate how crucial it is to create a good decision-making algorithm in order to address the FJSSP optimisation challenge. The goal is to test the proposed algorithm using data sets from a real-world manufacturing system supported by a simulation modelling method and to demonstrate its efficacy using test data sets to allow efficient comparison of the decision-making results of the proposed algorithm with the existing solutions. We want to provide a thorough optimisation system through the system's compatibility and adaptability, making it easier for users to plan their daily production in dynamic situations with the main goal of making the organisation competitive in terms of cost and time on the worldwide market.

In order to reduce the overall weighted flowtime, this study addresses the unique research topic of flexible job shop scheduling tasks across identical batch parallel machines. Numerous scholars have tackled the issue of parallel batch machine scheduling in various ways. Many sectors, like the production of paint, plastics, semi-conductors, chemicals, and paper, frequently use many resources with various capacities operating in parallel batch processing [Zhou, L., et al. (2019)]. The Pre-heat treatment phase of the production of automotive gears served as the inspiration for this paper.

The scheduling problems faced by these gear manufacturers during the Pre-Heat stage of gear manufacturing are the main subject of this study. The gear-shaving operation, which comes last in the Pre-HT stage, is the bottleneck operation because it requires a disproportionately longer processing time than the other procedures. Because we are only concentrating on the bottleneck operation in this study, all other processes can be thought of as a black box.

PROBLEM DESCRIPTION AND FORMULATION

An essential part of an automobile's gearbox system is the "gear train," as it is known in automotive terminology. Axles, a clutch system, oil seals, various other housings (castings), etc. are further components of the gearbox system, which is housed in a steel casting commonly referred to as the gearbox case. The gear train consists of a variety of gear types, couplings, and shifter forks. The vehicle gears can be categorised in a variety of ways. It can be categorised, for instance, according to the kind of teeth (spur, helical, hypoid, etc.), the geometry (module, diametric pitch, etc.), and the shape (solid, hollow, simple, cluster, shoulder, etc.). Steel forgings are used to make the gears and they have very strong quality requirements for the automobile (measured in terms of microns). Figure 1 shows the sample process' flow diagram of gear manufacturing process.

Pre heating treatment (Pre-HT), Heating treatment (HT), and post heating treatment (Post-HT) are the three main stages of the gear manufacturing process. Outsourcing a portion of the gear production process to subcontractors is a modern practise seen among companies that manufacture automobiles. Figure 1 shows the process' flow diagram

As a result, the Pre-HT and HT phases of gear manufacturing are contracted out. The two steps are carried out by the same subcontractor for a specific kind of gear. The vehicle manufacturing companies carry out the Post-HT stage internally because it is the final stage and crucial from a quality standpoint. As a result, subcontractors also known as gear manufacturers perform a large percentage of the work involved in making gears. The scheduling problems faced by gear manufacturers during the Pre-HT stage of gear manufacturing are the main subject of this paper. The gear-shaving operation, which comes last in the Pre-HT stage, is the bottleneck operation because it requires a disproportionately longer processing time than the other procedures.

