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Minimum Variance Run-to-Run Controller for General Stochastic Time-Series-Based Disturbances

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Keywords：stochastic disturbance, run-to-run control, disturbance observer, minimum variance control, time-series model.

ABSTRACT

Exponentially weighted moving average (EWMA) or double EWMA controllers are often employed to deal with various stochastic time-series disturbances for run-to-run control. If the disturbance model is known, then the best EWMA or double EWMA controller can be obtained by minimizing output variance with respect to controller parameters. However, from the theoretical view point, results are only sub-optimum because the control scheme may not be the best control scheme for the underlying stochastic disturbance. Therefore, investigating the best control scheme for the process disturbance following the general ARIMA time series process is worthwhile. In this paper, the predictive disturbance observer (PDOB) is developed based on minimum variance control for various ARIMA(p,r,q) stochastic disturbances. If the ARIMA(p,r,q) disturbance model is known, then the PDOB scheme generates one-step ahead prediction for this disturbance and then feeds it back to compensate the effect of stochastic disturbance on the system output; as a result, the system generates the minimum output variance or simply white noise variance.

INTRODUCTION

Run-to-run control (RtR) technology in advanced process control has been studied extensively. With the characteristic outputs of the previous runs being analysed, the RtR controller continuously modifies the model and then updates the input recipe for the next run to reduce variations between the process output and the target. Owing to the simplicity and robustness, the EWMA controller has been the most commonly used RtR controller in semiconductor manufacturing. In 1974, Box and MacGregor (1974) introduced the exponentially weighted moving average (EWMA) controller with proper weights to reduce variations between the process output and the target. Sachs et al. (1995) proposed the RtR control scheme based on the EWMA statistic. They used linear static models to design a feedback-based RtR controller, and the EWMA statistic was used as an estimate of the process. The scheme can reject shift disturbance but produce an offset in the process output by the drift disturbance. To compensate the offset produced by the EWMA controller, a predictor-corrector controller (PCC) and a double EWMA controller were developed by Bulter (1994) and Guo et al. (2000). The EWMA controller and the PCC controller can be represented by using the internal model control (IMC) structure. Adivikolanu and Zafiriou (2000) applied the IMC structure to extend the EWMA controller.

The system stability of several RtR controllers has been studied in the literature. Ingolfsson and Sachs (1993) studied the conditions of the stability of the EWMA controller and showed the allowable range of model mismatch where the process could remain stable. Tseng et al. (2002) extended the stability analysis to a MIMO EWMA controller. Del Castillo (1999) discussed the necessary conditions of weights to ensure closed-loop stability. Good and Qin (2006) investigated the effect of metrology delay on the closed-loop stability of a MIMO EWMA controller. Recently, Lee et al. (2011) unified the framework of EWMA, double EWMA, and PCC controllers on the basis of the concept of output disturbance observer (ODOB). They designed and analysed the tuning parameters of the ODOB controller to meet the performance and stability for the processes.

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With regard to the optimization issue, Box and Jenkins (1990) proved that the EWMA statistic is a minimum mean-square error (or minimum variance) controller for the process disturbance following an IMA(1,1) time series process. Del Castillo (1999) proposed a trade-off solution for the double EWMA weights between long-run variance and short-run transient performance. Tseng (2003) derived explicit expressions for the optimal variable discount factors when the disturbance follows the ARMA(1,1) process or the IMA(1,1) process. Ma and Li (2015) improved Tseng's method by using the auto-covariance of the time-series model to design the variable EWMA controller. However, the method can only be applied to a special type of ARIMA disturbances only. So far, many individuals, including researchers and practicing engineers, use EWMA or double EWMA controllers to deal with various stochastic time-series disturbances. If the disturbance model is known, which could be estimated by open-loop experimentations (Box and MacGregor, 1974; Pan and Del Castillo, 2001), then the best EWMA or double EWMA weights for the best controller can be obtained by minimizing the output variance with respect to weights. However, results may be sub-optimum from a theoretical point of view because the above control schemes may not be the best controller for the underlying stochastic disturbance. In the literature, the EWMA controller is the best control scheme that enables the IMA(1,1) process disturbance to produce the minimum variance output (1994); Del Castillo (1999) further extended this scheme to obtain optimum weights and included the factor of model mismatch. As for the double EWMA controller, Box and Jenkins (1990) have shown that it is a minimum variance controller for the process disturbance following an IMA(2,1) or IMA(2,2) time series process without considering the model mismatch. This paper investigated the best control scheme for the process disturbance following the general ARIMA time series process.

In this paper, the analysis is based on the ODOB structure (2011), which provides a unified framework of EWMA, double EWMA, and PCC controllers. The predictive disturbance observer (PDOB) structure illustrated in the box with grey-based color of Fig. 1 is adopted to develop a minimum variance RtR controller, which replaces the original Q'-filter in the original ODOB structure (2011) by using a one-step ahead predictor F-filter. As shown in Fig. 1, T is the process target, u_k is the process output (input recipe), y_k is the process output, η_k is the output disturbance, P is the process gain (or actual plant), P_n is the nominal plant, α is the process intercept term, and subscript k is the batch index. The one-step ahead predictor, F-filter, functions as an observer to estimate the disturbance one step ahead and then feeds it back to diminish the effect of stochastic disturbance on the system output. With the implementation of F-filter, the PDOB control scheme can produce minimum output variance or only white noise variance if the process disturbance follows the general ARIMA time series process as will be discussed in the following sections.

The rest of this paper is organized as follows: the second section explains the essence of the PDOB structure in the RtR control scheme and develops the one-step ahead predictor based on minimum variance control. Then, the third section analyses the system output mean and variance under the IMA(1,1), ARMA(1,1), and ARIMA(1,1,1) disturbance in the PDOB RtR control scheme. In simulation section, some cases are presented using the proposed control structure in comparison with the EWMA controller or double EWMA for the different disturbances. The conclusion is drawn in the final section.

