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Gear Root Bending Stress Reduction by Employing Dual Asymmetry – Asymmetry in Tooth Profile Shape and Root Fillet Form

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Keywords： dual asymmetry, asymmetric root fillet, root bending stress, finite element analysis.

ABSTRACT

Conventional gear designs produce symmetrical involute and symmetrical fillet forms on both drive and coast sides of the tooth. Asymmetry in involute form has been studied by researchers to increase tooth root thickness and hence to reduce the root bending stress. Similarly, asymmetry in fillet form has also been recently studied with resulting 10% reduction in bending stress of conventional 20degree symmetric tooth gears. In this paper, asymmetry both in involute and root fillet forms are employed together to further investigate any likely reduction in bending stress. Results for different levels of asymmetry in both tooth form (pressure angles) of drive and coast sides and also for different levels of cutter tip radii coefficients of drive and coast sides produced bending stress reductions between 9-15%.

INTRODUCTION

Gears under torque experience two different kinds of stress mainly; contact stress at gear tooth surface and bending stress at tooth root region. Gear tooth root strength and tooth surface durability relates to the ability to resist tooth breakage under bending stress and pitting and scuffing under contact stress respectively. Wilfred Lewis introduced an equation for estimating the bending stress in gear teeth based on the well-known bending equation in beams (Figure 1.a).

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Equation announced in 1892, still remains the basis for most gear tooth bending design today, with many factors, including tooth form factor and others, added to this basic equation later (Budynas, 2006). Referring to Figure 1.a, it is assumed that the maximum bending stress in a tooth root occurs at point “a”, tensile stress loaded side of the tooth (Budynas, 2006). Moment of inertia of critical section is very essential for gear performance in terms of bending stress. Root critical section thickness, “t”, at point a, is taken into consideration for calculation of inertia hence root bending stress.

Gear tooth profile is usually composed of two different curves; involute and root fillet curves. The generation of involute spur gears (as shown in Figure 1.b, c and d) by a rack-cutter (MAAG), hob cutter, or pinion cutter (Fellows shaping) is a widespread practice in industry (Davis, 2005). The gear root fillet profile is typically determined by the generating cutting tool (gear hob or shaper cutter) tip trajectory also called the trochoid (Litvin, 2004). Fillet radius at tooth root depends on the method by which the gear is cut (Kapelevich and Shekhtman, 2008). Radius of curvature at the fillet, created by generation method, is not a fixed value; it varies from point to point.

There is an increasing demand for high load carrying capacity of gears and therefore many researchers investigate different design alternatives to reduce root bending stress by searching alternative tooth and root profiles. Zhao, Zhang, Liu and Wang, (2014) made an optimization for gear root stress by using quadratic rational Bezier curve for the cutter tip and showed the nonlinear relationship between the design variable (cutter tip profile parameter) and tooth root bending stress. Their study focuses on optimization of cutting tool tip rather than the optimization of tooth root/fillet profile.

Hebbal, Ishwar, Rayannavar and Prakash, (2014) investigated gear tooth fillet stress by replacing the conventional trochoid fillet with polynomial curves.

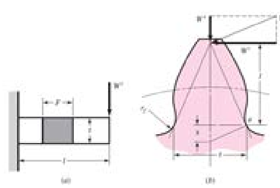
An initial circular root fillet profile is constructed by drawing an arc tangent to working profiles and root circle, and this arc is taken as reference root fillet to generate alternative root fillet profiles. The circular arc is divided into six segments. New fillet profiles are constructed by displacing the middle points radially using different relations and keeping the end points fixed. The improved root fillet profiles (of polynomial shapes derived by distorting the circular fillet form) resulted 9 to 12% reduction in bending stress compared to conventional trochoidal root profile. It looks advantageous in terms of bending stress level, but manufacturing feasibilty of such special fillet forms is another concern.

Ristić and Kramberger, (2014) gave attention to impact of gear tooth fillet radius on root stress value and root stress distribution at critical cross section. Their study focused on finding the optimal root fillet radius to minimize the root stress intensity. Results achieved by application of numerical methods and real working conditions simulation of tooth root fillet radius in two cases: with one tooth root fillet radius and with two fillet radii (“two-level approach” in a root). Two fillet radii are recommended to apply for the tensile side of the gear tooth profile in Ristić and Kramberger, (2014). This application will remain limited in practice, and it does not always produce low stresses as stated in the same paper, due to manufacturing difficulties about obtaining the same root profile for gear pairs which will work together.

