# Modified Smith Predictor with a Periodic Disturbance Reduction Method for Linear Small Delay Systems

Ming-Hau Tsai\*and Pi-Cheng Tung\*\*

**Keywords**: Disturbance observer, Disturbance reduction, Modified Smith predictor, Time delay.

#### ABSTRACT

A modified Smith predictor with a periodic disturbance reduction method for linear systems with small time delays is proposed in this paper. With this method it is not necessary to estimate unknown disturbance frequencies. The main control structure is provided by Astrom's modified Smith predictor. The proposed method consists of a disturbance reduction controller (DRC) and a residual disturbance observer (RDO). The DRC, which is composed of an inverse plant model and an integrator with a nonnegative gain, compensates for unknown load disturbances and exhibiting uncertainties in stable or unstable systems. The disturbance reduction performance of the proposed method is enhanced by combining the DRC with the RDO to suppress undesired residual signals including residual disturbances and residual uncertainties. Simulation examples demonstrate the effectiveness of the proposed periodic disturbance reduction method for linear uncertain systems with time delays, under periodic or non-periodic unknown load disturbances.

### **INTRODUCTION**

A method for periodic disturbance reduction is required in many industrial engineering applications, particularly for control systems of rotating machines. Obviously, the system performance is influenced by the periodic load disturbances. Many studies have been devoted to the issue of disturbance rejection and how to avoid the degradation of system performance that results

Paper Received July, 2018. Revised October, 2018. Accepted November, 2018. Author for Correspondence:Pi-Cheng Tung.

\* Graduate Student, Department of Mechanical Engineering, National Central University, Taoyuan, Taiwan 32054, ROC. from periodic load disturbances (Muramatsu, 2018; Chen, 2018; Tan, 2018; Shen, 2014; Karanam, 2013).

Muramatsu (2018) proposed an adaptive periodicdisturbance observer to compensates a frequency-varying periodic disturbances. An adaptive notch filter was used to estimate the fundamental frequency of the periodic disturbance. Chen et al. (2018) combined equivalentinput-disturbance (EID) theory with the internal model principle is applied to achieve periodic disturbance rejection in input-time-delay systems. The EID estimator was constructed by inserting appropriate time-delay elements to compensate for the total influence of the input-time-delay and the periodic disturbance. Tan (2018) proposed a multiple periodic disturbance rejection under the Smith predictor configuration for processes with long dead-time. One feedback loop was added to compensate periodic disturbance while retaining the advantage of the Smith predictor. Shen (2014) designed a periodic disturbance rejection controller based on the Smith predictor for process with long dead-time. By adding two feedback loops and the online spectrum analysis, multiple periodic disturbances can be suppressed effectively in existence of long dead-time. Karanam (2013) proposed a modified Smith predictor control scheme for step and periodic disturbance rejection for unstable processes with time delay. A periodic disturbance rejection controller was designed to improve nominal and robust performances for step and periodic disturbances and also improved closedloop performances.

In process control, small time delays between system output and sensor output are a common occurrence, however the occurrence of such time delays may complicate the design of the control system (Richard, 2003; Watanabe, 1981;). Smith (1959) proposed a predictor scheme for the control of stable processes with time delays. The Smith predictor scheme includes a plant model and a conventional controller. In the scheme, the time delay can be taken out of the closed-loop characteristic equation when the plant model perfectly matches the real plant. In short, the control performance of the Smith predictor is relative to the plant model. Modeling uncertainties, which are caused by the inevitable mismatch between the real plant and the plant model, are regarded as an additional load disturbance in

<sup>\*\*</sup> Professor, Department of Mechanical Engineering, National Central University, Taoyuan, Taiwan 32054, ROC.

the control system by Tian (1998). However, the Smith predictor shows poor capability for rejecting load disturbances, including modeling uncertainties. Although much work has been carried out to improve the disturbance rejection performance of the Smith predictor (Watanabe, 1981; Tian, 1998; Hang, 1979; Astrom et al., 1994; Mataušek, 1999; Chien et al., 2002; Stojić et al., 2001: Kava. 2004: Zheng et al., 2010: Chen et al., 2007: Tsai, 2010:), few methods based on the Smith predictor have been proposed to attenuate periodic load disturbances introduced into delay systems (Zheng et al., 2010; Chen et al., 2007; Tasi, 2010; Tsai, 2012; Tsai, 2012). Tsai (2010) designed a control structure based on Astrom's modified Smith predictor with a disturbance reduction scheme and an artificial neural network (ANN). Tsai (2012) presented a robust disturbance reduction scheme using an artificial neural network (ANN) for linear systems with small time delays. Tsai (2012) proposed an input disturbance reduction controller using an artificial neural network (ANN) to reduce unknown load disturbances and modeling uncertainties in stable systems and unstable systems.

