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New Soliton Solutions for KdV Equations by the Simplest and the Extended Simplest Equation Methods

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Keywords：KdV, simplest equation method, linearized, soliton-sliding.

ABSTRACT

Four new single-soliton solutions for the Korteweg and de Vries (KdV) equation are developed by the simplest equation method (SEM) with the Bernoulli equation being the simplest equation. These solutions overcome the long existing problem of discontinuity when the nonlinear term coefficient approaches zero and reveal a new phenomenon, named soliton-sliding. In addition, the multi-soliton solutions for the KdV and the potential KdV equations are shown to be obtainable from the SEM by choosing the Burgers equation as the simplest equation. Compared with Hirota's direct method, the proposed method is more simple and straightforward.

INTRODUCTION

The Korteweg and de Vries (KdV) equation is a typical nonlinear partial differential equation that provides soliton solutions. The KdV equation describes shallow water waves of long wavelength and small amplitude (Wazwaz, 2002; Korteweg and de Vries., 1895). It is the simplest nonlinear dispersive equation. In addition, there are other physical systems that can also be modeled by the KdV equation, such as acoustic waves in harmonic crystals and ion waves in plasmas (Wazwaz, 2002; Jeffrey and Kakutani, 1972).

Exact solitary solutions of the KdV equation with a variable nonlinear term coefficient have been developed by the tanh-coth method (Wazwaz, 2004, 2006; Soliman, 2006), the sine-cosine method (Wazwaz, 2006) and the Exp–function method (Ebaid, 2007), which all provide single-soliton solutions. Since the KdV equation is a completely integrable equation, its multi-soliton solutions were obtained by Hirota's direct method (Wazwaz, 2002; Hirota, 2004; Hereman, and Nuseir, 1997). However, Hirota's method requires a complex algorithm to construct the multi-soliton solutions.

From the literature (Wazwaz, 2002, 2006; Ebaid, 2007), it can be found that all exact solutions of the KdV equation, including solitary and multi-soliton solutions, will approach infinity and do not satisfy the continuity condition when the nonlinear term coefficient is zero. Obviously, they can not be reduced to linear solutions.

In this paper, four new solutions of the KdV equation are derived by the simplest equation method (SEM) with the Bernoulli equation as the simplest equation (Kudryashov, 2009, 2011, 2012; Vitanov, 2010, 2011; Kudryashov and Loguinova, 2008; Mohamad, Petkovic and Biswas, 2010; Kuo and Lee, 2015). These new solutions overcome the problem of discontinuity and can be successfully reduced to linear ones, while the nonlinear term coefficient of the differential equation approaches zero. Moreover, in order to construct the multi-soliton solutions of the completely integrable equations without the complex algorithm, the SEM is extended by choosing the Burgers equation as the simplest equation. Two completely integrable equations, the KdV equation and the potential KdV equation are handled and their general multi-soliton solutions formally obtained. Unlike Hirota's method, the multi-soliton solutions are constructed directly and easily. The results confirm that both the proposed linearized and multi-soliton solutions are good. Therefore, it can be said that the SEM and the extended SEM are concise and effective for constructing solitary and multi-soliton solutions.

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THE SIMPLEST EQUATION METHOD

Assume a partial differential equation. After a transformation by the wave variable $\xi = x - ct$, a nonlinear ordinary differential equation (ODE) results,

$$P(U, U_{\xi}, U_{\xi\xi}, ...) = 0 . \tag{1}$$

SEM is a method commonly used to develop the exact solutions to a number of ordinary nonlinear differential equations. Herein, the method is applied to develop exact solutions of the KdV equation.

For a large class of equations of the type represented in (1), the exact solution can be assumed to be in the form

$$U(\xi) = \sum_{i=0}^{L} a_i [Y(\xi)]^i , \tag{2}$$

where $L > 0$ and must be an integer; a_i are parameters; and $Y(\xi)$ is a solution of a certain nonlinear ordinary differential equation with an exact solution, referred to as the simplest equation. L is determined by substituting equation (2) into equation (1) and balancing the linear term of the derivative's highest order with the highest nonlinear term in equation (1) (Kudryashov, 2009, 2011, 2012; Vitanov, 2010, 2011; Kudryashov and Loguinova, 2008; Mohamad, Petkovic and Biswas, 2010).