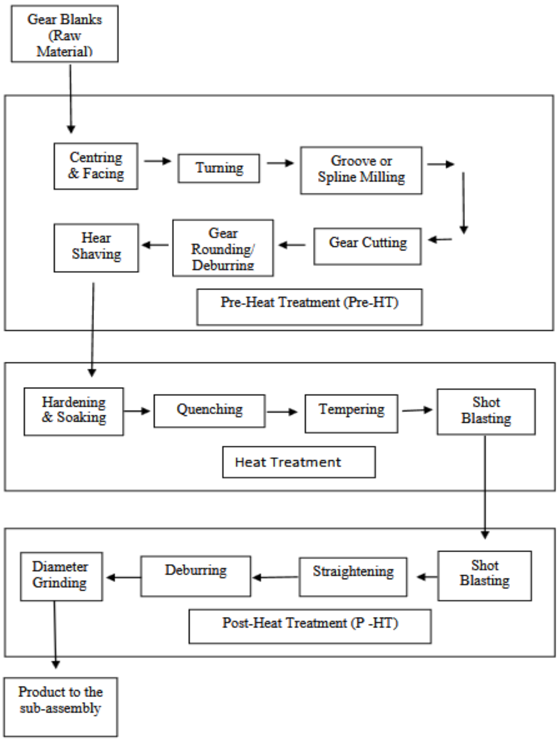

Figure 1: Sample Automobile gear manufacturing process

A gear manufacturer gets orders from various auto manufacturing companies, which are referred to as jobs in scheduling terminology. A specific quantity of identical gear blanks makes up a job. These jobs must be completed during the pre-HT and HT-stages of the gear manufacturing process. Finding bottleneck processes in the Pre-HT and HT stages is the main research challenge. One can anticipate having a competitive benefit among the gear makers and, in turn, among the organisations that build automobiles by addressing the bottleneck processes. Every gear manufacturer always strives to accomplish both throughput from their own company's perspective and due date compliance from the customer's, i.e., vehicle manufacturing organizations, perspective, as every gear manufacturer is involved in both the Pre-HT stage and the HT stage of gear manufacturing. These two goals heavily rely on the Pre-HT stage since it necessitates the best machine work assignment. Therefore, the Pre-HT stage is the research challenge in this paper for determining the best Cmax for throughput compliance and due date compliance.

The gear shaving process, which takes a comparatively longer time than other operations and is therefore the bottleneck activity, is where the research issue is found in the Pre-HT stage of gear manufacture. For processing this task, more than one machine is available. Jobs for this activity arrive dynamically, with varying setup requirements, priorities (which are typically determined depending on the type of jobs, type of vehicle manufacturing organisations, etc.), and schedule difficulty. Additionally, some machines might not be able to do a certain kind of job, placing additional restrictions on the job. From the perspective of the gear manufacturer, since this is the initial stage of the gear manufacturing process, increasing throughput is crucial. This circumstance aided in the definition of the study problem as the parallel batch processing machines of flexible job shop scheduling problem with Makespan(total completion time of the job) minimization as the goal.

Development of the Mathematical Model

The nomenclature of FJSSP with parallel batch machines processing model for the identified problem in gear manufacturing is discussed as follows.
The main sets:

J be the $number\ of\ jobs$ with numbers 1, 2 ... n

M $be\ the\ number\ of$ Machines with numbers 1, 2 ... m

Parameters

r_j $be\ the\ Release\ time\ of\ a\ job\ j$

w_j $be\ the\ weight\ of\ a\ job\ j$

p_j $be\ the\ \ Processing\ time\ of\ a\ job\ j$

s_{ij} $be\ the\ Setup$time from current setting of job i to subsequent of job j

$$e_{jk} = \begin{cases} 1 & if\ machine\ k\ is\ eligible\ to\ process\ job\ j \\ 0 & otherwise \end{cases}$$

Decision Variables

$$X_{ijk} = \begin{cases} 1 & if\ job\ j\ is\ processed\ immediately\ after\ job\ i\ on\ machine\ k \\ 0 & otherwise \end{cases}$$

Dependent Variables

C_j $Completion\ time\ of\ a\ job\ j$

P_{ijk} $Processing\ time\ of\ job\ j\ when\ it\ is\ processed\ immediately\ after\ job\ i\ on\ machine\ k$

Mathematical Model of the identified FJSSP with parallel batch machines in gear manufacturing is

$$Minimize\ \left(\max\left(C_j\right)\right) \tag{1}$$

Subject to:

$$P_{ijk} = s_{ij} + p_j + (1-e_{ik}) + (1-e_{jk}) \qquad \forall i \in J, \forall j \in J/\{1,2,\ldots,m\}, \forall k \in M \tag{2}$$

$$C_i + \sum_{k=1}^{m}(P_{ijk}X_{ijk}) + \left(\sum_{k=1}^{m} X_{ijk} - 1\right) \le C_j \qquad \forall i \in J; \forall j \in J/\{1,2,\ldots,m\} \tag{3}$$

$$r_i + \sum_{i=1}^{n}\sum_{k=1}^{m}(P_{ijk}X_{ijk}) \le C_j \qquad \forall j \in J/\{1,2,\ldots,m\} \tag{4}$$