The purpose of this study is to develop a periodic disturbance reduction method to reduce periodic or nonperiodic unknown load disturbances introduced into a delay system with modeling uncertainties. The control structure of the proposed scheme is based on Astrom's modified Smith predictor (Astrom et al., 1994). With the proposed method there is no need to estimate the disturbance frequencies when applied to control systems. A disturbance reduction controller (DRC) and a residual disturbance observer (RDO) are included in the proposed method. The DRC is composed of an inverse plant model and an integrator with a nonnegative gain and is proposed to reduce the unknown load disturbances and modeling uncertainties. The unknown real plant can be modeled as a stable plant model with modeling uncertainties even if the unknown real plant is unstable. Since the DRC cannot completely cancel out the unknown load disturbances and modeling uncertainties, residual disturbances and residual uncertainties exist in the control system which will affect the system performance. To enhance the disturbance reduction performance, an RDO, based on a disturbance observer (Sastry, 1994), is combined with the DRC. The RDO is designed to reduce residual signals including disturbances and uncertainties. As a consequence, the unknown load disturbances and modeling uncertainties are significantly compensated for by the proposed method used in the modified Smith predictor for delay systems.

The rest of the paper is organized as follows. The Smith predictor and the Astrom's modified Smith predictor are described in Section 2. The proposed periodic disturbance reduction method, consisting of the DRC and the RDO, is discussed in Section 3. In Section 4, we discuss simulation results obtained under different conditions that show the effectiveness of the proposed scheme. Some brief conclusions are presented in Section 5.

## THE SMITH PREDICTOR AND THE MODIFIED SMITH PREDICTOR

The predictor scheme proposed by Smith (1959) is an effective time-delay compensator and can be used to control stable processes with time delays. A block diagram of the Smith predictor is shown in Fig. 1(a), where C(s) is the main controller. When a delay-free plant model  $\hat{G}(s)$ with an estimated time delay  $\hat{L}$  is identical to a delay-free real plant G(s) with a real time delay L for this predictor, i.e.,  $\hat{G}(s)e^{\hat{L}s} = G(s)e^{-Ls}$ , the transfer function from the reference command R(s) to the system output Y(s) is given by

$$\frac{Y(s)}{R(s)} = \frac{C(s)G(s)e^{-Ls}}{1 + C(s)G(s)}.$$
(1)

It can be seen in Eq. (1) that the Smith predictor removes the time delay term in a characteristic equation from a closed-loop system. Therefore, the main controller C(s) can make use of a simple conventional controller (such as the PI or PID). Good set-point responses to reference commands are obtained using the Smith predictor scheme. The transfer function from the input disturbance D(s) to the system output Y(s) is given by

$$\frac{Y(s)}{D(s)} = \frac{G(s)e^{-Ls}}{1+C(s)G(s)} + \frac{C(s)G(s)e^{-Ls}}{1+C(s)G(s)} \cdot [G(s) - G(s)e^{-Ls}].$$
 (2)

In Eq. (2) it can be observed that the performance of the disturbance response depends on the poles of the delay-free plant G(s). When the poles of the delay-free plant G(s) are near the imaginary axis, the input disturbance D(s) introduces a steady-state error into the output response which will affect the system performance. Therefore, the input disturbance cannot be cancelled out by an integrator process which uses the Smith predictor (Hang, 1979). Watanabe (1981) proposed a modified Smith predictor scheme for the integrator processes. However, one drawback of the Watanabe's modified Smith predictor is that the resulting set-point response tends to be very slow. Astrom et al. (1994) also developed a new modified Smith predictor for the improvement of the control performance. Astrom's modified Smith predictor is shown in Fig. 1(b). The set-point transfer function and the disturbance transfer function are given respectively by

$$\frac{Y(s)}{R(s)} = \frac{C(s)G(s)e^{-Ls}}{1+C(s)G(s)} \cdot \frac{[1+M(s)G(s)e^{-Ls}]}{[1+M(s)G(s)e^{-Ls}]} = \frac{C(s)G(s)e^{-Ls}}{1+C(s)G(s)},$$
 (3)

$$\frac{Y(s)}{D(s)} = \frac{G(s)e^{-Ls}}{1 + M(s)G(s)e^{-Ls}}.$$
(4)

Since the Astrom's modified Smith predictor decouples the set-point transfer function Eq. (3) from the

disturbance transfer function Eq. (4), the two transfer functions can be optimized independently. The compensator M(s) in the Astrom's modified Smith predictor can be designed by the user to improve the disturbance response. In this study, the proposed periodic disturbance reduction method is employed in the Astrom's modified Smith predictor to deal with the input disturbance introduced into the process with the time delay.