In order to find the linearized solution, the Bernoulli equation was chosen as the simplest equation for the simplest equation method. The Bernoulli equation is in the form

$$Y_{\xi} = aY + bY^2 , \tag{3}$$

where a and b are constants. The exact solution to the equation above is (Spiegel, 1968)

$$Y = \frac{a}{e^{-a\xi} - b} . \tag{4}$$

Obviously, solution (4) can be reduced to a linear solution when the nonlinear coefficient b in equation (3) approaches zero. That is why the Bernoulli equation was chosen as the simplest equation for finding linearized solution.

After substituting equations (2-3) into (1), and equating the coefficients of the same powers of Y to zero in the resultant equation, a system of algebraic equations involving a_i, ($i = 0,..., L$) are derived. Having determined these parameters and using equation (4), an analytically closed-form solution can be obtained.

EXACT SOLUTIONS OF THE KDV EQUATION

Existing solutions

The KdV equation in dimensionless variables can be expressed as (Wazwaz, 2007)

$$u_t + \alpha u u_x + u_{xxx} = 0 , \tag{5}$$

where α is scaled to any real number. The delicate balance between uu_x and u_{xxx} defines the formulation of solitons that consist of single humped waves. Exact solutions of equation (5) developed by the tanh-coth method, the sine-cosine method (Wazwaz, 2004, 2006), and the Exp–function method (Ebaid, 2007) can be summarized as

$$u_1 = \frac{3c}{\alpha} \sec h^2 [\frac{\sqrt{c}}{2}(x - ct)] , \quad c > 0 , \tag{6}$$

$$u_2 = \frac{-3c}{\alpha} \csc h^2 [\frac{\sqrt{c}}{2}(x - ct)] , \quad c > 0 , \tag{7}$$

$$u_3 = \frac{-c}{\alpha}\left(1 + 3\cot^2 [\frac{\sqrt{c}}{2}(x - ct)]\right), \quad c > 0 , \tag{8}$$

$$u_4 = \frac{-c}{\alpha}\left(1 + 3\tan^2 [\frac{\sqrt{c}}{2}(x - ct)]\right), \quad c > 0 , \tag{9}$$

and

$$u_5 = \frac{3c}{\alpha} \sec^2 [\frac{\sqrt{-c}}{2}(x - ct)] , \quad c < 0 , \tag{10}$$

$$u_6 = \frac{3c}{\alpha} \csc^2 [\frac{\sqrt{-c}}{2}(x - ct)] , \quad c < 0 , \tag{11}$$

$$u_7 = \frac{-c}{\alpha}\left(1 - 3\coth^2 [\frac{\sqrt{-c}}{2}(x - ct)]\right), \quad c < 0 , \tag{12}$$

$$u_8 = \frac{-c}{\alpha}\left(1 - 3\tanh^2 [\frac{\sqrt{-c}}{2}(x - ct)]\right), \quad c < 0 , \tag{13}$$

where c is the wave speed. It is noted that only solutions u_1 and u_8 are soliton solutions.

It is well-known that the KdV equation is a completely integrable equation and that its multi-soliton solutions to equation (5) can be obtained by Hirota's method (Hirota, 2004). For example, the one-soliton solution for equation (5) is constructed as

$$u_9 = \frac{12}{\alpha} \frac{k^2 e^{k(x - k^2 t)}}{(1 + e^{k(x - k^2 t)})^2} . \tag{14}$$

Of note, for solution continuity, if the nonlinear term coefficient α in nonlinear differential equations (5) approaches zero, the equation could be reduced to a linear one, and the corresponding nonlinear solutions should be reducible to linear solutions. However, from the existing solutions above, it can be observed that all solutions of the KdV equation are proportional to $1/\alpha$. When the nonlinear term coefficient, α, is reduced to zero, the solutions become singular; therefore, none will satisfy the continuity condition at $\alpha = 0$. In the following, new exact solutions of the KdV equation with linearized solutions are developed.

