# Hunting Suppression for a Control System with Low Sampling Rate

### Pi-Cheng Tung\* and Wei-Hann Yao\*\*

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#### ABSTRACT

This paper presents hunting suppression for a control system with low sampling rate. In highprecision motion digital controllers, such as servo motor drivers, PLC-based controllers and PC-based motion control cards, the sampling rates may be fixed and sampling frequency may be as low as 100 Hz. The closed loop control system may become unstable and cause hunting if a low sampling rate is set with certain controller parameters. The proposed hunting suppression scheme is simple and ensures that the closed loop system remains stable at a fixed low sampling rate. The effectiveness of the proposed control scheme is verified by simulation a closed loop system with low sampling rate.

#### **INTRODUCTION**

Nowadays, the digital controller is widely used in closed loop control systems, such as automatic aircraft landing systems, servomotor motion control, and robot control systems. If the sampling rate of the digital controller can higher, the closed loop control system will approach a continuous system. But the sampling rates are fixed for most digital controllers. In highprecision motion digital controllers, the sampling rates may be fixed and sampling frequency may be as low as 100 Hz. The closed loop control system may become unstable and cause hunting if a low sampling rate is set with certain controller parameters. Time delay due to low sampling rates may cause systems unstable. Various controllers have been proposed to solve the time delay problems (Dunn, 2004; Huang, 2009; Lin, 2001; Liu, 2011; Tsai, 2012;).

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\* Professor, Department of Mechanical Engineering, National Central University, Taoyuan, Taiwan 32054, ROC.

\*\*Graduate Student, Department of Mechanical Engineering, National Central University, Taoyuan, Taiwan 32054, ROC.

Within the existing research on solving the problem of digital controller design, a high-speed track seek technique, based on two degree-of-freedom control in high density hard disk drives of head positioning control was proposed by Ishikawa et al. (1996). A novel robust feedback controller for a perfect tracking control method based on multi-rate feedforward control was proposed by Fujimoto et al. (2001). An adaptive repetitive control to track variable periodic signals with fixed sampling rate was proposed by Cao and Ledwich (2002). A novel multi-rate sampling method for an acceleration control system was proposed by Mizuochi et al. (2007). A new method of designing the generalized minimum variance control (GMVC) in a sampled-data control system, in which a continuous-time plant is controlled using a discrete-time controller was proposed by Sato (2001). Yim et al. (1996) investigated modified current control schemes for high-performance permanent-magnet AC drives with low sampling to operating frequency ratio. A network-based H<sub>∞</sub> control of systems with timevarying sampling period was proposed by Wang and Yang (2010). A robust  $H_{\infty}$  networked control for systems with uncertain sampling rates was proposed by Borges et al. (2010). A fast rate adaptive output feedback control of multi-rate sampled system with an adaptive output estimator was proposed by Mizumoto et al. (2011). A stability analysis of digital repetitive control systems under time-varying sampling period was proposed by Olm et al. (2011). An AI-based dynamic sampling planning system was proposed by Lee (2002). An economic design model of DS is developed based on Yang and Hancock's assumption of correlation and Lorenzen and Vance's cost model to determine sample size, sampling interval, An AI-based dynamic sampling planning system was proposed by Lee (2002). An economic design model of DS is developed based on Yang and Hancock's assumption of correlation and Lorenzen and Vance's cost model to determine sample size, sampling interval, and coefficients of control limits and warring limits was proposed by Torng et al. (2009).

In this paper, hunting suppression for a control system with low sampling rate is proposed. The proposed hunting suppression scheme remains inactive provided that the parameter values of the PD controller remains below certain designed values, which are equivalent to the upper bound of the system's stable range. The proposed hunting suppression scheme is designed based on the stability analysis of a discrete time system to ensure the system stability at low sampling rate. Simulations of a control system with the PD controller at low sampling rate are performed to show the feasibility of the proposed hunting suppression scheme.

#### PROPOSED HUNTING SUPPRESSION SCHEME

Consider a digital control system (Ogata, 1995) as shown in Figure 1. The stability of the digital control system depends on the sampling period. A large sampling period may cause the system unstable. For a given sampling period increasing the value of the controller gain will make the digital control system less stable and finally will make it unstable.