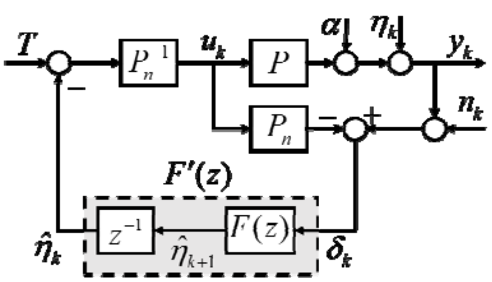

Fig. 1. Predictive disturbance observer structure.

PDOB STRUCTURE APPLIED TO RTR CONTROL

PDOB Structure

The PDOB structure is described in Fig. 1, where a unit delay operator is combined with the F-filter in the feedback loop because of the RtR process characteristic; this unit delay is different from any additional metrology delays. The closed-loop transfer functions in the z-domain can be obtained as

$$\begin{aligned} y(z) &= \frac{P}{P_n+(P-P_n)Fz^{-1}}T(z)+\frac{P_n(1-Fz^{-1})}{P_n+(P-P_n)Fz^{-1}}\alpha(z) \\ &+\frac{P_n(1-Fz^{-1})}{P_n+(P-P_n)Fz^{-1}}\eta_0(z)-\frac{PFz^{-1}}{P_n+(P-P_n)Fz^{-1}}n(z) \end{aligned} \quad . (1)$$

If the F-filter is designed as $F(z)\approx 1$ in the low frequency range with DC gain being equal to one, i.e., $F(z)\big|_{z=1}=1$, then the first term on the right-hand side (RHS) of Eq. (1) reveals that the transfer function from T to y_k will be equal to one in the low frequency range with the steady-state output y_k being equal to the process target T . Otherwise, if the DC gain of F-filter is not equal to one, then the process output will produce an offset from the target. In the second term on the RHS of Eq. (1), the process intercept term is an unknown constant. Thus,

$(1-Fz^{-1})\alpha = 0$ if $F(1)=1$ and therefore has no effect on the output. Otherwise, an offset will be present in the output. Furthermore, in the third term on the RHS of Eq. (1), if the F-filter can optimally estimate the disturbance one step ahead in a minimum variance sense, then this term becomes an innovation, i.e., it will constitute a white noise time series. Therefore, the PDOB structure provides the function of disturbance rejection. Furthermore, if the measurement noise exists, then the F-filter designed such that $F(z) \approx 0$ in the high frequency range will filter out the noise. Furthermore, without loss of generality, process target is set to zero due to the superposition property of linear systems.

F-filter Design

The one-step ahead predictor $F(z)$ in the PDOB structure is first developed based on minimum variance control (1997). An inherent unit delay exists in the feedback loop of the PDOB structure for the RtR control. Thus, the block diagram of the structure in Fig. 1 is converted into its equivalent diagram shown in Fig. 2 to make itself suitable for the development of the minimum variance controller scheme. Once the one-step ahead predictor $F(z)$ in the PDOB structure is obtained, we focus on how the predictor is composed in comparison with the EWMA and double EWMA controllers. Assume that general ARIMA time series disturbance η_k in (I.1) may be represented as the output of a linear system driven by white noise or

$$\eta(k) = \frac{C(q)}{A(q)}e(k) = \frac{C^*(q^{-1})}{A^*(q^{-1})}e(k), \tag{2}$$

where $C(q)$ and $A(q)$ are polynomials in forward-shift operator, $C^*(q^{-1})$ and $A^*(q^{-1})$ are polynomials in backward-shift operator, and $e(k)$ is a sequence of independent or uncorrelated random variables with zero mean and standard deviation σ. Consider a process represented as

$$y(k) = Pz^{-d}u'(k) + \frac{C^*(q^{-1})}{A^*(q^{-1})}e(k). \tag{3}$$

Then, for predicting d-step ahead at $y(k+d)$, it follows from (3) that

$$\begin{aligned} y(k+d) &= Pu'(k) + \frac{C^*(q^{-1})}{A^*(q^{-1})}e(k+d) \\ &= H^*(q^{-1})e(k+d) + \frac{G^*(q^{-1})}{A^*(q^{-1})}e(k) + Pu'(k) \end{aligned}, \tag{4}$$

where the polynomial $H^*(q^{-1})$ and $q^{-d}G^*(q^{-1})$ are the quotient and the remainder when dividing $C^*(q^{-1})$ by $A^*(q^{-1})$, i.e.,

$$C^*(q^{-1}) = A^*(q^{-1})H^*(q^{-1}) + q^{-d}G^*(q^{-1}) \ , \tag{5}$$

where

$$A^*(q^{-1}) = 1 + a_1 q^{-1} + \cdots + a_{p+r}q^{-p-r}, \tag{6}$$

$$C^*(q^{-1}) = 1 + c_1 q^{-1} + \cdots + c_q q^{-q}, \tag{7}$$

$$H^*(q^{-1}) = 1 + h_1 q^{-1} + \cdots + h_{d-1}q^{-d+1}, \tag{8}$$

$$G^*(q^{-1}) = g_0 + g_1 q^{-1} + \cdots + g_{n-1}q^{-n+1}, \tag{9}$$

$$n = \begin{cases} p+r & \text{if } p+r \ge q \\ q & \text{if } p+r < q \end{cases}. \tag{10}$$

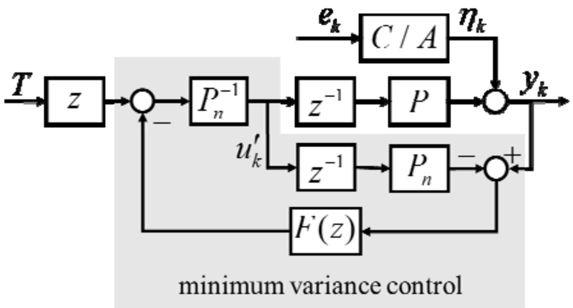

Fig. 2. Minimum variance control applied to the PDOB structure

The first term on the RHS of ,(4) is independent of the data available at time k and thus also of the second and third terms. The second term can be computed exactly in terms of data available at time k. To perform this computation, the variable $e(k)$ is obtained from Eq. (3), that is,

$$e(k) = \frac{A^*(q^{-1})}{C^*(q^{-1})}y(k) - \frac{A^*(q^{-1})}{C^*(q^{-1})}Pz^{-d}u'(k). \tag{11}$$