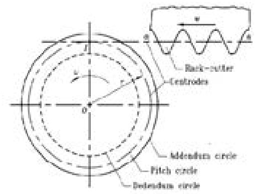
For both reversal and non-reversal (unidirectional) operation cases, load on driving flank of gear tooth is always much larger than coast side flank. For the sake of manufacturing, however, gear teeth are usually cut in symmetric form (having the same pressure angle on both driving and coast sides). Pressure angle has long been a design parameter to help designers reduce root bending stress (via a larger tooth root thickness) by employing larger pressure angles like 25degrees or more.

In old times both standard and larger pressure angle gears were also used as symmetric tooth gears but recently asymmetric tooth forms were also employed to help avoid using smaller cutter tip radii which result in smaller radius of curvature at tooth root. A fact is that for generating type of gear cut the larger the pressure angle the smaller the cutter tip radii allowed hence the smaller radius of curvature at tooth root. A smaller radius of curvature at tooth root means a larger concentration of the bending stress at tooth root. Symmetric tooth involute gears with standard and larger pressure angles are usually preferred for both unidirectional and reversal operation of gears while asymmetric tooth involute gears are preferred for unidirectional operations mostly. However, compared with larger pressure angle gears it is expected that at the expense of slightly increased bending stress, asymmetric tooth involute gears can be used for reversal operations too. Advantage of asymmetric tooth gears over symmetric tooth gears with standard pressure angles have already been proved by many researchers (Akpolat, Yildirim, Sahin, Yildirim, Karatas, Erdogan, 2017; Akpolat, 2017; Kapelevich and Shekhtman, 2008; Mallesh, Math, Ashwij, and Shanbhag, 2009; Mallesh, Math, Uttesh, 2009; Singh and Senthilvelan, 2007).

In this paper, in reference to standard tooth form defined by ISO, (1998), individual and combined effects of different parameters like, asymmetry in tooth profile, asymmetry in tooth root trochoidal fillet and the form of fillet (circular fillet created by not the generation processes but forming processes) on tooth root bending stress of external spur gears were investigated. All these parameters are somehow the tools to increase the thickness and the radius of curvature at critical section of the tooth.



a.Cantilever beam model



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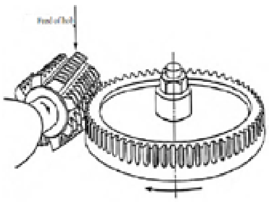
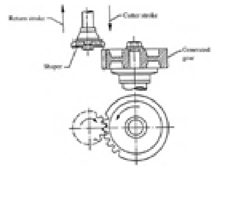


c.Hobbing



d.Shaping

Fig. 1. Cantilever beam model of a gear tooth under load (a) and gear generating methods (b, c, and d)

LITERATURE

There are many studies (Devendran and Vacca, 2016; Marimuthu and Muhtuveerappan, 2014, 2016; Mohan and Senthilvelan, 2014) focus on evaluating the performance of asymmetric gears and their performance related parameters. Many researchers studied on comparison of symmetric and asymmetric tooth gears with some advantages including reduced bending stress, contact stress, in some cases reduced sliding velocity and wear rate. Some of these studies (Akpolat, 2018; Alipiev, 2014; Devendran and Vacca, 2016; Kapelevich and Shekhtman, 2008; Mallesh, Math, Ashwij, and Shanbhag, 2009; Mallesh, Math, Uttesh, 2009; Marimuthu and Muhtuveerappan, 2014, 2016; Mohan and Senthilvelan, 2014; Spitas, Spitas, Amani, Rajabalinejad, 2014) were reviewed in paper of Akpolat, Yildirim, Sahin, Yildirim, Karatas, Erdogan, (2017) with detailed results.

In reference paper of Akpolat, Yildirim, Sahin, Yildirim, Karatas, Erdogan, (2017), the effect of symmetric and asymmetric cutter tip radii on root stress of symmetrical gears was studied by keeping all other gear parameters constant. Firstly, symmetric coefficient of cutter tip radii for both drive and coast sides were used, then asymmetric cutter tip radii coefficients for two (both drive and coast) sides of gear tooth profile were used to benefit from larger radii of curvature on drive or tensile side of the flank for a reduced bending stress. Almost 10–11% reduction in bending stress was obtained by using asymmetric cutter tip radii coefficients for two sides of gear tooth profile with standard center distance and no tooth interference. This type of asymmetric cutter tip radii (of circular shapes) generate trochoidal fillets on both sides of a tooth based on the conventional generating type gear cutting processes like hobbing, shaping, grinding etc. However, asymmetric root-generated by asymmetric cutter tip- was employed with only the symmetric tooth and not employed with the asymmetric tooth yet. It is well known that asymmetric tooth help reduce root bending stress too. Therefore, here in this study, “dual asymmetry”, both in tooth shape and the root fillet form are used together for further reduction in root bending stress of gear tooth under torque/load.