The system model  $G_p$  shown in Fig. 1 is of a second order system. The characteristic equation of the digital closed-loop control system with a proportional controller can be written as:

$$1 + \frac{Kz(1 - e^{-T})}{(z - 1)(z - e^{-T})} = 0, \qquad (1)$$

$$\xrightarrow{r(t)}_{R(z)} \overbrace{\delta_{\tau}} \longrightarrow \overbrace{G_{\mathcal{C}}(S)} \longrightarrow \overbrace{\frac{1 - e^{-Ts}}{s}}^{1} \overbrace{\frac{1}{s(s+1)}}^{c(t)} \overbrace{C_{p}(s)}^{c(t)}$$

Fig. 1. Block diagram of a digital closed-loop control system

where *K* is the gain of the proportional controller. Consider the sampling period of the closed-loop control system to be fixed and sampling period T=0.01(sec). The stability analysis shows that the value of the critical gain K is 400. Figures 2(a) and 2(b) show the simulation results for K=399 and K=401 respectively. It can be seen that increasing *K* will cause system unstable.



Fig. 2. Response of the P controller for (a) Kp=399 and (b) Kp=401.

However, in real application the system response cannot go to infinite due to nonlinear elements such as saturation existing in the system. Figure 3 shows the digital control system with saturation. The saturation value is  $\pm 500$ . For the closed loop control system, the saturation will cause the unstable system hunting.



Fig. 3. Block diagram of a digital closed-loop control system with saturation.

Figures 4(a) and 4(b) show the simulation results for the closed loop control system with saturation of  $\pm 500$  for K=399 and K=401 respectively. It can be seen that hunting occurs for K=401. It should be noted that stability problem due to saturation will not considered in this paper in order to focus on the proposed hunting suppression scheme.



Fig. 4. Response of the P controller with saturation for (a) K=399 and (b) K=401.

Similarly, Figures 5(a) and 5(b) show the simulation results for the PD closed loop control system with saturation of  $\pm$  500 for  $K_p$ =390  $\cdot$   $K_d$ =5.001 and  $K_p$ =390  $\cdot$   $K_d$ =5.002 respectively. It can be seen that hunting occurs for  $K_d$ =5.002.





Fig. 5. Response of the PD controller with saturation for (a)  $K_p=390 \times K_d=5.001$  and  $K_p=390 \times K_d=5.002$ .

However, the PD closed loop control system with saturation will become stable by increasing sample rate to 1Kz as shown in Figure 6.



Fig. 6. Response of the PD controller with saturation for  $K_p=390 \ K_d=5.002 \ T=0.001$ .

This paper proposes a hunting suppression scheme for suppressing hunting. Figure 7 presents the block diagram of the hunting suppression scheme. In Fig. 7, e denotes the input signal, u denotes the output signal of the controller,  $K_p$  denotes the proportional term parameter,  $K_d$  denotes the derivative term parameter of the PD controller,  $K_s$  is the upper limit gain value designed according to stability analysis,  $K_a$ is the adaptive gain parameter and determines the rate of convergence of the compensation scheme and h is the compensation signal produced by the proposed scheme. As shown in Fig. 7, the hunting suppression scheme consists of a dead-zone which is a function of e and  $K_s$ .



Fig. 7. Block diagram of the hunting suppression scheme.

The signal *h* has a non-zero value when the output signal *u* exceeds the value of the product of the input *e* and the gain  $K_s$  In other words, the equivalent gain of the controller such as P or PD controller is restricted to a value lower than  $K_s$  irrespective of the value assigned to the controller. If the output signal *u* is beyond the product of the input and the gain  $K_s$ , a non-zero value of the compensator signal *h* is employed to counteract the input signal *e* in order to keep the upper limit of the output *u*.

The relationship between the output signal u and the compensation signal h is therefore given by

$$\begin{cases}
 if \quad u > |e \cdot K_s| \text{ or } u < -|e \cdot K_s| \\
 h(s) = (u(s) - e(s) \cdot K_s) \cdot K_a, . \\
 else \quad h=0.
\end{cases}$$
(5)

In Fig. 7, the compensation signal, h can be expressed as

$$h(s) = (u(s) - e(s) K_s) K_a.$$
 (6)

The relationship between the input signal e and output signal u is given by

$$u(s) = (e(s) - h(s))(K_P + K_I / s) = [e(s) - (u(s) - e(s)K_s)K_a](K_P + K_I / s) = (1 + K_s K_a)(K_P + K_I / s)e(s) - K_a(K_P + K_I / s)u(s).$$
(7)

Equation (7) can be rewritten as

$$\frac{u(s)}{e(s)} = \frac{(1+K_S K_a)(K_P s + K_I)}{s + K_a(K_P s + K_I)}.$$
(8)

If the input signal has the form of a unit step response, the steady-state of Eq. (4.4) can be rewritten as

$$u(t)|_{t\to\infty} = \lim_{s\to0} \frac{1}{s} s \frac{(1+K_s K_a)(K_P s + K_I)}{s + K_a(K_P s + K_I)} = K_s + \frac{1}{K_a}.$$
 (9)

It should be noted that equations (6), (7), (8) and (9) hold only if  $u > |e \cdot K_s|$ . If  $K_a$  is designed to be larger, the relationship between the input signal e and output signal u can be replaced by a value of  $K_s$ . Figure 8 shows a block diagram of Fig. 3 with the proposed hunting suppression scheme.