Using this expression for $e(k)$, one can write Eq. (4) as

$$\begin{aligned} y(k+d) &= H^*(q^{-1})e(k+d) \\ &\quad + \frac{G^*(q^{-1})}{C^*(q^{-1})}y(k) - \frac{G^*(q^{-1})}{C^*(q^{-1})}Pz^{-d}u'(k) + Pu'(k) \\ &= H^*(q^{-1})e(k+d) + \frac{G^*(q^{-1})}{C^*(q^{-1})}y(k) \\ &\quad + \frac{A^*(q^{-1})H^*(q^{-1})}{C^*(q^{-1})}Pu'(k) \end{aligned}. \tag{12}$$

Now, let $u'(k)$ be an arbitrary function of $y(k)$, $y(k-1)$,… and $u'(k-1)$, $u'(k-2)$,…. Then,

$$\begin{aligned} Ey^2(k+d) &= E\left[H^*(q^{-1})e(k+d)\right]^2 \\ &\quad + E\left[\frac{G^*(q^{-1})}{C^*(q^{-1})}y(k) + \frac{A^*(q^{-1})H^*(q^{-1})}{C^*(q^{-1})}Pu'(k)\right]^2 \end{aligned}. \tag{13}$$

The mixed terms vanish because $e(k+d)$,…, $e(k+1)$ are independent of $y(k)$, $y(k-1)$,… and $u'(k-1)$, $u'(k-2)$,…. The last term in Eq. (13) is nonnegative. Therefore,

$$Ey^2(k+d) \ge [1 + h_1^2 + \cdots + h_{d-1}^2]\sigma^2 \ . \tag{14}$$

As shown in Fig. 2, the grey box represents the minimum variance controller that needs to be obtained. The delay, d, is equal to one in this case. Therefore, the inequality is expressed as $Ey^2(k+1) \ge \sigma^2$, where equality is obtained for

$$u'(k) = \frac{-G^*(q^{-1})}{A^*(q^{-1})H^*(q^{-1})P} y(k) = \frac{G(q)z}{(C(q)-G(q))P} y(k), \quad (15)$$

which is the desired minimum variance control law and $Ey^2(k+1) = \sigma^2$.

The control law from $y(k)$ to $u'(k)$ in the PDOB structure of Fig. 2 can be obtained as

$$u'(k) = -\frac{F(q)P_n^{-1}}{1-F(q)z^{-1}} y(k). \quad (16)$$

From Eqs. (15) and (16), the predictor is obtained as

$$F(q) = \frac{G(q)z}{A(q)H(q)\xi + G(q)}, \quad (17)$$

where ξ is the model mismatch defined as $\xi = P/P_n$.

In the following, the one-step ahead predictor $F(z)$ for various time-series disturbance will be investigated. If the disturbance follows IMA(1,1), then we obtains $C(z) = z-\theta$ and $A(z) = z-1$, so $H(z) = 1$ and $G(z) = 1-\theta$. Thus, the predictor for IMA(1,1) is

$$F(z) = \frac{\left((1-\theta)/\xi\right)z}{z+(1-\xi-\theta)/\xi} \triangleq \frac{\lambda z}{z-(1-\lambda)}, \quad (18)$$

where $\lambda \triangleq (1-\theta)/\xi$. This predictor is equivalent to the EWMA controller with the optimum weight for the minimum output variance; the same result is obtained in reference (Castillo, 1999). Note that the DC gain is always equal to one even when a model mismatch exists.

For the case of ARMA(1,1), we can obtain the predictor as

$$F(z) = \frac{\left((\phi-\theta)/\xi\right)z}{z+\left(\phi(1-\xi)-\theta\right)/\xi}. \quad (19)$$

This predictor is not equivalent to the EWMA controller, thereby revealing that EWMA controller is not the best controller for ARMA(1,1) stochastic disturbances. Also, note that the DC gain is not equal to one when a model mismatch exists, thereby indicating that an offset will be produced at the output.

For the case of IMA(2,2), the predictor is

$$F(z) = \frac{\frac{2-\theta_1}{\xi}z^2 + \frac{-1-\theta_2}{\xi}z}{z^2 + \frac{2-\theta_1-2\xi}{\xi}z + \frac{-1-\theta_2+\xi}{\xi}} \triangleq \frac{(\lambda_1+\lambda_2)z^2-(\lambda_1)z}{z^2-(2-\lambda_1-\lambda_2)z+(1-\lambda_1)}, \quad (20)$$

where $\lambda_1 \triangleq (1+\theta_2)/\xi$ and $\lambda_2 \triangleq (1-\theta_1-\theta_2)/\xi$. This predictor is equivalent to the double EWMA controller with optimum weights λ_1 and λ_2 for the minimum output variance. If $\theta_2 = 0$ in the case of IMA(2,1), then Eq. (20) is equivalent to the double EWMA controller with optimum weights. The special results were demonstrated in reference (Box et al., 1994) without the model mismatch. Also, the DC gain is always equal to one even when a model mismatch exists.

Next, the predictor for ARIMA(1,1,1) is

$$F(z) = \frac{\frac{1+\phi-\theta}{\xi}z^2 - \frac{\phi}{\xi}z}{z^2 + \frac{1+\phi-\theta-\xi(\phi+1)}{\xi}z + \frac{\phi(\xi-1)}{\xi}}. \quad (21)$$

This predictor is not equivalent to the double EWMA controller, thereby implying that the double EWMA controller is not the best controller for ARIMA(1,1,1) stochastic disturbances. Also, the DC gain is always equal to one even when a model mismatch exists.