New Solutions

After a transformation by the wave variable ξ, the KdV equation (5) is transformed into the following nonlinear ordinary differential equation

$$-cU + \frac{\alpha}{2}U^2 + U_{\xi\xi} = 0\,. \tag{15}$$

Substituting equation (2) into equation (15) and balancing the linear term of the derivative's highest order yields $L = 2$. Therefore, the solution can be constructed as

$$U(\xi) = a_0 + a_1 Y + a_2 Y^2\,. \tag{16}$$

Substituting equations (3) and (16) into (15), and setting equal power coefficients of Y to zero, leads to a system of nonlinear relationships among the parameters of the solution and the parameters of the solved equation class.

$$Y^0 : -ca_0 + \frac{\alpha}{2}a_0^2 = 0\,, \tag{17}$$

$$Y^1 : -ca_1 + \alpha a_0 a_1 + a_1 a^2 = 0\,, \tag{18}$$

$$Y^2 : -ca_2 + \alpha a_0 a_2 + \frac{\alpha}{2}a_1^2 + 3a_1 ab + 4a_2 a^2 = 0\,, \tag{19}$$

$$Y^3 : \alpha a_1 a_2 + 2a_1 b^2 + 10 a_2 ab = 0\,, \tag{20}$$

$$Y^4 : \frac{\alpha}{2}a_2^2 + 6a_2 b^2 = 0\,. \tag{21}$$

Solving equations (17-21) yields the following four cases.

Case1.

$a = \sqrt{c}, b = \alpha, a_0 = 0, a_1 = -12\sqrt{c}, a_2 = -12\alpha$,

Case2.

$a = -\sqrt{c}, b = \alpha, a_0 = 0, a_1 = 12\sqrt{c}, a_2 = -12\alpha$,

Case3.

$a = \sqrt{c}, b = -\alpha, a_0 = 0, a_1 = 12\sqrt{c}, a_2 = -12\alpha$,

Case4.

$a = -\sqrt{c}, b = -\alpha, a_0 = 0, a_1 = -12\sqrt{c}, a_2 = -12\alpha$.

As a result, the four exact solutions are derived as

$$u_{10} = \frac{-12ce^{-\sqrt{c}(x-ct)}}{(e^{-\sqrt{c}(x-ct)} - \alpha)^2}\,, \tag{22}$$

$$u_{11} = \frac{-12ce^{\sqrt{c}(x-ct)}}{(e^{\sqrt{c}(x-ct)} - \alpha)^2}\,, \tag{23}$$

$$u_{12} = \frac{12ce^{-\sqrt{c}(x-ct)}}{(e^{-\sqrt{c}(x-ct)} + \alpha)^2}\,, \tag{24}$$

$$u_{13} = \frac{12ce^{\sqrt{c}(x-ct)}}{(e^{\sqrt{c}(x-ct)} + \alpha)^2}\,. \tag{25}$$

Clearly, by observing the forms of equations (22-25), it can be found that all can be reduced to linear solutions as $\alpha = 0$. Moreover, equation (22) will equal (23), and equation (24) will equal (25) as $\alpha = \pm 1$, all of which can be presented as soliton solutions.

It is noted that equation (22) with $\alpha = -1$, and equation (24) with $\alpha = 1$ could be equivalent to equation (6). This means, under certain conditions, that equations (22-23) and (24-25) are the same as (6). And for the continuous case, the soliton solution could be presented by equations (22-25) instead of equation (6).

It should be noted the existing solutions will approach infinity and become singular as α approaches zero, as shown in Figure 1.

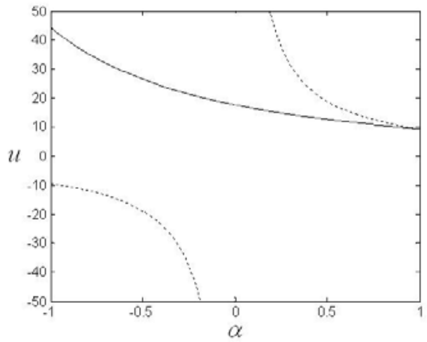

Fig. 1: Comparison between the new solution and the existing solution as the nonlinear coefficient α varies from -1 to 1. [$t = 2$, $x = 7.5$, $c = 4$; ——: equation (24); ……: equation (6).]

One can observe that the new solution (24) is a continuous function of α, while solution (6) has discontinuity at $\alpha = 0$. It is noted that the steep slope of solutions (6) and (13) remain in the same location; however, this is not the case for the new solutions.

Moreover, the location of the steep slope of the new solutions will slide and change as α is changed. As shown in Figures 2-3, the location of the steep slope of new solutions (24) and (25) slide to the right/left as α is changed from 2 to 0.2, respectively. We have named this peculiarity the soliton-sliding phenomenon.