CASE STUDIES

Based on the literature review and the bending stress theory of gear tooth, it is rather clear that asymmetry of gear tooth profile, the fillet geometry of tooth root profile and eventually the cutter tip geometry (cutting the tooth root fillet itself) has a significant potential to effectively reduce the tooth root bending stress.

Manufacturing of non-standard (circular, optimized, spline, two fillets etc.) fillet geometries are usually more expensive and more difficult than geometries generated by using conventional gear manufacturing methods. Similarly non-standard cutter geometries (Bezier like cutting tips etc.) requires special tooling and may cause unexpected fillet profiles if the cutting tool designer is not well experienced. For years, involute gear tooth including fillet with trochoid form has been successfully manufactured by using conventional rack and hob type cutting tools with circular tool tip having a radius of ρ_F.

$$\rho_F = \rho_F^* . m_n \quad (1)$$

Where ρ_F^* may vary between two (lower and upper) limits depending on dedendum and clearance values.

Long experience gathered over the years has also perfected the design of both rack and hob type cutters for mass production of such gears. However, most cutters were designed as symmetric regarding the radii of tool tip on drive and coast sides as seen in Spitas, Spitas, Amani, Rajabalinejad, (2014) and international gear standards ISO, (1998) and DIN, (1986).

In this study, as an extension of study of Akpolat, Yildirim, Sahin, Yildirim, Karatas, Erdogan, (2017) different forms of both tooth and the root will be studied in a systematic way. Regarding tooth and root forms, both symmetric and asymmetric tooth forms will be studied in combination with three different forms of root shape namely the full circular, symmetric trochoid and asymmetric trochoid (see Table 1 for abbreviations). Asymmetric root forms are generated by rack cutters with asymmetric rack tip radii and no cutter design, so far, with asymmetric tool tip radii has been noted in standards to effectively reduce the bending stress in tooth root. It is intended to use maximum limit of cutting tip radius coefficient for symmetric and asymmetric root fillet without sharpening of cutting tool tip to obtain unpointed tip tool. Spur gear pair parameters studied for bending stress reductions through tooth shape and root form modifications are given in Table 2. Case studies are performed for constant torque and reference center distance values. For each case, interference is checked for meshing of gear pairs.

Table 1. Abbreviation of case study alternatives in terms of tooth shape and root fillet profiles

Case Study Alternatives	Abbreviations
Symmetric Tooth - Circular Root	ST-CR
Symmetric Tooth - Symmetric Root	ST-SR
Symmetric Tooth - Asymmetric Root	ST-AR
Asymmetric Tooth - Symmetric Root	AT-SR
Asymmetric Tooth - Asymmetric Root	AT-AR

Symmetric Tooth Profile with Circular, Symmetric and Asymmetric Root Fillets

Symmetric tooth profile gear with different fillet profile types such as circular, symmetrical and asymmetrical root fillets are studied to evaluate the effects of root profile and cutter tip radius coefficients on bending stress (cases ST-CR, ST-SR and ST-AR). For 2-D Finite Element analysis of the spur gears, MSC Software MARC is used to simulate the working conditions (meshing under design load) of different gears with consistent boundary and loading conditions. Tooth load is applied at the highest point of single tooth contact (HPSTC) as pressure from element edge and gears are fixed inside the bore diameter as shown in Figure 2. Element types are quadratic eight node (Q8) and triangular six node (T6) elements. Element size on the tooth surface contour is about 0.10 mm and total numbers of elements and nodes are around 36485 and 111770 respectively. FEA model and bending stress result of finite element analysis for case of ST-SR3 are presented in Figure 2 and 3 respectively.

Table 2. Spur gear parameters used in case studies

Parameter	Value	Unit
Module	3.00	mm
Face Width	20.00	mm
Teeth Number of Driving Gear	40	–
Teeth Number of Driven Gear	40	–
Addendum	1.00*m	mm
Dedendum	1.25*m	mm
Center Distance	120.00	mm
Torque	160.43	Nm
Pitch diameter	120.00	mm
Rim thickness (Root Dia.-Bore Dia.)/2	31.25 ((112.5-50.0) / 2)	mm
Rim backup ratio (Rim thickness/tooth height)	4.63	-

DXXCYY is the representation of gear tooth profile shape; D and C refer to drive and coast sides of gear tooth flank whereas XX and YY show pressure angle values of the drive and coast side respectively. For example, D20C25 refer to a case where tooth has pressure angles of 20degree on drive side and 25degree on coast side.