Finally, we provide a general expression for the predictor when the stochastic disturbance is ARMA(p, q) or

$$F(z) = \frac{\sum_{i=1}^{k} \frac{\phi_i-\theta_i}{\xi} z^{k-i+1}}{z^k + \sum_{i=1}^{k} \frac{\phi_i-\theta_i-\phi_i\xi}{\xi} z^{k-i}}. \quad (22)$$

The general expression of the predictor for general ARIMA(p, r, q) is quite complex. In particular, the special cases for ARIMA(p,1,q) and ARIMA(p,2,q) are given as follows:

$$F(z) = \frac{\frac{1+\phi_1-\theta_1}{\xi}z^k + \sum_{i=1}^{k-1}\frac{-\phi_i+\phi_{i+1}-\theta_{i+1}}{\xi}z^{k-i}}{\left(\begin{array}{l} z^k + \frac{1+\phi_1-\theta_1-\xi(1+\phi_1)}{\xi}z^{k-1} \\ +\sum_{i=1}^{k-1}\frac{\phi_{i+1}-\phi_i-\theta_{i+1}-\xi(\phi_{i+1}-\phi_i)}{\xi}z^{k-i-1} \end{array}\right)} \quad (23)$$

$$F = \frac{\left(\begin{array}{l} \frac{2+\phi_1-\theta_1}{\xi}z^k + \frac{-1-2\phi_1+\phi_2-\theta_2}{\xi}z^{k-1} \\ +\sum_{i=1}^{k-2}\frac{\phi_i-2\phi_{i+1}-\theta_{i+2}}{\xi}z^{k-i-1} \end{array}\right)}{\left(\begin{array}{l} z^k + \frac{2+\phi_1-\theta_1-\xi(2+\phi_1)}{\xi}z^{k-1} \\ +\frac{-1-2\phi_1+\phi_2-\theta_2-\xi(-1-2\phi_1+\phi_2)}{\xi}z^{k-2} \\ +\sum_{i=1}^{k-2}\frac{\phi_i-2\phi_{i+1}-\theta_{i+2}-\xi(\phi_i-2\phi_{i+1})}{\xi}z^{k-i-2} \end{array}\right)}, \quad (24)$$

where k is the order of the predictor. If $p+r \ge q$, then the order of the predictor is $p+r$; if $p+r < q$, then the order of the predictor is q. The derivations of Eqs. (22), (23), and (24) are given in Appendix.

VARIANCE ANALYSIS

IMA(1,1)

To prove that the PDOB scheme that can produce the system minimum output variance, the state-space approach is used to analyze the closed-loop system. Consider the IMA(1,1) stochastic disturbance

$$\eta(k+1) = \eta(k) + e(k+1) - \theta e(k), \quad (25)$$

where θ is the coefficient of the moving average term and $e(k) \in N(0,\sigma^2)$ is a zero mean white noise with variance σ^2. Recalling Fig. 1 and Eq. (18), the relationship between the input and output of the F-filter for IMA(1,1) in time domain is written as

$$\hat{\eta}(k+1) = -\frac{(1-\xi)-\theta}{\xi}\hat{\eta}(k) + \frac{1-\theta}{\xi}\delta(k) \quad . \tag{26}$$

Furthermore, the signal of the predictor input is

$$\delta(k) = (1-\xi)\hat{\eta}(k) + (\xi-1)T + \eta(k) \quad . \tag{27}$$

Substituting Eq. (27) into Eq. (26) obtains

$$\hat{\eta}(k+1) = \theta\hat{\eta}(k) + \frac{1-\theta}{\xi}\eta(k) + \Delta_{IMA}T \, , \tag{28}$$

where the expression $\Delta_{IMA} = (1-\theta)(\xi-1)/\xi$. To solve the system output property, we define state variable $x(k) = [\hat{\eta}(k) \quad \eta(k)]^T$ and combine the disturbance model and system model represented by Eq. (25) and Eq. (28) into a state-space model as follows:

$$\begin{aligned} x(k+1) &= Ax(k) + w(k) \\ y(k) &= C^T x(k) + R \end{aligned} \, , \tag{29}$$

where $w(k) = \begin{bmatrix}\Delta_{IMA}T & e(k+1) - \theta e(k)\end{bmatrix}^T$, $R = \xi T$, $A = \begin{bmatrix}\theta & (1-\theta)/\xi \\ 0 & 1\end{bmatrix}$, and $C^T = \begin{bmatrix}-\xi & 1\end{bmatrix}$.

Solving the Eq. (29) for $x(k)$, we obtain

$$x(k) = \begin{bmatrix} \sum_{j=0}^{k-1}\theta^{k-j-1}\Delta_{IMA}T + \sum_{j=0}^{k-1}(1-\theta^{k-j-1})(e_{j+1}-e_j)/\xi \\ \sum_{j=0}^{k-1}(e_{j+1}-\theta e_j) \end{bmatrix} . \tag{30}$$

The system output is

$$y(k) = \sum_{j=0}^{k-1}\theta^{k-j-1}(e_{j+1}-\theta e_j) - \sum_{j=0}^{k-1}\xi\theta^{k-j-1}\Delta_{IMA}T + \xi T \quad . \tag{31}$$

Thus, the output asymptotic mean and variance can be obtained as

$$\lim_{k\to\infty} E[y(k)] = T \quad , \tag{32}$$

$$\lim_{k\to\infty} \mathrm{Var}[y(k)] = \sigma^2 \quad . \tag{33}$$

The system output is on-target with minimum output variance equalling to white noise variance.

ARMA(1,1)

Consider the case of ARMA(1,1) time series disturbance represented by

$$\eta(k+1) = \phi\eta(k) + e(k+1) - \theta e(k) \, , \tag{34}$$

where ϕ is coefficient of autoregressive term. The input/output relationship of the F' predictor with batch delay for ARMA(1,1) in time domain is written as

$$\hat{\eta}(k+1) = -\frac{\phi(1-\xi)-\theta}{\xi}\hat{\eta}(k) + \frac{\phi-\theta}{\xi}\delta(k) \quad . \tag{35}$$

Substituting (27) into (35) results in

$$\hat{\eta}(k+1) = -\theta\hat{\eta}(k) + \frac{\phi-\theta}{\xi}\eta(k) + \Delta_{ARMA}T \quad . \tag{36}$$

where the expression $\Delta_{ARMA} = (\phi-\theta)(\xi-1)/\xi$. Follow the above steps to get the system output, or

$$y(k) = \sum_{j=0}^{k-1}\theta^{k-j-1}(e_{j+1}-\theta e_j) - \sum_{j=0}^{k-1}\xi\theta^{k-j-1}\Delta_{ARMA}T + \xi T \, . \tag{37}$$

The output asymptotic mean and variance can be obtained as

$$\lim_{k\to\infty} E[y(k)] = \left(\frac{(\theta-\phi)(\xi-1)}{1-\theta} + \xi\right)T \quad , \tag{38}$$

$$\lim_{k\to\infty} \mathrm{Var}[y(k)] = \sigma^2 \quad . \tag{39}$$

The system output apparently always produces the minimum output variance but with an offset from the target when a model mismatch exists.