Fig. 1 (a) A schematic diagram of the Smith predictor controller and (b) A schematic diagram of the Astrom's modified Smith predictor controller.

### PROPOSED PERIODIC DISTURBANCE REDUCTION METHOD

#### **Disturbance reduction controller (DRC)**

A block diagram of the proposed periodic disturbance reduction method consisting of the DRC and the RDO is shown in Fig. 2(a). It is not necessary to measure unknown disturbance frequencies when the proposed method is applied to the control system. In Fig. 2(a), the unknown real plant P(s) can be modeled as a stable plant model  $\hat{P}(s)$  with modeling uncertainties even if the unknown real plant P(s) is unstable. The DRC should be properly designed in order to reject the measurement noise (Sastry, 1994). The DRC consists of an inverse plant model  $\hat{P}^{-1}(s)$  and an integrator with a nonnegative gain  $K_1$ . The compensative force  $\hat{D}_l(s)$  generated from the DRC can be given by

$$\hat{D}_{1}(s) = \frac{\kappa_{I}}{s} \hat{P}^{-1}(s)[Y(s) - R(s)\hat{P}(s) + \hat{D}_{2}(s)\hat{P}(s)]$$

$$= \frac{\kappa_{I}}{s} [(D_{L}(s) - \hat{D}_{1}(s)) + \frac{\Delta(s)}{\hat{P}(s)}(R(s) + D_{L}(s) - \hat{D}_{1}(s) - \hat{D}_{2}(s)), \quad (5)$$

where  $\hat{D}_2(s)$  is a compensative force of the RDO; and  $D_2(s)$  is an unknown load disturbance. For convenience of derivation, in Eq. (5), a modeling error function  $\Delta(s)$  and a parametric error function  $\rho(s)$  are defined respectively as

$$\Delta(s) = P(s) - \hat{P}(s), \tag{6}$$

$$\rho(s) = \frac{\Delta(s)}{\hat{P}(s)} (R(s) + D_L(s) - \hat{D}_1(s) - \hat{D}_2(s)), \tag{7}$$

where  $\Delta(s)$  is assumed to satisfy the matching condition. Therefore, Eq. (5) can be rewritten as

$$\hat{D}_{1}(s) = \frac{K_{I}}{s} [(D_{L}(s) - \hat{D}_{1}(s)) + \rho(s)] = \frac{K_{I}}{s + K_{I}} [D_{L}(s) + \rho(s)].$$
(8)

For convenience of interpretation, the sum of the unknown load disturbance  $D_L(s)$  and the unknown parametric error function  $\rho(s)$  in Eq. (8) is defined as an input disturbance D(s), i.e.,  $D(s) = D_L(s) + \rho(s)$ . Hence, a simplified form of Eq. (8) can be written as

$$\hat{D}_1(s) = \frac{K_I}{s + K_I} D(s).$$
<sup>(9)</sup>

Since the transient response of Eq. (9) resembles that obtained by a first-order system, the compensative force  $\hat{D}_I(s)$  can track the unknown input disturbance D(s) when a nonnegative gain  $K_I$  is properly specified. This is to say, the compensative force  $\hat{D}_I(s)$  can compensate for both the unknown load disturbance  $D_L(s)$  and the modeling uncertainties that arise due to the parametric error function  $\rho(s)$ . The residual signal H(s) introducing into the process can be expressed as

$$H(s) = D(s) - \hat{D}_1(s) = \frac{s}{s + K_1} D(s).$$
(10)

A block diagram equivalent to that in Fig. 2(a) is shown in Fig. 2(b). It can be seen in Eq. (10) and Fig. 2(b) that the DRC performs similarly to a high-pass filter introduced between the process and the unknown input disturbance. Hence, the DRC can reduce an unknown input disturbance whose frequency is less than the nonnegative gain  $K_1$ .



Fig. 2 (a) A schematic diagram of the proposed method composed of the DRC and the RDO and (b) An equivalent structure of (a).