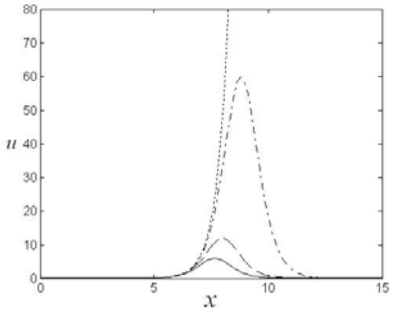

Fig. 2: Influence of the nonlinear term coefficient α on equation (24). [$t=2$, $c=4$; ——— : $\alpha=2$; – – – : $\alpha=1$; – · – : $\alpha=0.2$; ....: $\alpha=0$.]

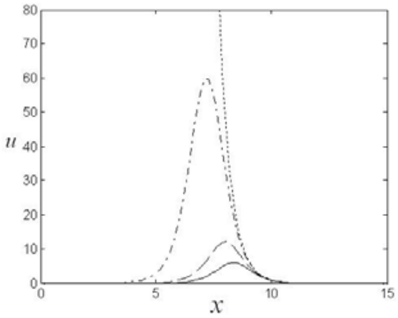

Fig. 3: Influence of the nonlinear term coefficient α on equation (25). [$t=2$, $c=4$; ——— : $\alpha=2$; – – – : $\alpha=1$; – · – : $\alpha=0.2$; ....: $\alpha=0$.]

THE EXTENDED SEM

The second aim of this paper is to illustrate a new method for constructing multi-soliton solutions, which are the main feature of the completely integrable equations.

It is well known that Hirota's method is commonly used to construct multi-soliton solutions. However, it is extremely difficult to find the Hirota's transformation for solved PDEs. Therefore, the Burgers equation was chosen as the simplest equation to construct multi-soliton solutions due it being a completely integrable equation (Wazwaz, 2007, 2007). In the following, the algorithm of the new method, called the extended SEM, is illustrated.

Consider the Burgers equation

$$u_t+\beta uu_x-u_{xx}=0\,, \tag{26}$$

where β is an arbitrary nonlinear coefficient. Using Cole-Hopf transformation the multi-soliton solution of equation (26) can be constructed as (Wazwaz, 2007)

$$u=\frac{-2}{\beta}\frac{\sum_{i=1}^{N}k_i e^{k_i x-c_i t}}{1+\sum_{i=1}^{N}e^{k_i x-c_i t}}\,, \tag{27}$$

where N is a positive integer.

The traveling wave variable is $\eta=kx-ct$, and the Burgers equation (26) is thus transformed into

$$-cY_\eta+k\beta YY_\eta-k^2Y_{\eta\eta}=0\,. \tag{28}$$

Integrating equation (28) once with respect to η, and making the integral constant zero, it becomes

$$Y_\eta=\frac{-c}{k^2}Y+\frac{\beta}{2k}Y^2\,. \tag{29}$$

It is well known that the dispersion relation of equation (29) is

$$c=-k^2\,, \tag{30}$$

therefore, equation (29) is rewritten as

$$Y_\eta=Y+\frac{\beta}{2k}Y^2\,, \tag{31}$$

and its multi-soliton solution general form is

$$Y=\frac{-2}{\beta}\frac{\sum_{i=1}^{N}k_i e^{\eta_i}}{1+\sum_{i=1}^{N}e^{\eta_i}}\,. \tag{32}$$

Equation (31) is the final form of the simplest equation which will be used to construct the multi-soliton solutions for the investigated class of nonlinear PDEs.

In the following, the extended SEM is applied to construct the multi-soliton solutions for two completely integrable equations, namely the KdV equation and the potential KdV equation.

The KdV Equation

Use the traveling wave variable $\eta=kx-ct$ to transform equation (5) into

$$-cF_\eta + \alpha kFF_\eta + k^3 F_{\eta\eta\eta} = 0\,. \tag{33}$$

Integrating equation (33) once with respect to η, and making the integral constant zero, it becomes

$$-cF + \frac{\alpha k}{2}F^2 + k^3 F_{\eta\eta} = 0\,. \tag{34}$$

Substituting equations (2) and (31) into (34) and by means of the balanced equation yields $L = 2$. Accordingly, the exact solution of equation (34) is assumed as

$$F(\xi) = a_0 + a_1 Y + a_2 Y^2\,. \tag{35}$$

Substituting equations (31) and (35) into (34), and setting equal power coefficients of Y to zero, leads to a system of nonlinear relationship among the parameters of the solution and the parameters of the solved equation class