$$\sum_{i=1}^{n}\sum_{k=1}^{m} X_{ijk} = 1 \qquad \forall j \in J/\{1,2,\ldots,m\} \tag{5}$$

$$\sum_{a=1}^{n} X_{ajk} + \sum_{b=m+1}^{n} \sum_{h \in M/\{k\}} X_{jbh} \leq 1 \qquad \forall j \in J/\{1,2,...,m\}, \forall k \in M \qquad (6)$$

$$\sum_{j=m+1}^{n} \sum_{k=1}^{m} X_{ijk} \leq 1 \qquad \forall j \in J/\{1,2,...,m\} \qquad (7)$$

$$X_{ijk} \in \{0,1\} \qquad \forall i \in J, \forall \in J/\{1,2,...,m\}, \forall k \in M \qquad (8)$$

The goal of the under-consideration research challenge is given in equation (1). The goal is to reduce the considered problem's Makespan as much as possible. The final job in the job sequence's completion time can be the Makespan of Flexible job shop scheduling problem.

The number restrictions in the problem under consideration are given in equations (2) to (8). The value of the dependent variable, which represents the processing time including the setup time of a work that immediately follows a specific job on a specific machine, is obtained using the formula (2). This value is calculated using the processing time, the setup time between the two jobs, and the eligibility of the job. A realistic value is only returned by the equation when both of the jobs in the pair are eligible for the machine, according to the structure of constraint (2).

To calculate a job's completion time and guarantee that the jobs run concurrently on the machines, utilise constraint (3). Constraint (4) makes sure that the sum of the release time of the job and the processing time with the setup time of the present job is not less than the finishing time of a task on a machine. The assignment constraint (constraint 5) makes sure that all n existing jobs are handled on a single machine and are immediately preceded by a single job. Limitation (6) guarantees that there can only be one replacement for each position. The restriction that if a task is available and succeeds a work on a certain machine, it cannot thereafter immediately precede a job on any other machine is handled by constraint (7) and constraint (8) then gives the choice variable binary values.

EXPERIMENTAL DESIGN

The goal of this study was to create an effective scheduling algorithm to address the Flexible Job Shop Scheduling Problem with batch parallel machines in the manufacturing sector. In a changing and continuous production setting, the Clonal Selection Algorithm from the Artificial Immune System is seen as a technique for resolving the Flexible Job Shop Scheduling Problem with batch parallel machines. The created scheduling method is an enhanced version of the traditional Clonal Selection method, or CLONALG in OR society, which is based on the Artificial Immune System. improvements made in CLONALG's initialization, mutation, termination, and memory stages, among others. Positive Selection based Modified Clonal Selection method (PSMCSA) was the moniker given to the improved method. These advancements were made possible by the ongoing research that was done at every stage of CLONALG.

There are two stages to the enhanced scheduling algorithm. For initialising the first-generation population of the Clonal Selection Algorithm, the first stage comprises of a T-cell based Positive Selection Algorithm. In most cases, initialization is carried out using chaotic operation or random population. The Modified Clonal Selection method, which generates the Final Solution Library for the given problem, builds the second stage of the method.

The standard Clonal Selection Algorithm has been modified in the following ways:

1. Positive selection method inspired by the immune system used for improved initialization of the clonal selection algorithm to start the search for the ideal solution from a better position.

2. Effective mutation with a fixed mutation rate and the Log-Normal mutation operator

3. Society-based receptor editing, in which the antibody society is first created from the antigen families. This prevented the loss of numerous superior antibodies produced by a single antigen. The algorithm is compelled by this tweak to find the ideal answer during the early iterations.

4. Modified the algorithm termination criteria such that even while the algorithm finds the best answer in the first few runs, it keeps looking for other, individually optimum solutions for the predetermined number of runs. Due to this, the method produces a strong solution library that contains numerous optimal solutions.