In order to apply the DRC to the Astrom's modified Smith predictor for the delay system  $(P(s)=G(s)e^{-Ls})$ , the compensator M(s) in Eq. (4) could be selected as follows:

$$M(s) = \frac{K_I}{s\hat{G}(s)}.$$
(11)

Then the corresponding disturbance response can be written as

$$Y(s) = \frac{s\hat{G}(s)e^{-\hat{L}s}}{s + K_I e^{-Ls}} D(s).$$
 (12)

Since the nonnegative gain  $K_I$  is related to the control system stability, an appropriate nonnegative gain value can be obtained using the root locus stability method. Similarly, the compensator M(s) in Eq. (4) could be chosen in another manner as follows:

$$M(s) = \frac{K_I}{s\hat{G}(s)}e^{\hat{L}s}.$$
(13)

Now the corresponding disturbance response has the form,

$$Y(s) = \frac{s\hat{G}(s)e^{-\hat{L}s}}{s+K_{I}}D(s) = H(s)\hat{G}(s)e^{-\hat{L}s}.$$
 (14)

However, the leading time term  $e^{\hat{L}s}$  in Eq. (13) requires future data. Since future data cannot be estimated

exactly, a proper approximation equation Eq. (15) is used to substitute the leading time term (Hong, 1998),

$$e^{\hat{L}s} \approx \frac{1}{(1+\tau s)^n} \sum_{k=0}^n \frac{(\hat{L}s)^k}{k!},$$
 (15)

where  $\tau$  is a small positive number. Since the residual signal H(s) in Eq. (10) cannot be canceled out completely by the DRC, the RDO is combined with the DRC to deal with the residual signal H(s). The development of the RDO is described in the next section.

#### Residual disturbance observer (RDO)



Fig. 3 A schematic diagram of the RDO.

A block diagram of the RDO is shown in Fig. 3, where  $\widehat{G}^{-1}(s)$  is an inverse delay-free plant model. Since the compensative force  $\widehat{D}_1(s)$  of the DRC cannot cancel out the input disturbance D(s), this compensative force  $\widehat{D}_2(s)$  is used to reduce the residual signal H(s), including residual disturbances and residual uncertainties. The transfer functions from the reference command R(s)and the residual signal H(s) to the system output Y(s) as shown in Fig. 3, can be given respectively by

$$\frac{Y(s)}{R(s)} = \hat{G}(s)e^{-\hat{L}s}.$$
(16)

$$\frac{Y(s)}{H(s)} = \hat{G}(s)e^{-\hat{l}s}(1 - Q(s)e^{-\hat{l}s}).$$
(17)

From Eq. (17), it can be observed that Q(s) should be chosen to have a unit dc gain for attenuation of the residual signal H(s). Moreover, in order to reject measurement noise, Q(s) should be selected to be a low-pass filter. In view of the two design demands mentioned above, therefore, Q(s) is assumed to be a low-pass filter with a unit dc gain. Furthermore, the relative degree of Q(s)should be designed appropriately such that  $Q(s)\widehat{G}^{-1}(s)$  is proper, as shown in Fig. 3 (Hong, 1998).

Figure 4 shows a combination of the Astrom's modified Smith predictor and the proposed periodic disturbance reduction method consisting of the DRC and the RDO. During the implementation of the predictor, the RDO provides suitable compensation for various residual signals, including approximation errors due to Eq. (15), residual disturbances, residual uncertainties, and so on.



Fig. 4 A schematic diagram of a modified Smith predictor with the proposed method consisting of the DRC and the RDO.

#### SIMULATION RESULTS

In this section, Example 1 illustrates the disturbance reduction ability of the proposed method in Fig. 2(a) and the rest of examples show the control performance of the proposed scheme in Fig. 4. In the simulation, the sampling time is 1ms. The leading time term  $e^{\hat{L}s}$  in the following examples is approximated using Eq. (15). Moreover, the parameters for the approximation equation Eq. (15),  $\tau$  and n, are chosen to be 0.01 and 1, respectively. Therefore, the approximation equation can have the simplified form,

$$e^{\hat{L}s} \approx \frac{1+\hat{L}s}{1+0.01s}.$$
 (18)

**Example 1.** Consider an unstable first order plus dead time (FOPDT) process:

$$P(s) = G(s)e^{-Ls} = \frac{1}{s - 0.3}e^{-0.4s}.$$
(19)



Fig. 5 Results of Example 1 for an unstable FOPDT process. (a) The random unknown load disturbance  $d_L(t)$ , (b) the set-point response obtained using the DRC alone and (c) the set-point response obtained using the DRC and the RDO.