In Table 3, bending stress values calculated by the finite element analysis for symmetric gears of D20C20, D25C25 and D30C30 with different fillet geometries such as circular, symmetrical and asymmetrical ones are presented. When case studies are grouped based on pressure angle values for D20C20, D25C25 and D30C30 in itself, it is easier to compare effect of root fillet type and coefficient of tool tip radius on bending stress. With symmetric tooth profile of D20C20, root bending stress value decreases from 113.34 MPa to 108.69Mpa and to 105.98MPa for the root shapes of symmetric with cutting tool tip radius coefficient of 0.38, (suggested by ISO standard), circular root fillet and symmetric with cutting tool tip radius coefficient of 0.47 respectively. Here it is seen that on the contrary to expectations circular root fillet does not produce the least stress, because symmetric root with cutting tool tip radius coefficient of 0.47 has resulted in a better performing fillet with possibly larger curvature and less stress concentration. However, circular root fillet does still produce less bending stress compared to the case of symmetric root with cutting tool tip radius coefficient of 0.38. It is clear from results in Table 4 that increasing cutting tool tip radius coefficient from 0.38 to 0.47 provides a stronger tooth root section (with 6.5% less stress) than what a full circular root form provides (with 4.1% less stress) for the current gear data. Compared with the circular root form, almost 2.5% more reduction in stress (due to increased cutter tip radii) will certainly improve the gear service life in bending. Larger cutting tool tip radius coefficient of 0.47 is suggested by Akpolat, (2018), Akpolat, Yildirim, Sahin, Yildirim, Karatas, Erdogan, (2017), Spitas, Spitas, Amani, Rajabalinejad, (2014) as a means to reduce bending stress. No interference problem has been experienced with the cutting tool tip radius coefficients up to 0.47 as stated in studies of Akpolat, (2018), Akpolat, Yildirim, Sahin, Yildirim, Karatas, Erdogan, (2017), Spitas, Spitas, Amani, Rajabalinejad, (2014) too. Such an improvement obtained by cutter tip radius change naturally causes an inclination to further increase the tip radii but increasing radius beyond 0.47 is not limitless. There is a limit for the radius beyond which cutting tool tip start to sharpen at the midpoint and a tool form dis-order occurs as shown in Figure 4.a. Such problems regarding cutting tool form dis-order were also explained by Akpolat, (2018), Akpolat, Yildirim, Sahin, Yildirim, Karatas, Erdogan, (2017). Due to limitation of tool tip sharpening and form dis-order, it is not possible to make tool tip radius larger beyond the limit of 0.47 for symmetric fillets. However any further increase in tool tip radius without harm to tool itself and the tooth would be likely to help reduce bending stress further. An alternative means of increasing the tip radius (without harming the tool) is to change the form of the cutter and fillet by shifting from symmetric tool tip to asymmetric one. Asymmetrical cutting tool tip (without any dis-order as seen in Figure 4.b) will help increase critical tooth thickness and decrease stress concentration more to decrease bending stress at tooth critical section. Coefficients of asymmetric cutting tool tip of 0.58 and 0.36 for drive and coast side (Case ST-AR1) are the maximum limitations for D20C20 type gears. There is a significant reduction of 10.2 % when Case ST-AR1 is preferred instead of Case ST-SR1 (in Table 4). Beyond these limits of root fillet coefficients (stated for Case ST-AR1), any value will cause interference as shown in Figure 5.

These limit values, however, change with pressure angle of the tooth (Akpolat, 2018). Increase in pressure angle causes a thicker tooth at root section while a reduced tooth space near root land. This then will decrease maximum limit of the fillet radius hence the tool tip radius. The upper limit for usable root fillet radius (or cutting tool tip radius) depends upon pressure angles of both flanks of involute tooth profile as shown in Figure 6. For D25C25, having a pressure angle of 25degrees, 0.31 (Case ST-SR3) is the maximum limit for coefficient of symmetric cutter tip radius with a root stress of 107.70MPa. A circular fillet with radius of 1.166mm (Case ST-CR2) can be used but with a resulting root stress slightly more than the trochoidal fillet stress (111.37MPa). A stress reduction of almost 14 % (95,98MPa), with reference to circular fillet, can be obtained if an asymmetric trochoidal fillet (coefficients of 0.53 and 0.10 for drive and coast sides) is used as in Case

ST-AR3, (Table 5).