$$Y^0 : -ca_0 + \frac{k\alpha}{2}a_0^2 = 0\,, \tag{36}$$

$$Y^1 : -ca_1 + \frac{k\alpha}{2}2a_0 a_1 + k^3 a_1 = 0\,, \tag{37}$$

$$Y^2 : -ca_2 + \frac{k\alpha}{2}(2a_0 a_2 + a_1^2) + k^3\left(\frac{3a_1\beta}{2k} + 4a_2\right) = 0\,, \tag{38}$$

$$Y^3 : \frac{k\alpha}{2}(2a_1 a_2) + k^3\left(\frac{5a_2\beta}{k} + \frac{a_1\beta^2}{2k^2}\right) = 0\,, \tag{39}$$

$$Y^4 : \frac{k\alpha}{2}a_2^2 + k^3\frac{3a_2\beta^2}{2k^2} = 0\,. \tag{40}$$

Solving equations (36-40) yields

$$\begin{aligned} a_0 &= 0, \\ a_1 &= \frac{-6k\beta}{\alpha}, \\ a_2 &= \frac{-3\beta^2}{\alpha}, \\ c &= k^3. \end{aligned} \tag{41}$$

As a result, the general form of the multi-soliton solution is derived as

$$u = \frac{12}{\alpha}\left(\frac{\sum_{i=1}^{N} k_i^2 e^{(k_i x - k_i^3 t)}}{1+\sum_{i=1}^{N} e^{(k_i x - k_i^3 t)}}\right) - \frac{12}{\alpha}\left(\frac{\sum_{i=1}^{N} k_i e^{(k_i x - k_i^3 t)}}{1+\sum_{i=1}^{N} e^{(k_i x - k_i^3 t)}}\right)^2. \tag{42}$$

For the one-soliton solution, equation (42) is exactly the same as equation (14). The corresponding two-soliton solution is constructed as

$$u = \frac{12}{\alpha}\left(\frac{k_1^2 e^{(k_1 x - k_1^3 t)} + k_2^2 e^{(k_2 x - k_2^3 t)}}{1+e^{(k_1 x - k_1^3 t)} + e^{(k_2 x - k_2^3 t)}}\right) - \frac{12}{\alpha}\left(\frac{k_1 e^{(k_1 x - k_1^3 t)} + k_2 e^{(k_2 x - k_2^3 t)}}{1++e^{(k_1 x - k_1^3 t)} + e^{(k_2 x - k_2^3 t)}}\right)^2, \tag{43}$$

as show in Figure4.

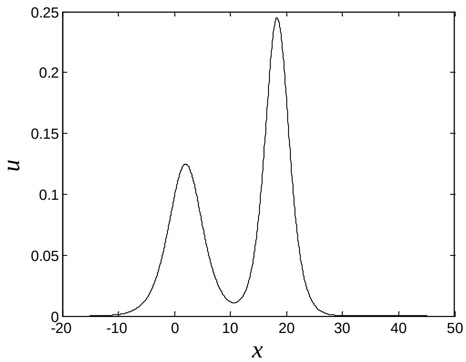

Fig. 4: The two-soliton solution for equation (43) with $k_1 = 1.2$, $k_2 = 0.5$, $t = 8$ and $\alpha = 6$.

The Potential KdV Equation

Consider the potential KdV equation (Wazwaz, 2007)

$$u_t + 3\gamma u_x^2 + u_{xxx} = 0\,, \tag{44}$$

where γ is an arbitrary nonlinear coefficient.

Use the traveling wave variable $\eta = kx - ct$ to transform equation (44) into

$$-cF_\eta + 3\gamma k^2 F_\eta^2 + k^3 F_{\eta\eta\eta} = 0\,. \tag{45}$$

Substituting equations (2) and (31) into (45) and processing as before, we have $L = 1$. Therefore, the exact solution of equation (45) is assumed as

$$F(\xi) = a_0 + a_1 Y\,. \tag{46}$$

Substituting equations (31) and (46) into (45), and setting equal power coefficients of Y to zero, leads to a system of nonlinear relationship among the parameters of the solution and the parameters of the solved equation class

$$Y^1 : -ca_1 + k^3 a_1 = 0\,, \tag{47}$$

$$Y^2 : \frac{-ca_1\beta}{2k} + 3\gamma k^2 a_1^2 + \frac{7a_1\beta k^2}{2} = 0\,, \tag{48}$$

$$Y^3 : 3ka_1^2\gamma\beta + 3ka_1\beta^2 = 0\,, \tag{49}$$

$$Y^4 : \frac{3\gamma a_1^2\beta^2}{4} + \frac{3\gamma a_1\beta^3}{4} = 0\,. \tag{50}$$