A reference command is set to be zero. A random unknown load disturbance  $d_L(t)$  shown in Fig. 5(a) is introduced at time t=0 and has the form,

$$d_L(t) = \sin(0.1t + \mu), \quad t \ge 0.$$
 (20)

where  $\mu$  is a normally distributed random variable and is given by the routine "randn" in MATLAB. In addition, the parameters of the routine are normally distributed by a mean with an amplitude of 0, a variance with an amplitude of 1 ( $\sigma^2 = 1$ ), and a standard deviation with an amplitude of 1 ( $\sigma = 1$ ). In the proposed method, the plant model is chosen to be stable with modeling uncertainties,

$$\hat{P}(s) = \hat{G}(s)e^{-\hat{L}s} = \frac{1}{s+0.2}e^{-0.5s}.$$
(21)

Moreover, a 25% error is set between the real time delay and the estimated time delay. Therefore, the real time delay and the estimated time delay are 0.4 and 0.5 seconds, respectively. In the DRC, the root locus stability method is used to specify the nonnegative gain  $K_I$  to be 0.737. The leading time term  $e^{\hat{L}s}(\hat{L}=0.5)$  is approximated by Eq. (18). Hence, the compensator M(s) in the proposed method Eq. (13) can be expressed as

$$M(s) = \frac{K_I}{s\hat{G}(s)}e^{\hat{L}s} = \frac{0.737(s+0.2)(1+0.5s)}{s(1+0.01s)}.$$
 (22)

The low-pass filter Q(s) of the RDO is selected to have the following form:

$$Q(s) = \frac{1}{0.5s + 1}.$$
(23)

In Fig. 5(b), it can be seen that the unknown load disturbance and modeling uncertainties are effectively reduced using the DRC alone. The disturbance reduction performance obtained using both of the DRC and the RDO is shown in Fig. 5(c). It can be observed in Figs. 5(b) and 5(c) that the system performance is enhanced by combining the DRC with the RDO. As shown in Fig. 5, therefore, the proposed method in Fig. 2(a) can effectively reduce the random unknown load disturbance introduced into an unstable delay system.

**Example 2.** Consider an integrator plus dead time (IPDT) process:

$$P(s) = G(s)e^{-Ls} = \frac{1}{s}e^{-0.5s}.$$
(24)

A reference command with a unit step signal is introduced at time t=0. A periodic unknown load disturbance  $d_L(t)$  is introduced and has the form

$$d_{L}(t) = \begin{cases} 0, & \text{if } 20 > t \ge 0.\\ 0.1\sin(0.2t), & \text{if } t \ge 20. \end{cases}$$
(25)

Under a perfect model match ( $\widehat{G}(s)e^{-\widehat{L}s} = G(s)e^{-Ls}$ ), the set-point response of the proposed scheme is compared with those obtained by the Astrom's modified Smith predictor (Astrom et al., 1994) and the PID controller. The conventional controller C(s) in (Astrom et al., 1994) was designed as a proportional controller,

$$C(s) = 0.5.$$
 (26)

Moreover, the compensator M(s) in Astrom et al. (1994) had the form,

$$M(s) = \frac{k_4 + \frac{k_3}{s}}{1 + k_1 + \frac{k_2}{s} + \frac{k_3}{s^2} - (\frac{k_4}{s} + \frac{k_3}{s^2}) - (\frac{k_4}{s} + \frac{k_3}{s^2})e^{-Ls}}.$$
 (27)

where  $k_4=k_2+k_3$ ,  $k_1=4$ ,  $k_2=3$  and  $k_3=0$ . An effective rule given by the Ziegler-Nichols tuning method is used to tune the gain values of the PID controller. According to the Ziegler-Nichols rule, therefore, the three gain values are  $K_P=1.88$ (proportional gain),  $T_I=1.00$  (reset time) and  $T_D=0.25$  (derivative time). In the proposed method, the conventional controller C(s) and the compensator M(s) are designed respectively as,

$$C(s) = 9.5 + \frac{25}{s},\tag{28}$$

$$M(s) = \frac{K_I}{s\hat{G}(s)}e^{\hat{L}s} = \frac{0.737(1+0.5s)}{1+0.01s},$$
(29)

where the nonnegative gain  $K_I$  is specified to be 0.737 by using the root locus stability method; and the leading time term  $e^{\hat{L}s}(\hat{L}=0.5)$  is approximated by Eq. (18). Moreover, the low-pass filter Q(s) in the RDO has the same transfer function as that in Eq. (23). The set-point responses depicted in Fig. 6 show that the proposed scheme in Fig. 4 provides the better set-point response and disturbance rejection performance.



Fig. 6 Results of Example 2 for an IPDT process.