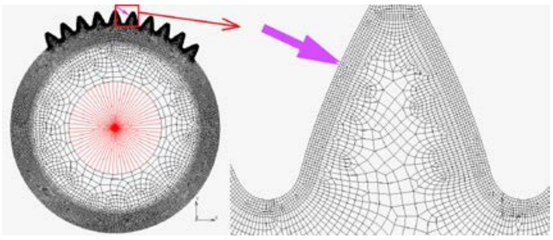


Fig. 2. Finite element model of symmetric tooth profile gear with symmetric root fillet (Case of ST-SR3)

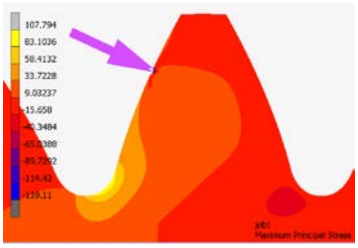
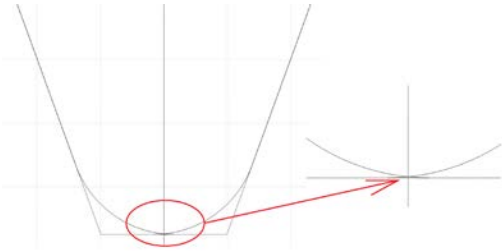


Fig. 3. Finite element results of root bending stress (maximum principal stress) for symmetric tooth profile gear with symmetric root fillet (Case of ST-SR3)

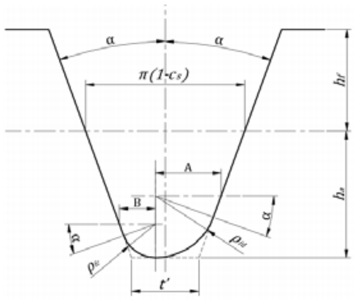
Table 3. Root bending stress of different root shapes with symmetric tooth profile

Case	Tooth Profile (DXXCYY)	Tooth Force (N)	ρ_f^*		σ_b
			ρ_{fd}^*	ρ_{fc}^*	FEA (MPa)
ST-CR1	D20C20	2845.40	Circular		108.7
ST-CR2	D25C25	2950.21	Circular		111.4
ST-CR3	D30C30	3087.44	Circular		125.2
ST-SR0	D20C20	2845.40	0.30		120.8
ST-SR1	D20C20	2845.40	0.38		113.3
ST-SR2	D20C20	2845.40	0.47		106.0
ST-SR3	D25C25	2950.21	0.31		107.8
ST-SR4	D30C30	3087.44	0.11		123.5
ST-AR1	D20C20	2845.40	0.58	0.36	101.7
ST-AR2	D25C25	2950.21	0.43	0.20	100.7
ST-AR3	D25C25	2950.21	0.53	0.10	96.0
ST-AR4	D30C30	3087.44	0.12	0.10	121.5

Similar circular, symmetrical and asymmetrical fillets could be applied for symmetric tooth profile gear of D30C30 with pressure angle of 30degrees. Tooth root bending stresses calculated by FEA are presented in Table 6 for cases of ST-CR3, ST-SR4 and ST-AR4. With reference to circular fillet, symmetrical trochoidal fillet will have 1.32 % reduction while asymmetrical trochoidal fillet will have nearly 3.0 % reduction in bending stress.



a. Symmetric cutting tool tip with $\rho^*_{fd} = 0{,}55$; $\rho^*_{fc} = 0{,}55$ (15)



b. Asymmetric cutting tool tip, Akpolat (2017)

Fig. 4. Cutting tool configurations

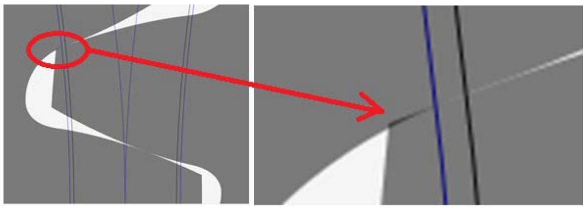


Fig. 5. Interference simulations of symmetric tooth gear of D20C20 with asymmetric root fillet (ρ^*_{fd} = 0.70, ρ^*_{fc} = 0.24) (Interference occurs)