ARIMA(1,1,1)

Consider the ARIMA(1,1,1) time series disturbance represented by

$$\eta(k+1) = (1+\phi)\eta(k) - \phi\eta(k-1) + e(k) - \theta e(k-1) \, . \tag{40}$$

Then, the input/output relationship of the F' predictor with batch delay for ARIMA(1,1,1) in time domain can be written as

$$\begin{aligned} \hat{\eta}(k+1) = &-\frac{1+\phi-\theta-\xi(\phi+1)}{\xi}\hat{\eta}(k) - \frac{\phi(\xi-1)}{\xi}\hat{\eta}(k-1) \\ &+ \frac{1+\phi-\theta}{\xi}\delta(k) - \frac{\phi}{\xi}\delta(k-1) \end{aligned} \, . \tag{41}$$

Substituting (27) into . (41) yields

$$\begin{aligned} \hat{\eta}(k+1) = \theta\hat{\eta}(k) &+ \frac{1+\phi-\theta}{\xi}\eta(k) - \frac{\phi}{\xi}\eta(k-1) \\ &+ \frac{\Delta_{ARIMA}}{\xi}T \end{aligned} \, , \tag{42}$$

where the expression $\Delta_{ARIMA} = (1-\theta)(\xi-1)/\xi$. Now, we define the state variable $x(k) = [\hat{\eta}(k) \quad \hat{\eta}(k-1) \quad \eta(k) \quad \eta(k-1)]^T$ and follow the above steps to get the system output

$$y(k) = \sum_{j=0}^{k-1}\theta^{k-j-1}(e_{j+1}-\theta e_j) - \sum_{j=0}^{k-1}\xi\theta^{k-j-1}\Delta_{ARIMA}T + \xi T \, . \tag{43}$$

The output asymptotic mean and variance can be obtained as

$$\lim_{k\to\infty} E[y(k)] = T \quad , \tag{44}$$

$$\lim_{k\to\infty} \mathrm{Var}[y(k)] = \sigma^2 \quad . \tag{45}$$

The system output is on-target with minimum variance.

So far, we have demonstrated that the proposed PDOB structure can obtain the minimum output variance under general ARIMA disturbance. Table 1 summarizes an assessment of on-target and output variance for EWMA, double EWMA, and PDOB under various stochastic time-series disturbances. One can see that the steady-state output produces an offset from the target if the stochastic disturbance is ARMA(p, q) when model mismatch occurs. This problem can be easily solved by employing a PI controller in the outer loop of the PDOB scheme to compensate for the offset. The control structure is shown in Fig. 3.

Table 1. The output properties of different controllers under the different stochastic disturbances

EWMA controller				
	IMA(1,1)	ARMA(1,1)	ARIMA(1,1,1)	Other ARIMA(p, r, q)
On-target	O	O	O	X(except r=0, 1)
Minimum variance	O	X	X	X
Double EWMA controller				
On-target	O	O	O	X(except r=0, 1, 2)
Minimum variance	O	X	X	X (except IMA(2,1) and IMA(2,2))
PDOB				
On-target	O	X ($\xi \neq 1$) O ($\xi = 1$)	O	O (except ARMA(p, q), $\xi \neq 1$)
Minimum variance	O	O	O	O

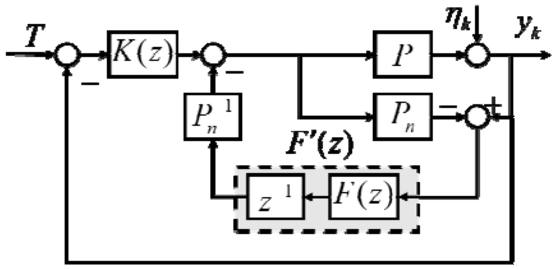

Fig. 3. The PDOB with PI outer controller structure

SIMULATION

In this section, some simulations are presented to illustrate the differences of output performance using the EWMA, double EWMA, and PDOB controllers under different time-series based disturbances. Table 2 presents the numerical results for given values of model mismatch ξ , optimal weights for the EWMA and double EWMA controllers, the target T , a zero mean white noise with variance, and the coefficients of the time series model. An estimate of the variance is

$$\text{variance} = \frac{1}{n}\sum_{k=1}^{n}(y_k - \hat{y}_k)^2 \quad , \tag{46}$$

where n is the batch of the process, y_k is the process output, $\hat{y}_k$ is the mean of the process output, and k is the number batch of the process.

For the IMA(1,1) disturbance in Case I of Table 2, the system output achieved minimum output variance no matter what the model mismatch is. The EWMA controller with optimum parameters is equivalent to the PDOB for the IMA(1,1). Thus, the results of the two controllers are identical.

For the ARMA(1,1) disturbance in Case II of Table 2, the output responses of the system using the EWMA controller and PDOB are shown in Fig. 4. We can observe that the performance of the controlled system using the PDOB has faster transient response and less oscillation under model mismatch; moreover, it produces minimum output variance. However, the system output produces an offset, as we noted before.

For the ARIMA(2,2,2) disturbance in Case III of Table 2, the output responses of the system using the double EWMA controller and PDOB, respectively, are shown in Fig. 5. The results depict different performances of the two controllers; the output of PDOB achieves minimum variance and is on target no matter what the model mismatch is. According to the internal model principle, the controlled output rejects disturbance without steady-state error if the reciprocal of the disturbance is modelled into the controller. Note that the proposed PDOB is a fourth-order controller in this case rather than a second-order controller in double EWMA.

The ARI(3,1) disturbance observed in the sputter deposition process (Chen et al., 2007) and the historical data series is plotted in Fig. 6. In this illustration, the model mismatch is one, and the reference target is zero. The variance for double EWMA and PDOB is 1.2777 and 1.1725, respectively, with a variance reduction of approximately 8%.