**Example 3.** Consider an open-loop stable FOPDT process such as the one studied in Chen et al. (2007):

$$P(s) = G(s)e^{-Ls} = \frac{1}{s+0.5}e^{-0.2s}.$$
(30)

A reference command with a unit step signal is introduced at time t = 0. Two unknown load disturbances in (Chen et al., 2007) are introduced:

**Disturbance 1.** Assume that the unknown load disturbance is given by,

$$d_L(t) = 0.2\sin(0.15t), \quad t \ge 0.$$
 (31)

**Disturbance 2.** Assume that the unknown load disturbance is given by,

$$d_{L}(t) = 0.15\sin(0.06t) + 0.2\sin(0.08t) + 0.12\cos(0.15t), \quad t = 0.$$
(32)

In Chen et al. (2007), it was assumed that the real plant and the plant model match each other perfectly, i.e.,  $\hat{G}(s)e^{\hat{L}s} = G(s)e^{-Ls}$ . The conventional controller C(s) and the compensator M(s) in Chen et al. (2007) was given by

$$C(s) = 9.5 + \frac{25}{s}.$$
(33)

$$M(s) = \frac{K_I}{s\hat{G}(s)}e^{\hat{I}s} = \frac{1.8(s+0.5)}{s}e^{0.2s},$$
(34)

where the nonnegative gain  $K_I$  is estimated to be 1.8 by using the root locus stability method; and the leading time term  $e^{0.2s}$  is approximated by the grey predictor. In comparison, the conventional controller C(s) and the nonnegative gain  $K_I$  for the proposed method are the same as those obtained in Chen et al. (2007). In the proposed method, however, the leading time term  $e^{\hat{L}s}(\hat{L}=0.2)$  is replaced by using the approximation in Eq. (18). Therefore, the compensator M(s) in Eq. (13) M.-H. Tsai and P.-C. Tung: Modified Smith Predictor with a Periodic Disturbance Reduction Method.

can be given by,



Fig. 7 Results of Example 3 for a stable FOPDT process. (a) The periodic unknown load disturbance  $d_L(t)$  and the compensative force  $\hat{d}_L(t)$ , (b) The periodic unknown load disturbance  $d_L(t)$  and compensative forces  $\hat{d}_1(t) + \hat{d}_2(t)$  and (c) the reference command r(t) and the system output y(t).



Fig. 8 Results of Example 3 for a stable FOPDT process. (a) The multiple periodic unknown load disturbance  $d_L(t)$  and the compensative force  $\hat{d}_I(t)$ , (b) the multiple periodic unknown load disturbance  $d_L(t)$  and compensative forces  $\hat{d}_I(t) + \hat{d}_2(t)$  and (c) the reference command r(t) and the system output y(t).

In the RDO, the low-pass filter Q(s) has the same form as that in Eq. (23). The results in Figs. 7(a)-8(a) are given to show the disturbance-tracking performance obtained by applying the DRC alone with Disturbance 1 and Disturbance 2 respectively introduced into the control system. The effects of using both the DRC and the RDO to reduce Disturbance 1 and Disturbance 2 are illustrated in Figs. 7(b)-8(b), respectively. In addition, the set-point responses presented in Figs. 7c–8c are similar to those obtained in Chen et al. (2007); see Figs. 6(b)-7(b) in Chen et al. (2007). A comparison of Figs. 7(b)-8(b) with those in Chen et al. (2007) shows that the proposed scheme in Fig. 4 gives better convergence of the compensative forces to the unknown load disturbances; see Figs. 6(a)-7(a) in Chen et al. (2007).

In the next example we consider the open-loop stable FOPDT process with a time-varying delay (Tsai, 2010) and Disturbance 1, such that

$$P(s) = G(s)e^{-Ls} = \frac{1}{s+0.5}e^{-Ls},$$
(36)

$$L = 0.2 + 0.1\sin(0.2t). \tag{37}$$



Fig. 9 Results of Example 3 for a stable FOPDT process with a time-varying delay. (a) The periodic unknown load disturbance  $d_L(t)$  and the compensative force  $\hat{d}_I(t)$ , (b) The periodic unknown load disturbance  $d_L(t)$ and compensative forces  $\hat{d}_I(t) + \hat{d}_2(t)$  and (c) The reference command r(t) and the system output y(t).

The simulation conditions are the same as those mentioned above. The disturbance-tracking results obtained using the DRC alone, and using both DRC and the RDO, are shown in Fig. 9(a) and 9(b), respectively. Moreover, Fig. 9(c) indicates the set-point response. A comparison of Fig. 9(b) with Fig. 14 in Tsai (2010) shows that the proposed scheme gives better disturbancetracking performance to deal with the time-varying delay.