#### SIMULATION RESULTS OF THE PROPOSED HUNTING SUPPRESSION SCHEME

Simulation was performed to compare the performance of the PD controller with the hunting suppression scheme shown in Fig. 8. with that of the PD controllers without the hunting suppression scheme shown in Fig. 5(b).



Fig. 8. Block diagram of Fig. 3 with proposed hunting suppression scheme.

The conditions of simulation are the same as in previous section. The PD controller parameters in the simulation are all the same, for which  $K_s = 390 \times K_s = 5.002$ . The sampling period *T* in the simulation is 0.01sec.

Considering that PD controller can be equivalent to a gain K and in order to make the control system stable, the gain K must be satisfied with K < 400.

The parameters of hunting suppression scheme in the simulation are designed as  $K_s = 399 < 400$  and  $K_a = 1$ . In the simulation, the input commands have two different kinds: the step input and the sin-wave. Figure 9(a) shows the step response of the PD controller without the hunting suppression scheme in sampling period T=0.01(sec). The step input command is 0.2. The control system with the PD controller parameters in the sampling period and saturation term will cause hunting. The hunting will reduce the system performance. Figure 9(b). indicates the large variation of tracking error of the PD controller without the hunting suppression scheme. The tracking error cannot converge to zero due to hunting. The step response of the PD controller with the hunting suppression scheme is shown in Figure 9(c). In Fig. 9(c), since the hunting is suppressed by the hunting suppression scheme, the response can converge to the value of the command. Figure 9(d) shows that the tracking error of the PD



controller with the hunting suppression scheme is able to converge near to zero.



Fig. 9. (a)Step response of the PD controller without the hunting suppression. (b) Tracking error without hunting suppression scheme. (c) Step response of the PD controller with the hunting suppression. (d) Tracking error with hunting suppression scheme.

Figures 10(a) and 10(c) show a comparison of sinwave response of the PD controller without the hunting suppression scheme and sin-wave response of the PD controller with the hunting suppression scheme in sampling period T=0.01(sec).

Figure 10(b) shows that the tracking error of the PD controller without the hunting suppression scheme cannot converge to zero. Figure 10(d) shows the tracking error of the PD controller with the hunting suppression scheme; the hunting is suppressed by the hunting suppression scheme. Therefore, the tracking error of the PD controller with the hunting suppression scheme can converge near zero, as shown in Fig. 10(d).





Fig.10. (a) Sin-wave response of the PD controller without the hunting suppression. (b) Tracking error without hunting suppression scheme. (c) Sin-wave response of the PD controller with the hunting suppression. (d) Tracking error with hunting suppression scheme.

The simulation results have shown the good effect of the hunting suppression scheme. The hunting is effectively suppressed by the hunting suppression scheme at a low sampling period.

#### CONCLUSION

Hunting suppression for a control system with low sampling rate has been presented. The proposed hunting suppression scheme is designed based on the stability analysis of a discrete time system to ensure system stability at low sampling rate. Since the closed loop control system may become unstable and cause hunting at a low sampling rate with certain PD controller parameters, we propose a hunting suppression scheme that can limit the parameter values of the PD controller to an upper bound value corresponding to the upper limit of the stability range of the system at a fixed sampling period. The simulation results show that the proposed hunting suppression scheme can successfully suppress hunting at a fixed low sampling rate.