Gear manufacturing is the issue that is being looked at to determine whether the proposed algorithm is appropriate for a real-world scenario. As we've seen, one of the sectors propelling the automotive industry is gear production. Several auto manufacturing businesses place job orders with a gear manufacturer. The window for scheduling can be expressed in terms of weeks, days, or hours. The systems should allow for the collection of the necessary data while avoiding delayed decision-making and expensive continual monitoring. The hour unit appears to be the optimum choice as a trade-off between complexity and efficacy based on the industry's observed pattern of task arrivals and processing. As a result, job characteristics like release, processing, setup times, etc. are expressed in hours. In this investigation, the hour is used as the measurement unit. After this introduction, the following part will describe the factors taken into account in the suggested experimental design as well as the various levels for each aspect.

The major goal of the experimental design is to produce adequate test data for evaluating PSMCSA's applicability to real-world scenarios. One needs to be aware of a variety of elements that may influence scheduling decisions in order to produce appropriate test results. Additionally, distinct levels of each element must be defined. These standards must be based on the commonplace procedures seen in the sector. As a result, the following seven variables are noted:

The term "number of jobs" (N) refers to how many jobs we run into within the scheduling window. According to the industry, depending on the size of the gear manufacturer, seasonality, and other factors, the number of jobs inside the scheduling window may range from approximately 25 to 150. However, a small size is assumed for the factor number of jobs n and three tiers for the same based on the knowledge gained in validating the suggested model. The three levels have values of n=8, n=10, and n=12, respectively.

Machine count (M): A total of 12 jobs are possible. As a result, two levels for this component with the values m=2 and m=3 are chosen in connection to that. With this, the system is balanced between being overloaded and being underutilised.

Machine eligibility (e): This element determines if a machine is qualified to carry out a specific task. This variable has two facets. The first is the number of machines that can process a job. The next is whether or not the job can be processed on a certain machine. A maximum of three machines may be present. This factor will have two levels as a result. The first level will have distribution V{1,2} eligibility, meaning that a job can only be processed on a maximum of two machines. With a distribution of V{1, number of machines}, the second level will be eligible. Here, the selection probabilities for all machines will be equal. The uniform distribution was chosen because, due to its relatively high variation, it enables the testing of problem-solving approaches in challenging environments.

Job release time (r): It was noted that orders (jobs) come in steadily and at regular intervals. However, there is no preference for the rate of task arrival during the scheduling window. Therefore, it is expected that the times of job release are evenly spread over the entire scheduling window. As a result, there is just one level of release times with a uniform distribution of V{0,23}. This period was calculated using the 24 hour scheduling window as the basis.

Job weight (w): Three sub-factors that affect the weight (or "priority") of a job have been established in practise:

a. The history of interactions with the auto industry to assess how significant it is to the gear manufacturer.

b. The cost of the job, which varies depending on the processes involved, the level of precision needed, the raw materials used, etc.

c. The task's urgency for the next operation

Each of the three sub-factors may have a maximum of two values (low and high). The weight can therefore vary from 1 to 8 (2x2x2). For this component, two levels are taken into account. Weights for the first level will come from the V{1,8} distribution. The second level will depict a situation that is more accurate. It was noted from the sector that there is a strong likelihood for the weight's central figures. As a result, weights in the second level will often come from the distribution V{3,6} in 70% of the cases. Additionally, weights will come from either V{1,2} or V{7,8} for the remaining 30% of the period.

Job Processing Time (p): As previously established, a job's processing time is determined by the number of gears in the job (lot size), the processing time of each gear in the job, and other factors. It was found that processing a lot often requires 2 to 5 work shifts of 8 hours each. That is, typically, processing takes 16 to 40 hours each process. As a result, two levels of processing time are taken into account (in hours). Processing time for the first level will come from a distribution of V{16,40}. The second level will typically capture a situation that is more realistic. The distribution V{24,32} accounts for 60% of the processing times at this level, whereas V{16,23} and V{33,40} each account for 40% of the processing times.

Job setup time (s): Sequence setup time is dependent on the pair of jobs that make up the predecessor and successor. The actual setup time ranges from two to eight hours. Two levels for the setup time are taken into consideration in this impact. The setup time for the first level will come from a distribution of V{2,8}. The second level will be like the real world. When it comes to setup time, it has been noted that each given pair of jobs is typically either nearby or far away from another pair. V{2,3} or V{7,8} will thus provide 70% of the preparation time for this scenario in the second level, and V{4,6} will provide the remaining 30% of the setup time at the appropriate moments. A MATLAB programme is created in MATLAB 2020a to produce a variety of problem situations according to the setup made by the above-mentioned parameters.