**Example 4.** Consider the following open-loop stable second order plus dead time (SOPDT) process as studied in Takehara et al. (1996):

$$P(s) = G(s)e^{-Ls} = \frac{0.04936s + 0.00197}{s^2 + 0.0401s + 0.000169}e^{-0.3s}.$$
 (38)



Fig. 10 Results of Example 4 for a stable SOPDT process. (a) The periodic unknown load disturbance  $d_L(t)$ , (b) the set-point response obtained using the DRC alone and (c) the set-point response obtained using the DRC and the RDO

The reference command is set to be zero. A sine unknown load disturbance  $d_L(t)$  shown in Fig. 10(a) is introduced at time t = 0 and has the form,

$$d_{I}(t) = \sin(t), \quad t \ge 0. \tag{39}$$

The conventional controller in Takehara et al. (1996) was designed as follows:

$$C(s) = 100 + \frac{10}{s}.$$
 (40)

To compare with Takehara et al. (1996), the proposed method uses the same conventional controller as that in Eq. (40). When the plant model is exact, the compensator M(s) of the proposed method Eq. (13) is given by

$$M(s) = \frac{K_I}{s\hat{G}(s)} e^{\hat{L}s} = \frac{1.36(s^2 + 0.0401s + 0.000169)(1 + 0.3s)}{s(0.04936s + 0.00197)(1 + 0.01s)}.$$
 (41)

where the nonnegative gain  $K_I$  is specified by the root locus stability method to be 1.36; and the leading time term  $e^{\hat{L}s}(\hat{L}=0.3)$  is approximated by Eq. (18). The lowpass filter Q(s) in the RDO has the same transfer function as that in Eq. (23). Fig. 10(b) demonstrates the effective disturbance reduction provided by the DRC alone. It can be seen in Fig. 10(c) that the disturbance reduction performance has been enhanced by combining the DRC with the RDO. It can be seen in Fig. 10(c) that the proposed scheme in Fig. 4 gives the better disturbance reduction performance for reducing the sine unknown load disturbance; see Fig. 9 in Takehara et al. (1996).

#### CONCLUSION

This paper proposes a modified Smith predictor for controlling linear delay systems with unknown load disturbances and modeling uncertainties. A PI controller and the proposed periodic disturbance reduction method are included in this modified Smith predictor. The proposed method does not necessarily estimate the disturbance frequencies when it is applied to the control system. In addition, the proposed method is composed of the DRC and the RDO. The DRC is used to reduce unknown load disturbances and modeling uncertainties. The residual signals are effectively reduced by the RDO. The closed-loop system performance shows that the proposed method gives robustness against modeling uncertainties, even if the time delay is varying. Simulations demonstrate that the proposed scheme provides effective disturbance reduction performance and simplicity of structure for controlling stable or unstable delay systems with periodic or non-periodic unknown load disturbances.

#### REFERENCE

- Astrom, K. J., Hang, C. C., and Lim, B. C., "A new smith predictor for controlling a process with an integrator and long dead-time", IEEE Trans. Automat. Contr., Vol. 39, No. 2, pp. 343-345. (1994).
- Chen, X., Cai, W. J., "A new approach for periodic disturbance rejection in input-time-delay systems", T. I. Meas. Control., Vol. 40, No. 8, pp. 2589-2598. (2018).
- Chen, Y. D., Tung, P. C., Fuh, C. C., "Modified Smith predictor scheme for periodic disturbance reduction in linear delay systems", J. Process Control, Vol. 17, No. 10, pp. 799-804. (2007).
- Chien, I. L., Peng, S. C., Liu, J. H., "Simple control method for integration processes with long deadtime", J. Process Control, Vol. 12, No. 3, pp. 391-404. (2002).
- Hang, C. C., Wong, F. S., "Modified Smith predictors for the control of processes with dead time", Proc. ISA Annual Conf., Vol. 34, No. 2, pp. 33-44. (1979).
- Hong, K., Nam, K., "A load torque compensation scheme under the speed measurement delay", IEEE Trans. Ind. Electron., Vol. 45, No. 2, pp. 283-290. (1998).
- Karanam, K. C., "Modified Smith predictor design for unstable processes with step and periodic disturbances", P. I. Mech. Eng. I-J Sys., Vol. 227, No. 2, pp. 146-160. (2013).
- Kaya, I., "IMC based automatic tuning method for PID controllers in a Smith predictor configuration, Comput", Comput. Chem. Eng., Vol. 28, No. 3, pp. 281-290. (2004).