Furthermore, simulation results are presented when a PI outer loop is added to the PDOB structure to compensate for the offset. Fig. 7 illustrates the controlled performance using the EWMA controller or double EWMA and the PI plus the PDOB controller, respectively, under IMA(1,1), ARMA(1,1), and ARIMA(2,2,2) disturbances, as used in the previous study. The simulation considered the case that the model mismatch is $\xi = 1.5$ and the PI controller is $5z/(z-1)$. The results showed that the PI plus the PDOB controller not only can supresses the offset but also reduces the output variance.

Table 2. Simulation results for various controllers under various disturbances

Controller	ξ	λ^*	Variance	Performance Improvement $\frac{\text{Var}[EWMA]-\text{Var}[PDOB]}{\text{Var}[EWMA]}\times 100\%$
Case I: $\theta=0.7,\ \phi=0,\ T=20,\ e(k)\sim N(0,1)$, IMA(1,1)				
EWMA controller & minimum variance controller	0.5	0.6	1.0206	0%
	1	0.3	1.0206	0%
	1.5	0.2	1.0206	0%
Case II: $\theta=0.7,\ \phi=0.8,\ T=20,\ e(k)\sim N(0,1)$, ARMA(1,1)				
EWMA controller	0.5	0.045	1.3114	N/A
	1	0.023	1.1785	N/A
	1.5	0.015	1.2611	N/A
minimum variance controller	0.5	N/A	1.0206	22.17%
	1	N/A	1.0206	13.40%
	1.5	N/A	1.0206	19.07%
Case III: $\theta_1=\theta_2=0.1,\ \phi_1=0.2,\ \phi_2=0.7,\ T=20,\ e(k)\sim N(0,1)$, ARIMA(2,2,2)				
Double EWMA controller	1	$\lambda_1=0.7693$ $\lambda_2=0.99$	2.46	N/A
	1.5	$\lambda_1=0.3655$ $\lambda_2=0.99$	1.5799	N/A
minimum variance controller	1	N/A	1.0213	58.48%
	1.5	N/A	1.0213	35.36%
Case IV: $\phi_1=-0.021,\ \phi_2=-0.088,\ \phi_3=0.088,\ T=20,\ e(k)\sim N(0,1)$, ARI(3,1)				
Double EWMA controller	1	$\lambda_1=0.97$ $\lambda_2=0.01$	1.2777	N/A
minimum variance controller	1	N/A	1.1725	8.23%

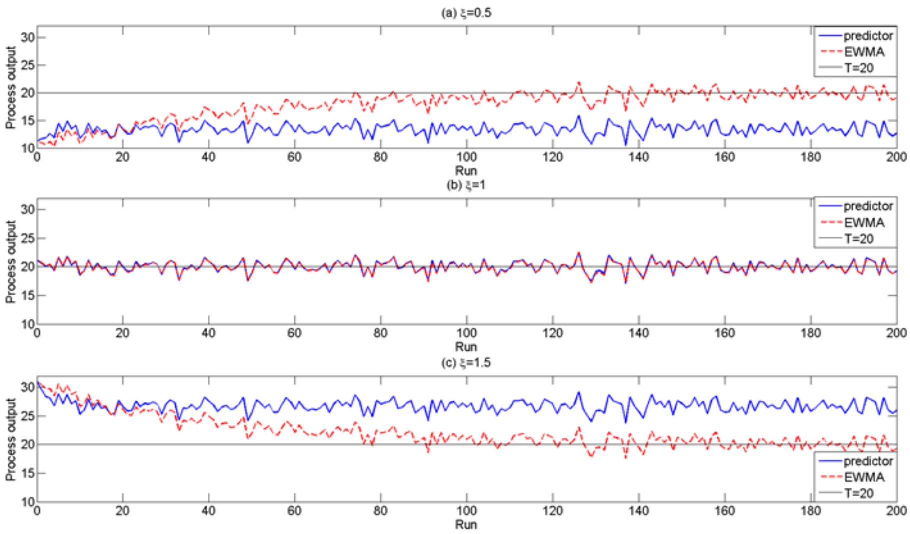

Fig. 4. The process output of PDOB and EWMA for different model mismatch under ARMA(1,1)

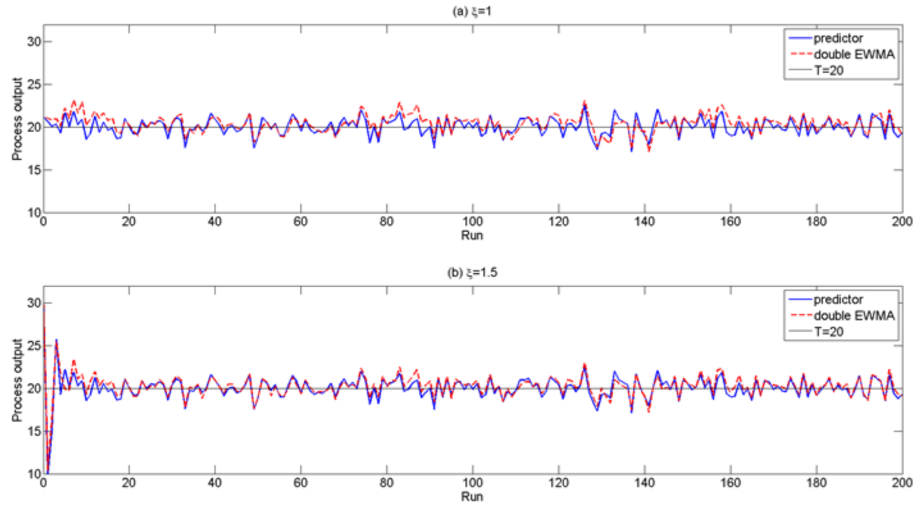

Fig. 5. The process output of PDOB and EWMA for different model mismatch under ARIMA(2,2,2)