NUMERICAL RESULTS AND DISCUSSION

Sample instances are constructed using the settings listed in the previous section in order to investigate the applicability of the proposed scheduling algorithm PSMCSA for the gear manufacturing sectors. To accommodate the FJSSP with parallel batch processing machines mathematical model used in the gear manufacturing business, the established scheduling programme has been changed. Tables 1 and 2 provide one numerical example. Table 3 lists the PSMCSA simulation parameters and table 4 describe the Optimal schedule for the numerical example of the research problem.

Table 1: Data on job release time, weight, processing time and machine eligibility for the numerical example

Job (j)	Release time (r_j)	Weight (W_j)	Processing Time(p_j)	Eligibility for		
				$M/c1(e_{j1}$	$M/c2(e_{j2}$	$M/c3(e_{j3}$
$j1$	9	3	32	1	0	1

$j2$	23	4	24	1	1	0
$j3$	0	4	23	0	0	1
$j4$	22	1	35	1	1	1
$j5$	17	3	22	0	1	1
$j6$	15	2	33	1	1	1
$j7$	10	3	28	1	1	1
$j8$	4	4	24	1	1	1

Table 2: Data on setup time between pairs of jobs for the numerical example

$Job\ i$	$Job\ j$							
	$j1$	$j2$	$j3$	$j4$	$j5$	$j6$	$j7$	$j8$
$j1$	0	3	2	8	4	4	7	5
$j2$	2	0	3	5	6	3	3	3
$j3$	3	2	0	4	5	8	6	4
$j4$	6	4	7	0	4	6	4	2
$j5$	6	5	8	8	0	5	6	5
$j6$	8	6	6	7	5	0	5	2
$j7$	4	7	2	2	7	6	0	4
$j8$	2	3	6	4	4	5	1	0

Table 3: Simulation parameters of PSMCSA for the sample problem

S. No.	Parameters	Values
1	Antibody Length (8*3)	24
2	Initial Population size	100
3	Makespan threshold for PSA	200
4	Antigen library size	$N = 5(\,an\ instance)$
5	Number of clones per antigen	$n_c = 50$
6	Number of mutation per antibody	$mu = 10points$
7	Antibody generation size	$NC = N * nc = 250$
8	Number of generations per iteration	$K = 50$
9	Number of iterations	$M = 10$
10	Final Solution library size	$N * M = 50$

Table 4: Optimal schedule for the numerical example of research problem

Job (j)	**Release Time (r_j)**	**Weight (w_j)**	**Best Solution generated by PSMCSA and ILP**				
			Machine allocated	**Start time**	**Completion time**	**Flow time**	**Weighted flow time**
j1	9	3	3	34	72	63	189
j2	23	4	1	31	54	31	124
j3	0	4	3	3	34	34	136
j4	22	1	2	70	103	81	81
j5	17	3	2	40	70	53	159
j6	15	2	1	54	87	72	144
j7	10	3	2	10	40	30	90
j8	4	4	1	4	31	27	108

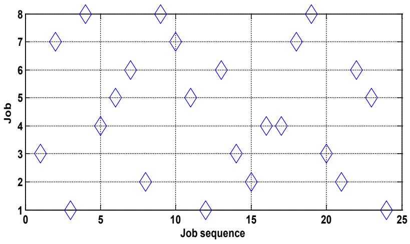

Figure 2: Antibody corresponds to an optimal schedule

Table 5: Basic performance metrics for a sample instance in gear manufacturing industry

Sl. No.	**Parameters**	**Proposed PSMCSA**
1	Makespan of Best Solution produced by ILP	103
2	Makespan of Best Solution produced by PSMCSA	103
3	Mean of Makespan	130
4	Time taken to reach the first optimal schedule	4s
5	Algorithm run time	30s
6	Number of times the algorithm reached the best solution over 10 iterations	7/10
8	Number of unique optimal schedules in FSL	28