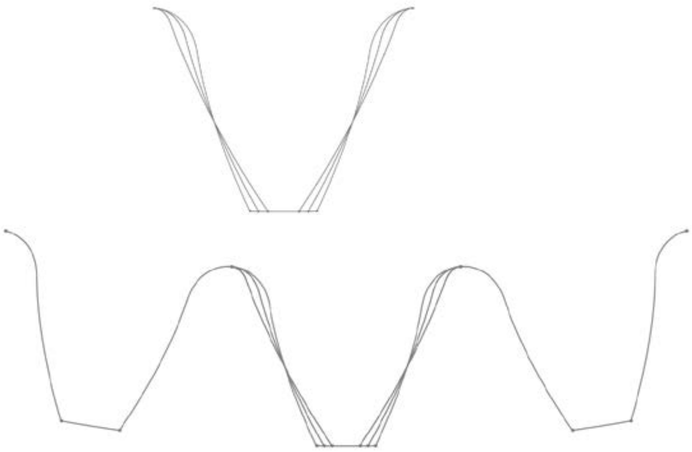


Fig. 6. Root fillets and root regions for cases of ST-SR2, ST-SR3 and ST-SR4
(Having different symmetric pressure angles with limiting symmetric cutter tip radii coefficients)

Table 4. Root bending stress reduction of symmetric tooth profile (D20C20) in cases of ST-SR1, ST-CR1, ST-SR2 and ST-AR1

Case	Tooth Profile (DXXCYY)	ρ_f^*		σ_b	Reduction %
		ρ_{fd}^*	ρ_{fc}^*	FEA (MPa)	
ST-SR1	D20C20	0.38		113.34	Ref. value
ST-CR1	D20C20	Circular		108.69	4.10
ST-SR2	D20C20	0.47		105.98	6.50
ST-AR1	D20C20	0.58	0.36	101.72	10.25

Table 5. Root bending stress values of symmetric tooth profile(D25C25) in cases of ST-CR2, ST-SR3, ST-AR2 and ST-AR3

Case	Tooth Profile (DXXCYY)	ρ_f^*		σ_b	Reduction %
		ρ_{fd}^*	ρ_{fc}^*	FEA (MPa)	
ST-CR2	D25C25	Circular		111.37	Ref. value
ST-SR3	D25C25	0.31		107.79	3.21
ST-AR2	D25C25	0.43	0.20	100.67	9.61
ST-AR3	D25C25	0.53	0.10	95.98	13.82

Table 6. Root bending stress values of symmetric tooth profile (D30C30) cases of ST-CR3, ST-SR4 and ST-AR4

Case	Tooth Profile (DXXCYY)	ρ_f^*		σ_b	Reduction %
		ρ_{fd}^*	ρ_{fc}^*	FEA	
ST-CR3	D30C30	Circular		125.19	Ref. value
ST-SR4	D30C30	0.11		123.54	1.32
ST-AR4	D30C30	0.12	0.11	121.49	2.96

With symmetric profile spur gears (having different pressure angles of 20, 25 and 30degrees), an analysis of the effect of different root fillet forms have been performed above. Circular roots and trochoidal roots generated by different cutter tip designs were compared in terms of maximum tooth root bending stress under a constant transmitted torque. In both circular and trochoidal roots, maximum allowable root radii were employed. Followings are the summary of bending stress reductions obtained for symmetric tooth profiles by different root fillet designs:

- 10.25 % for D20C20, (for design change from ST-SR1 to ST-AR1)
- 13.82 % for D25C25, (for design change from ST-CR2 to ST-AR3)
- 2.96 % for D30C30, (for design change from ST-CR3 to ST-AR4).

Asymmetric Tooth Profile with Symmetric and Asymmetric Root Fillets

As stated earlier, the main purpose of this study is to investigate influence of fillet region and radii (cutter tip radii) on root bending stress for circular, symmetrical and especially for asymmetrical root fillets. In previous section, there were some case studies for symmetric tooth profile with different fillet design alternatives. In this section, asymmetric tooth with symmetric fillet is first used to reduce root stress. Next, asymmetric tooth with asymmetrical fillet design is investigated to improve gear bending life performance. General gear pair parameters for asymmetric tooth cases are same as the symmetric tooth profile cases, and given in Table 2. Case studies of asymmetric tooth profile gears are performed for constant torque and reference/nominal center distance value for interference free meshing. FEA model and bending stress result of finite element analysis for case of AT-AR2 are presented in Figure 7 and 8 respectively. Tooth root bending (principal) stress values are presented in Table 7 for all of asymmetric tooth cases.