M.-H. Tsai and P.-C. Tung: Modified Smith Predictor with a Periodic Disturbance Reduction Method.

- Lee, H. S., "Implementation of adaptive feedforward cancellation algorithms for pre-embossed rigid magnetic disks", IEEE Trans. Magn., Vol. 33, No. 3, pp. 2419-2423. (1997).
- Mataušek, M. R., Micić, A. D., "On the modified Smith predictor for controlling a process with an integrator and long dead time", IEEE Trans. Automat. Contr., Vol. 44, No. 8, pp. 1603-1606. (1999).
- Muramatsu, H., Katsura, S., "An Adaptive Periodic-Disturbance Observer for Periodic-Disturbance Suppression", IEEE T. Ind. Inform., Vol. 14, No. 10, pp. 4446-4456. (2018).
- Richard, J. P., "Time-delay system: an overview of some recent advances and open problems", Automatica., Vol. 39, No. 10, pp. 1667-1694. (2003).
- Sastry, S., Bodson, M., "Adaptive Control Stability: Convergence and Robustness", Prentice-Hall, New Jersey, USA. (1994).
- Shen, P., Li, H. X., "A multiple periodic disturbance rejection control for process with long dead-time", J. Process Control, Vol. 24, No. 9, pp. 1394-1401. (2014).
- Smith, O. J., "A controller to overcome dead time", ISA Journal, Vol. 6, No. 2, pp. 28-33. (1959).
- Stojić, M. R., Matijević, M. S., and Draganović, L. S., "A robust Smith predictor modified by internal models for integrating process with dead time", IEEE Trans. Automat. Contr., Vol. 46, No. 8, pp. 1293-1298. (2001).
- Takehara, T., Kunitake, T., Hashimoto, H., and Harachima, F., "The control for the disturbance in the system with time delay", Proc. of 4<sup>th</sup> Int. Work. Adv. Mot., AMC '96-Mie, Mie, Japan. (1996).
- Tan, F., Li, H. X., "Smith predictor-based multiple periodic disturbance compensation for long deadtime processes", J. Process Control, Vol. 91, No.5, pp. 999-1010. (2018).
- Tian, Y. -C., Gao, F., "Double-controller scheme for control of processes with dominant delay", IEE P-Contr. Theor. Ap., Vol. 145, No. 5, pp. 479-484. (1998).
- Tsai, M. H., Tung, P. C., "A disturbance reduction scheme for linear small delay systems with modeling uncertainties", J. Process Control, Vol. 20, No. 6, pp. 777-786. (2010).
- Tsai, M. H., Tung, P. C., "Modified Smith predictor with a robust disturbance reduction scheme for linear systems with small time delays", Expert Syst., Vol. 29, No. 4, pp. 394-410. (2012).
- Tsai, M. H., Tung, P. C., "A robust disturbance reduction scheme for linear small delay systems with disturbances of unknown frequencies", ISA T., Vol. 51, No. 3, pp. 362-372. (2012).
- Watanabe, K., M. Ito, "A process-model control for linear systems with delay", IEEE Trans. Automat. Contr., Vol. 26, No. 6, pp. 1261-1269. (1981).

Zheng, D., Fang, J., and Ren, Z., "Modified Smith

predictor for frequency identification and disturbance rejection of single sinusoidal signal," ISA Trans., Vol. 49, No. 1, pp. 95-105. (2010).

# 修正史密斯估測器使用週期 干擾降低方法用於線性

# 時延系統

蔡銘浩 董必正 國立中央大學機械工程學系

#### 摘要

本文提出一改善之史密斯估測器及一具有降低週期 性干擾的方法,適用於短時間延遲之線性系統,使用這 種方法,不需要估測未知的干擾頻率。主要控制結構是 由Astrom改善的史密斯估測器提供。本文所提出的方法 由干擾降低控制器(DRC)與殘留干擾觀察器(RDO)組成, 其中干擾降低控制器是由逆模型和具備非負增益的積分 器組成,能夠補償在穩定或不穩定系統中之未知負載干 擾及模型之不確定性。本文之擾動降低性能是使用干擾 降低控制器結合殘留干擾觀測器來抑制不需要的殘留訊 號,包括殘留干擾訊號與殘留的不確定性訊號。模擬範 例驗證在週期性或非週期性的未知負載干擾下,使用本 文所提出的方法能對短時延遲之線性不確定系統的週期 性擾動進行有效的抑制。