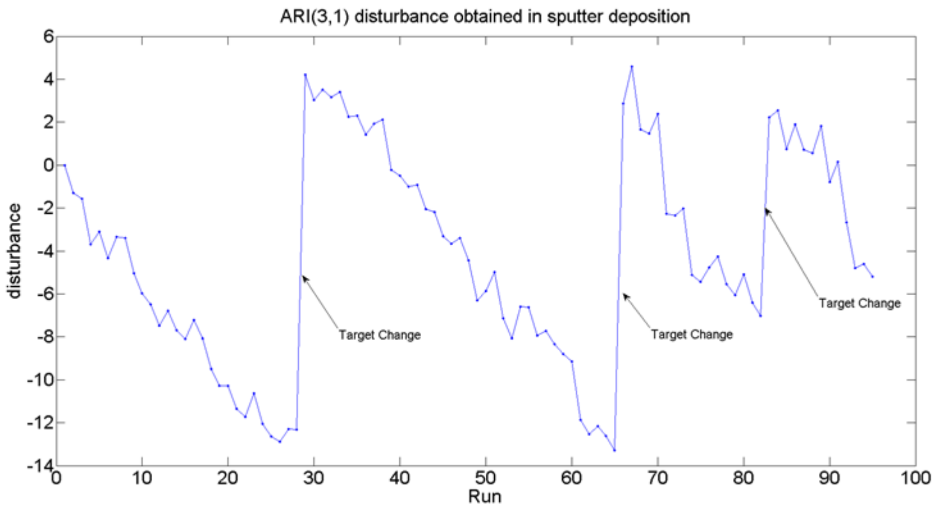

Fig. 6. ARI(3,1) disturbance obtained in sputter deposition process (Chen et al., 2007)

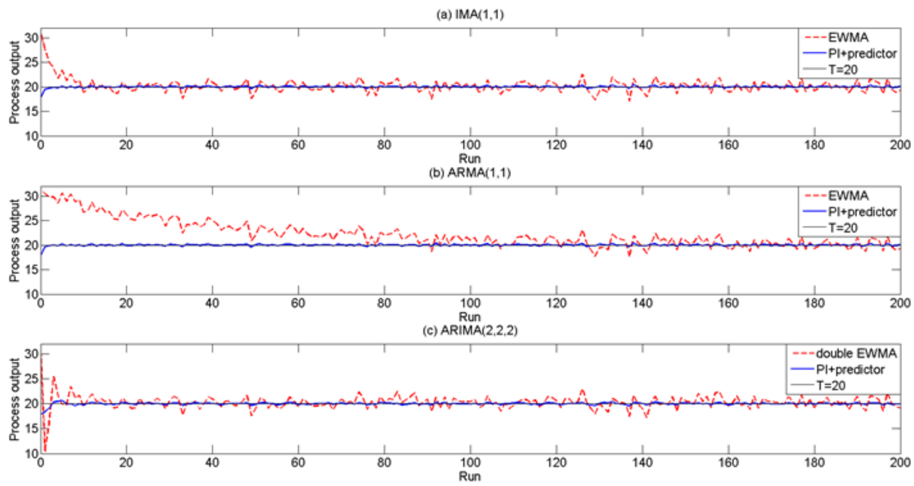

Fig. 7. The process output using EWMA (or double EWMA) and PI+PDOB respectively, under (a) IMA(1,1), (b) ARMA(1,1), and (c) ARIMA(2,2,2)

CONCLUSION

A traditional EWMA controller produces the system minimum output variance only under an IMA(1,1) disturbance, and a double EWMA controller produces the system minimum output variance only under an IMA(2,1) or IMA(2,2) disturbance. For general stochastic time-series based disturbances, both controllers are not the best control scheme from the output variance viewpoint. In this paper, we developed a PDOB control scheme based on the minimum variance control to deal with the ARIMA(p,r,q) stochastic disturbance, thereby producing minimum output variance or only white noise variance if the disturbance model is known. For an ARMA(p,q) disturbance, a EWMA controller cannot obtain the minimum output variance and has worse transient response when a model mismatch exists. By contrast, a PDOB controller can produce minimum output variance but with an offset under an ARMA(p,q) disturbance when model mismatch occurs, which can be compensated by the PI outer loop.

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APPENDIX

The ARIMA(p,r,q) disturbance can be expressed as

$$\eta_{ARIMA} = \frac{1-\sum_{k=1}^{q}\theta_k z^{-k}}{(1-z^{-1})^r\left(1-\sum_{k=1}^{p}\theta_k z^{-k}\right)} e = G_{ARIMA} e \,. \tag{I.1}$$

Because the predictor $F(z)$ can produce the minimum output variance, it follows that the disturbance term in (1) can be expressed as

$$y = \frac{P_n(1-Fz^{-1})}{P_n+(P-P_n)Fz^{-1}}\eta_0(z) = \frac{P_n(1-Fz^{-1})G_{ARIMA}}{P_n+(P-P_n)Fz^{-1}} e = e \,. \tag{I.2}$$

Let

$$F(z) = \frac{b_0 z^k + b_1 z^{k-1} + \cdots + b_k}{z^k + a_1 z^{k-1} + a_2 z^{k-2} + \cdots + a_k} \,. \tag{I.3}$$

Substituting (I.1), and (I.3) into (I.2), we can get by the coefficient comparison method.

For the ARIMA(p,1,q) disturbance,

$$G_{ARIMA} = \frac{1-\sum_{k=1}^{q}\theta_k z^{-k}}{(1-z^{-1})^1\left(1-\sum_{k=1}^{p}\theta_k z^{-k}\right)}$$
$$= \frac{z^k-\theta_1 z^{k-1}-\theta_2 z^{k-2}-\cdots-\theta_k z^{k-k}}{z^k+(-1-\phi_1)z^{k-1}+(\phi_1-\phi_2)z^{k-2}+\cdots+(\phi_p-\phi_{p+1})z^{k-k}} \quad . \text{(I.4)}$$

The following equation can be obtained

$$\frac{z^k+(-1-\phi_1)z^{k-1}+\cdots+(\phi_p-\phi_{p+1})z^{k-k}}{z^k-\theta_1 z^{k-1}-\theta_2 z^{k-2}-\cdots-\theta_k z^{k-k}}$$
$$= \frac{z^k+(a_1-b_1)z^{k-1}\cdots+(a_k-b_k)z^{k-k}}{z^k+(a_1-b_1+b_1\xi)z^{k-1}\cdots+(a_k-b_k+b_k\xi)z^{k-k}} \quad . \text{(I.5)}$$

Then, $F(z)$ can be obtained by the coefficient comparison method.