When case studies are grouped based on pressure angles it is easier to compare effects of root fillet type and coefficient of tool tip radius on bending stress. Therefore it is better to present stress values in different groups based on pressure angles of both sides of asymmetric tooth profile gear. There are significant stress reductions like:

- 7.27 % for D25C20, (for design change from AT-SR1 to AT-AR1),
- 11.27 % for D30C20, (for design change from AT-SR2 to AT-AR2),
- 10.17 % for D30C25, (for design change from AT-SR3 to AT-AR3),
- 8.84 % for D25C30, (for design change from AT-SR4 to AT-AR6),
- 8.47 % for D20C25, (for design change from AT-SR5 to AT-AR4) and
- 10.70 % for D20C30 (for design change from AT-SR6 to AT-AR5).

Table 7. Root bending stress values of asymmetric tooth (AT) with symmetric root (SR) and asymmetric root (AR) profiles calculated by finite element analysis

Case	Tooth Profile (DXXCYY)	Tooth Force (N)	ρ_f^*		σ_b
			ρ_{fd}^*	ρ_{fc}^*	FEA (MPa)
AT-SR1	D25C20	2950.2	0.395		106.6
AT-SR2	D30C20	3087.4	0.30		110.6
AT-SR3	D30C25	3087.4	0.21		114.0
AT-SR4	D25C30	2950.2	0.21		112.2
AT-SR5	D20C25	2845.4	0.395		105.5
AT-SR6	D20C30	2845.4	0.30		107.8
AT-AR1	D25C20	2950.2	0.59	0.22	98.8
AT-AR2	D30C20	3087.4	0.56	0.10	98.1
AT-AR3	D30C25	3087.4	0.35	0.10	102.4
AT-AR4	D20C25	2845.4	0.58	0.19	96.6
AT-AR5	D20C30	2845.4	0.48	0.10	96.3
AT-AR6	D25C30	2950.2	0.32	0.10	102.3

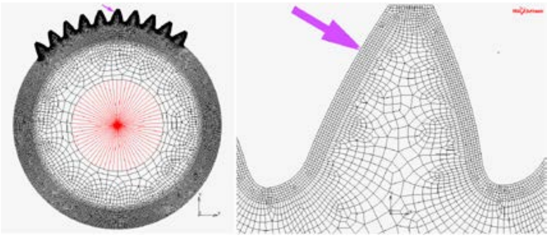


Fig. 7. Finite element model of asymmetric tooth profile gear with asymmetric root fillet (Case of AT-AR2)

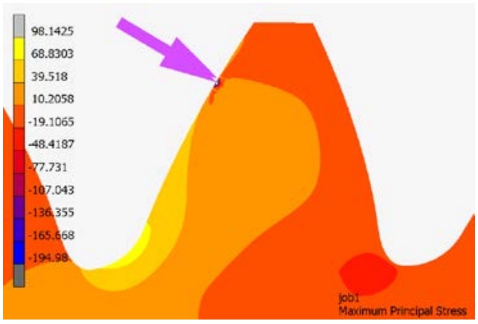


Fig. 8. Finite element results of root bending stress (max principal stress) for asymmetric tooth profile with asymmetric root fillet (Case of AT-AR2)

These results show that there is significant stress reduction (up to of 11.27 % for greater pressure angle of drive side and almost 10.70 % for greater pressure angle of coast side) for asymmetric tooth profile spur gears with asymmetric root fillets. All calculations and analysis up to here are based on constant torque loading. Under constant torque, tooth load is calculated based on base radius of drive side flank (hence pressure angle of drive side). Each gear pair having different pressure angles on drive side will be carrying a different tooth load. Increase in pressure angle decreases base radius hence increases load on tooth (under constant torque).

Compared with constant torque loading, under constant tooth load/force condition, comparison of bending stress for pairs having different pressure angles of drive flanks will not be a reliable method. Because tooth load value (Table 7) will be different for each case of pressure angles of drive flanks and it is not suggested to analyze the bending stress under constant tooth load.

Therefore, it is important that enough attention is given to the conditions of tooth form and the fillet geometry of the drive flanks of meshing pairs to clearly understand what causes change in bending stress results. For example, gear pairs of D20C20 with different types of fillet (circular, symmetric and asymmetric) geometries will usually show the effect of fillet form on the bending stress. Whereas gear pairs of D20C20 and D25C20 with a fixed type of fillet (circular, symmetric or asymmetric) geometry will usually show the effect of tooth form on the bending stress despite a slight difference in the coefficients of cutter tip radius due to tooth root space differences for different pressure angles.