#### REFERENCE

- Borges, R.A., Oliveira1, R.C.L.F., Abdallah, C.T., and Peres, P.L.D., "Robust H∞ networked control for systems with uncertain sampling rates", *IET Contr. Theory Appl.*, Vol. 4, No. 1, pp.50-60(2010).
- Cao, Z., and Ledwich, G.F., "Adaptive repetitive control to track variable periodic signals with fixed sampling rate", *IEEE/ASME Trans. Mechatronics*, Vol. 7, No. 3, pp.378-384(2002).
- Dunn, A.C., Shieh, L.S., and Guo, S.M., "Digital redesign of analog smith predictor for systems with input time delays", *ISA Trans.*, Vol. 43, No. 1, pp.33-47(2004).
- Fujimoto, H., Hori, Y., and Kawamura, A., "Perfect tracking control based on multirate feedforward control with generalized sampling periods", *IEEE Trans. Ind. Electron.*, Vol. 48, No. 5, pp.636-644(2001).
- Huang, G., and Wang, S., "Use of uncertainty polytope to describe constraint processes with uncertain time-delay for robust model predictive control applications", *ISA Trans.*, Vol. 48, No. 4, pp.503-511(2009).
- Ishikawa, J., Yanagita, Y., Hattori, T., and Hashimoto, M., "Head positioning control for low sampling rate systems based on two degree-offreedom control", *IEEE Trans. Magn.*, Vol. 32, No. 3, pp.1787-1792(1996).
- Lee, J.H., "Artificial intelligence-based sampling planning system for dynamic manufacturing process", *Expert Syst. Appl.*, Vol. 22, pp.117-133(2002).
- Lin, T.C., Wang, C.H., and Teng, C.C., "Quantitative feedback synthesis of sampled-data systems with time-delay by approximate Z-transform", *ISA Trans.*, Vol. 40, No. 4, pp.325-332(2001).
- Liu, A., Yu, L., and Zhang, W.A., "One-step receding horizon H-infinity control for networked control systems with random delay and packet

disordering", *ISA Trans.*, Vol. 50, No.1, pp.44-52(2011).

- Mizuochi, M., Tsuji, T., and Ohnishi, K., "Multirate Sampling Method for Acceleration Control System", *IEEE Trans. Ind. Electron.*, Vol. 54, No. 3, pp.1462-1471(2007).
- Mizumoto, I., Fujimoto, Y., Watanabe, N., and Iwai, Z., "Fast rate adaptive output feedback control of multi-rate sampled system with an adaptive output estimator", *Int. J. Innov. Comp. Inf. Control*, Vol. 7, No. 7B, pp.4377-4394(2011).
- Olm, J.M., Ramos, G.A., and Costa-Castello', R., "Stability analysis of digital repetitive control systems under time- varying sampling period", *IET Contr. Theory Appl.*, Vol. 5, No. 1, pp.29-37(2011).
- Ogata, K., *Discrete-Time Control Systems*, Prentice Hall, New Jersey (1995)
- Sato, T., "Design of a generalized minimum variance control in sampled-data control systems", *Int. J. Innov. Comp. Inf. Control*, Vol. 5, No. 10B, pp.3295-3302(2009).
- Torng, C. C., Lee, P. H., Laio, H. S., and Laio, N. Y., "An economic design of double sampling X over-bar charts for correlated data using genetic algorithms", *Expert Syst. Appl.*, Vol. 36, pp.12621-12626(2009).
- Tsai, S.H., Chen, F.M., and Yu, T.Y., "Efficient decentralized iterative learning tracker for unknown sampled-data interconnected largescale state-delay system with closed-loop decoupling property", *ISA Trans.*, Vol. 51, No. 1, pp.81-94(2012).
- Wang, Y.L., and Yang, G.H., "Network-based H(infinity) control of systems with timevarying sampling period", *Int. J. Innov. Comp. Inf. Control*, Vol. 6, No. 4, pp.1833-1842(2010).
- Yim, J.S., Sul, S.K., Bae, B.H., Patel, N.R., and Hiti, S., "Modified Current Control Schemes for High-Performance Permanent-Magnet AC Drives with Low Sampling to Operating Frequency Ratio", *IEEE Trans. Ind. Appl.*, Vol. 45, No. 2, pp.763-771(2009).
- Yao, W.H., Tung, P.C., Fuh, C.C., and Chou, F.C., "Suppression of hunting in an ILPMSM driver system using hunting compensator", *IEEE Trans. Ind. Electron.*, Vol. 60, No. 7, pp.2586-2594(2013).

## 低取樣頻率控制系統的 振盪抑制

董必正 姚維翰 國立中央大學機械工程學系

#### 摘要

本文提出了一種低取樣頻率控制系統的振盪 抑制方案,在高精度運動數位控制器中,例如伺服 電機驅動器,基於 PLC 的控制器和基於 PC 的運動 控制卡,取樣頻率可能是固定,並可能低至 100 Hz。 在較低的取樣頻率且控制器參數設定在某一數值 下,閉迴路控制系統可能變得不穩定而導致震盪現 象發生。本文所提出之創新震盪抑制方案,不但架 構簡單並可確保閉迴路控制系統在固定的低取樣 頻率下仍能保持穩定,並藉由電腦模擬驗證所提控 制方案之有效性。