Solving equations (47-50) yields

$$a_1 = \frac{-\beta}{\gamma}, \quad c = k^3, \tag{51}$$

here a_0 is an arbitrary constant, and is set as $a_0 = 0$. As a result, the general form of the multi-soliton solution is derived as

$$u = \frac{2}{\gamma} \frac{\sum_{i=1}^{N} k_i e^{(k_i x - k_i^3 t)}}{(1 + \sum_{i=1}^{N} e^{(k_i x - k_i^3 t)})}. \tag{52}$$

For the one-soliton solution and setting $\gamma = 1$, equation (52) is the same result as in Wazwaz's book (Wazwaz, 2007), namely

$$u = \frac{2k_1 e^{(k_1 x - k_1^3 t)}}{1 + e^{(k_1 x - k_1^3 t)}}. \tag{53}$$

The corresponding two-soliton solution is constructed as

$$u = \frac{2(k_1 e^{(k_1 x - k_1^3 t)} + k_2 e^{(k_2 x - k_2^3 t)})}{1 + k_1 e^{(k_1 x - k_1^3 t)} + k_2 e^{(k_2 x - k_2^3 t)}}, \tag{54}$$

as shown in Figure 5.

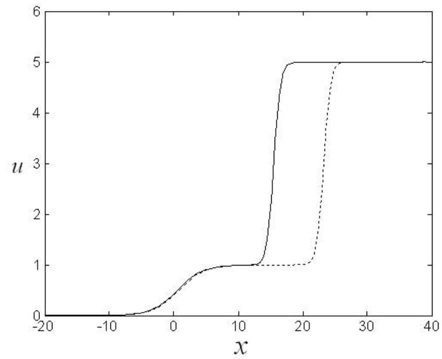

Fig. 5: The two-soliton solution in traveling kink wave for equation (54) with $k_1 = 0.5$, $k_2 = 2.5$ and $\gamma = 1$. [——: $t = 2$; ⋯⋯: $t = 3$]

Up to now, four new linearized solutions of the KdV equation have been obtained by the SEM; and two completely integrable equations have been solved by the extended SEM. Unlike Hirota's method, the multi-soliton solutions are easily obtained and the four new linearized solutions reveal a new phenomenon, soliton-sliding. Moreover, it should be mentioned that the new phenomenon can also be observed from the Burgers equation, namely kink-sliding (Kuo and Lee, 2015).

CONCLUSIONS

Based on the results presented in sections 3 and 4, two conclusions are given.

1. The SEM was employed to solve the KdV equation, and four new linearized solutions were obtained. The proposed derived solutions are solitary solutions, which can be successfully reduced to linearity, while the nonlinear term coefficient becomes zero. Under the same physical conditions, the new solutions will not become singular when α is zero. Moreover, the locations of the steep slopes in the new solutions slide, which is termed the soliton-sliding phenomenon. The reason the new solutions slide is entirely due to the influence of linearity.
2. The SEM was extended by choosing the Burgers equation as the simplest equation. Two completely integrable equations, namely the KdV and the potential KdV equations, were handled and their general multi-soliton solutions formally established. Unlike Hirota's method, the results confirm the extended SEM is concise and effective for constructing multi-soliton solutions.

Accordingly, we believe that solitary solutions and multi-soliton solutions existing for other classes of nonlinear mathematic physics models can be easily solved by the SEM and the extended SEM. Further work on these aspects is recommended.

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以 SEM 及 Extended SEM 求解 KdV 方程式的新孤子解

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摘 要

本文利用 the simplest equation method (SEM)搭配 Bernoulli 方程式求得 KdV 方程式的 4 個新精確孤波解。新解克服了懸宕已久的精確解不連續悖論，且由數值軟體 MATLAB 的模擬結果可觀察到，其對非線性項係數的變化相較於以往的孤波解而言，產生一新現象，暫稱為孤波滑移。

另一方面，利用 Burgers 方程式具有 multi-soliton solution 的特徵，將 Burgers 方程式轉換為 simplest equation，可用以求解完全可積分方程式的 multi-soliton solutions。文中以 KdV 及 potential KdV 兩個經典方程式為例，其 multi-soliton solutions 均在簡潔的數學運算下求得。

由上述的結果確認 SEM 及 extended SEM 極具效率且簡單，可直接針對各類型有關非線性波動偏微分方程式求解。