Table 5 is an example of an ideal schedule created by PSMCSA using the Final Solution Library. Figure 2 shows one of the antibodies from FSL. The horizontal axis indicates the job's position in the job sequence, while the vertical axis displays the job number. The antibody has a length of 64 and a job number that ranges from 1 to 8 depending on the problem at hand. The length FJSSP is 8x8. In Table 6, the fundamental performance indicators are listed. According to Table 6, PSMCSA can get at the best answer that Integer Linear Programming method (ILP) came up with Within 4 seconds, the first ideal solution is found, and the procedure is completed in 30 seconds. 28 ideal solutions are contained in the Final Solution Library produced by PSMCSA. Table 6 lists the performance results of PSMCSA for different situations together with the outcomes produced by ILP. With the exception of instances 16, 22, 45, 61, 63, 67, and 93, practically all instances show that PSMCSA performs better than the ILP.

Table 4 shows the machine sequence from top to bottom on the vertical axis. The schedule begins with machine 3 and ends with machine 1 in accordance with the job sequence. Every machine has a strict schedule that leaves idle time in either the front or back end.

Multiple jobs can be rearranged to reach the best solution because there is idle time at the end of the schedule. However, no single job can be left shifted in any machine without interfering with the order of other machines. Thus, the schedule that has been provided is both active and ideal.

Table 6: Numerical results generated by ILP and PSMCSA for the designed test problems

Problem instance	Best Makespan		Problem instances	Best Makespan		Problem instance	Best Makespan	
	ILP	PSMCSA		ILP	PSMCSA		ILP	PSMCSA
1	1398	1245	33	5184	5008	65	3486	3245
2	1347	1301	34	5168	4968	66	3478	3354
3	1314	1298	35	4626	4509	67	3861	3898
4	1340	1305	36	4605	4569	68	3972	3308
5	2552	1895	37	3004	2994	69	5050	5014
6	2460	2210	38	3026	2896	70	4879	4783
7	2240	2078	39	3283	3276	71	4360	4230
8	2360	2740	40	3228	3045	72	4398	4349
9	2012	1806	41	1527	1456	73	3102	3011
10	2099	1733	42	1500	1250	74	3172	3035
11	2601	2243	43	1835	1560	75	3103	3089
12	2594	2256	44	1876	1866	76	3145	3124
13	2864	2690	45	3122	3126	77	6016	5904
14	2917	3001	46	3080	2988	78	5840	5760
15	3017	3007	47	3799	3699	79	2686	2567
16	2948	2698	48	4043	4004	80	5164	5126
17	3089	2907	49	2907	2897	81	3405	3398
18	3125	3123	50	2917	2864	82	3487	3384
19	3108	2968	51	3037	2987	83	3360	3209
20	3147	3042	52	3082	2988	84	3358	3254
21	1451	1243	53	2987	2973	85	3819	3818
22	1391	1401	54	2973	2893	86	3876	3878
23	1562	1478	55	2921	2845	87	3772	3767
24	1596	1467	56	2824	2698	88	3716	3674
25	2354	2254	57	1832	1802	89	4049	4005
26	2278	2179	58	1906	1845	90	3153	3045
27	2121	2012	59	1823	1783	91	4106	4056
28	2212	2111	60	1826	1820	92	3956	4016
29	1195	1004	61	2583	2840	93	4509	4458
30	1124	1015	62	1768	1698	94	3420	3246
31	1031	984	63	2189	2220	95	3379	3364
32	1037	964	64	2072	2034	96	2802	2796

CONCLUSION

To determine whether the suggested scheduling algorithm, PSMCSA, is appropriate for use with real-world issues, the scheduling problem in the automobile gear manufacturing industry was chosen. The Pre-HT stage bottleneck was discovered, modelled as a Flexible Job Shop Scheduling Problem with parallel batch processing machines, and resolved with PSMCSA. Numerous instances of the modelled FJSSP were given numerical results, and those results were contrasted with those obtained using the Integer Linear Programming (ILP) method. The combined data revealed that PSMCSA performs better than ILP. This study demonstrated PSMCSA's ability to resolve FJSSP in the context of the automotive manufacturing industries.

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