$$F = \frac{\frac{1+\phi_1-\theta_1}{\xi}z^k+\sum_{i=1}^{k-1}\frac{-\phi_i+\phi_{i+1}-\theta_{i+1}}{\xi}z^{k-i}}{\left(\begin{array}{l} z^k+\frac{1+\phi_1-\theta_1-\xi(1+\phi_1)}{\xi}z^{k-1} \\ +\sum_{i=1}^{k-1}\frac{-\phi_i+\phi_{i+1}-\theta_{i+1}-\xi\left(-\phi_i+\phi_{i+1}\right)}{\xi}z^{k-i-1}\end{array}\right)} \quad . \text{(I.6)}$$

For the ARIMA(p,2,q) disturbance,

$$G_{ARIMA} = \frac{1-\sum_{k=1}^{q}\theta_k z^{-k}}{(1-z^{-1})^2\left(1-\sum_{k=1}^{p}\phi_k z^{-k}\right)}$$
$$= \frac{z^k-\theta_1 z^{k-1}-\theta_2 z^{k-2}-\cdots-\theta_k z^{k-k}}{z^k+(-2-\phi_1)z^{k-1}+(1+2\phi_1-\phi_2)z^{k-2}\cdots+(-\phi_p+2\phi_{p+1}-\phi_{p+2})z^{k-k}} \quad .\text{(I.7)}$$

The following equation can be obtained

$$\frac{\left(\begin{array}{l} z^k+(-2-\phi_1)z^{k-1}+(1+2\phi_1-\phi_2)z^{k-2} \\ +\cdots+(-\phi_p+2\phi_{p+1}-\phi_{p+2})z^{k-k}\end{array}\right)}{z^k-\theta_1 z^{k-1}-\theta_2 z^{k-2}+\cdots-\theta_k z^{k-k}} = \quad . \quad \text{(I.8)}$$
$$\frac{z^k+(a_1-b_1)z^{k-1}\cdots+(a_k-b_k)z^{k-k}}{z^k+(a_1-b_1+b_1\xi)z^{k-1}\cdots+(a_k-b_k+b_k\xi)z^{k-k}}.$$

Then, $F(z)$ can be obtained as

$$F(z) = \frac{\left(\begin{array}{l} \frac{2+\phi_1-\theta_1}{\xi}z^k+\frac{-1-2\phi_1+\phi_2-\theta_2}{\xi}z^{k-1} \\ +\sum_{i=1}^{k-2}\frac{\phi_i-2\phi_{i+1}+\phi_{i+2}-\theta_{i+2}}{\xi}z^{k-i-1}\end{array}\right)}{\left(\begin{array}{l} z^k+\frac{2+\phi_1-\theta_1-\xi(2+\phi_1)}{\xi}z^{k-1} \\ +\frac{-1-2\phi_1+\phi_2-\theta_2-\xi(-1-2\phi_1+\phi_2)}{\xi}z^{k-2} \\ +\sum_{i=1}^{k-2}\frac{\phi_i-2\phi_{i+1}+\phi_{i+2}-\theta_{i+2}-\xi\left(\phi_i-2\phi_{i+1}+\phi_{i+2}\right)}{\xi}z^{k-i-2}\end{array}\right)} \quad . \text{(I.9)}$$

For the ARMA(p,q) disturbance,

$$G_{ARIMA} = \frac{1-\sum_{k=1}^{q}\theta_k z^{-k}}{\left(1-\sum_{k=1}^{p}\phi_k z^{-k}\right)} \quad . \quad \text{(I.10)}$$
$$= \frac{z^k-\theta_1 z^{k-1}-\theta_2 z^{k-2}-\cdots-\theta_q z^{k-k}}{z^k-\phi_1 z^{k-1}-\phi_2 z^{k-2}-\cdots-\phi_p z^{k-k}}$$

The following equation can be obtained

$$\frac{z^k-\phi_1 z^{k-1}-\phi_2 z^{k-2}-\cdots-\phi_p z^{k-k}}{z^k-\theta_1 z^{k-1}-\theta_2 z^{k-2}-\cdots-\theta_k z^{k-k}}$$
$$= \frac{z^k+(a_1-b_1)z^{k-1}+\cdots(a_k-b_k)z^0}{z^k+(a_1-b_1+b_1\xi)z^{k-1}+\cdots(a_k-b_k+b_k\xi)z^0} \quad . \text{(I.11)}$$

Then, $F(z)$ can be obtained as.

$$F(z) = \frac{\sum_{i=1}^{k}\frac{\phi_i-\theta_i}{\xi}z^{k-i+1}}{z^k+\sum_{i=1}^{k}\frac{\phi_i-\theta_i-\xi\phi_i}{\xi}z^{k-i}} \quad . \quad \text{(I.12)}$$

NOMENCLATURE

T	process target
u_k	process output (input recipe)
y_k	process output
η_k	output disturbance
P	process gain (or actual plant)
P_n	nominal plant
α	process intercept term
k	batch index.
$F(z)$	one-step ahead predictor
$e(k)$	a sequence of independent random variables
ξ	model mismatch

通用隨機時間序列程序干擾之最小方差批次控制器

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摘 要

指數加權移動平均值(EWMA)或雙EWMA控制器經常被使用來處理各種隨機時間序列擾動，以實現批次控制。如果已知擾動模型，則可以通過將控制器參數的輸出差異最小化來獲得最佳EWMA或雙EWMA控制器。但是，從理論角度來看，結果只是次優化，因為控制架構可能不是當下隨機干擾的最佳控制方案。因此，研究一般ARIMA時間序列之程序擾動的最佳控制方案是值得的。本文基於最小方差控制，針對各種ARIMA(p,r,q)隨機擾動開發了預測擾動觀測器(PDOB)。如果已知ARIMA(p,r,q)干擾模型，則PDOB方案會針對該干擾生成單步提前預測，然後將其反饋以補償隨機干擾對系統輸出的影響，以致系統生成最小輸出方差或僅產生白噪聲方差。當模型不匹配時，在某些ARMA干擾模型下，受控系統的輸出可能會產生偏移。但此現象可以通過使用PI控制器等外環控制器輕鬆糾正此偏移。