DISCUSSION AND CONCLUSIONS

A systematic analysis of the root bending stress for cases of:

- Symmetric tooth and symmetric trochoidal root
- Symmetric tooth and symmetric circular root
- Symmetric tooth and asymmetric trochoidal root
- Asymmetric tooth and symmetric trochoidal root
- Asymmetric tooth and asymmetric trochoidal root

has shown that employing asymmetric trochoidal root via an asymmetric tipped cutter with two different tip radii coefficients, help decrease bending stress further compared with the symmetric trochoidal root and circular root for both symmetric and asymmetric tooth profiles. Referring to Table 8, root stress reductions of D20C20 and D25C25 with asymmetric trochoidal root (AR) are considerable (10-15%) compared to symmetric trochoidal root (SR) and circular root (CR) cases. However, a stress increase (instead of reduction) in bending stress occurs for D30C30 with asymmetric trochoidal root due to increased tooth load with decreasing base radius (under constant torque condition).

When asymmetric cutter tip radius coefficients are applied to asymmetric tooth profile cases (Table 9), a stress reduction between 9 to 15 % is obtained depending on the pressure angles of drive and coast sides. Based on all these analysis results it could be concluded that for both symmetrical and asymmetrical tooth profile gears, cutting tool with asymmetric tip provides stronger gear tooth in terms of bending stress. The suggested asymmetric cutter tip hence asymmetric trochoidal root form design results in a significant amount of stress reduction (up to 15 %) in the tooth root. Such a design change, employed on external spur gears in this study, is likely to provide similar advantageous power transmission units with helical and other gear types too.

Asymmetry in tooth profile (in terms of pressure angle) provides lesser root bending stress for gears in addition to root asymmetry. It can be concluded that dual asymmetry yields in lesser root stress than symmetric tooth with symmetric root and symmetric tooth with asymmetric root. Making gear tooth flank profile (involute) and root (trochoid) section asymmetric at same time results in more stress reduction.

Table 8. Root bending stress reduction of different symmetric tooth profiles with asymmetric root profiles

Case	Tooth Profile DXXCYY	Tooth Force (N)	ρ_f^*		σ_b	Reduction %
			ρ_{fd}^*	ρ_{fc}^*	FEA (MPa)	
ST-SR1	D20C20	2845.4	0.38		113.3	Ref
ST-CR1	D20C20	2845.4	Circular		108.7	4.10
ST-AR1	D20C20	2845.4	0.58	0.36	101.7	10.25
ST-AR3	D25C25	2950.2	0.53	0.10	96.0	15.31
ST-AR4	D30C30	3087.4	0.12	0.10	121.5	-7.19 (increase)

Table 9 Root bending stress reduction of different asymmetric tooth profiles with asymmetric root profiles

Case	Tooth Profile (DXXCYY)	Tooth Force (N)	ρ_f^*		σ_b	Reduction %
			ρ_{fd}^*	ρ_{fc}^*	FEA MPa)	
ST-SR1	D20C20	2845.4	0.38		113.3	Ref
AT-AR1	D25C20	2950.2	0.59	0.22	98.8	12.78
AT-AR2	D30C20	3087.4	0.56	0.10	98.1	13.41
AT-AR3	D30C25	3087.4	0.35	0.10	102.4	9.64
AT-AR6	D25C30	2950.2	0.32	0.10	102.3	9.75
AT-AR4	D20C25	2845.4	0.58	0.19	96.6	14.78
AT-AR5	D20C30	2845.4	0.48	0.10	96.3	15.07

Table 10 Root bending stress reduction of dual asymmetry

Case	Tooth Profile (DXXCYY)	Tooth Force (N)	ρ_f^*		σ_b	Reduction %
			ρ_{fd}^*	ρ_{fc}^*	FEA (MPa)	
ST-SR1	D20C20	2845.4	0.38		113.3	Ref
ST-AR1	D20C20	2845.4	0.58	0.36	101.7	10.25
AT-AR1	D25C20	2950.2	0.59	0.22	98.8	12.78

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NOMENCLATURE

α pressure angle of basic rack, degrees

α_d pressure angle of basic rack for drive side, degrees

α_c pressure angle of basic rack for coast side, degrees

ρ^*_f, ρ^*_F coefficient of fillet radius of basic rack

ρ^*_{fd} coefficient of fillet radius of basic rack for drive side

ρ^*_{fc} coefficient of fillet radius of basic rack for coast side

σ bending stress, MPa

M bending moment, Nm

I moment of inertia, mm^4

c distance to neutral axis, mm

s_{Fn} tooth root critical section thickness, mm