

# Optimal Contact Analysis of Meshing Pair for Single Screw Compressor

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**Keywords:** single screw compressor, contact analysis, meshing pair, induced curvature.

## ABSTRACT

In order to improve the transmission performance and lubrication condition of the meshing pair of single screw compressor, the contact analysis of single screw compressor is carried out based on tooth contact analysis. The mathematical model of meshing pair contact is constructed, and the contact line equation is obtained based on conjugate principle. The contact analysis of meshing pair is carried out. The induced curvatures of points chosen on contact line are calculated, and results show that the contact surface is close, which is benefit for improving contact strength of tooth surface, and then the optimal center distance is obtained based on calculation induced angle between contact line direction and relative speed. In addition, the effect of meshing angle and closed angle on meshing performance of single screw compressor is obtained, results can offer effective basis for manufacturing the single screw compressor with better performance.

## INTRODUCTION

Single screw compressor is a kind of rotary machine with excellent performance of the total structure. Since the 1960s, the single screw compressor has been developed, and some new types of single screw compressors have entered into industrial market. Some companies in American and Japan have manufactured single screw compressors, however the core technology of single screw compressor is not made public, which is meshing line design between gate rotor and screw rotor. The core technologies of single screw compressor are secret.

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The working process of single screw compressor mainly relies on the accurate meshing motion between gate rotor and screw rotor, which conclude the "heart" of the single screw compressor. Due to the complex geometry of the meshing pair, the design rationality of the meshing line is beneficial for processing, and they are important affecting factors of manufacturing and assembly precision, and these factors mentioned above affect the performance of single screw compressor greatly Huang et al. (2015).

In recent years some scientists have concerned the meshing pair analysis, and some good methods have been established. Tsuji I. et al. (2013) proposed the mathematical model of straight bevel gears by complementary crown gears considering manufacture on multitasking machine, and analyzed the tooth contact pattern and transmission errors of these straight bevel gears with modified tooth surfaces. Lin and Fong (2015) developed a numerical tooth contact analysis technology, simulation results showed that the calculation speed of NTCA was fast for the multiple tooth contact because the structure of the proposed numerical algorithm was fit for the parallel computing. Wang et al. (2014) put forward a tooth contact analysis (TCA) method based on high precision digital real tooth surface of spiral bevel gears, and the effectiveness of new method was verified through the comparison analysis between TCA software analysis and tooth wear test results [4].

In order to grasp the transmission performance of the meshing pair for the single screw compressor, the contact analysis of the meshing pair for single screw compressor is carried out from angle of tooth contact line. Because the center distance is a most active parameter of affecting the position of the meshing limit line, the effect of the center distance on contact line between the gate rotor and screw rotor is analyzed, in addition, the meshing angle and closed angle of meshing pair are also main affecting factors of performance for singles screw compressor. The research can provide effective basis for obtaining good meshing performance of meshing pair.

## THEORY MODEL

The shape of the gate rotor is designed as straight teeth, the radius of the gate rotor

is defined by  $r$ , the contour of the gate rotor is divided into four regions, which concludes  $AB$ ,  $BC$ ,  $CD$ ,  $AI$ , which are shown in figure 1, and the tooth surface equation of the gate rotor is constructed in coordinate  $S(O_1, X_1, Y_1)$  (Yang, 2004).

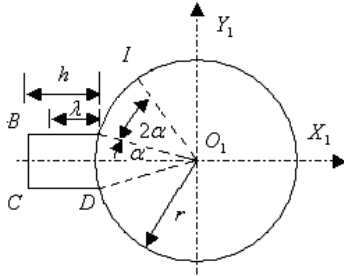


Fig. 1. Diagram of gate rotor tooth surface

Where  $AB$  and  $CD$  are the tooth side surface, which meshes with two side surfaces of screw rotor,  $\lambda$  is the design parameter of gate rotor surface, the equations of regions  $AB$  and  $CD$  are expressed as follows:

$$R_{1AB} = \begin{bmatrix} v \\ \lambda - w \\ \delta \end{bmatrix}, \quad 0 < \lambda < h \quad (1)$$

$$R_{1CD} = \begin{bmatrix} -v \\ \lambda - w \\ \delta \end{bmatrix}, \quad 0 < \lambda < h \quad (2)$$

where  $v = r \sin \alpha$ ,  $w = r \cos \alpha + h$ ,  $\alpha = \pi / N$ ,  $N = 11$ ,  $\alpha$  is the graduation angle,

$N$  is the number of gate rotor teeth,  $\delta$  is the thickness of tooth,  $h$  is the length of tooth.

$BD$  is the top side surface of gate rotor, which meshes with bottom of screw rotor during the procession of working, and the equation of region is expressed as follows:

$$R_{1BD} = \begin{bmatrix} -\lambda \\ -w \\ \delta \end{bmatrix}, \quad -v < \lambda < v \quad (3)$$

$AI$  is the transitional region between two teeth of the gate rotor, which meshes with tooth top surface of screw rotor,  $\alpha_r$  is the design parameter of the gate rotor, the equation of region  $AI$  is expressed as follows:

$$R_{1AI} = \begin{bmatrix} r \sin \alpha_r \\ -r \cos \alpha_r \\ \delta \end{bmatrix}, \quad \alpha < \alpha_r < 3\alpha \quad (4)$$

### Transition matrix of coordinate system

According to the meshing principle, the screw rotor tooth surface and gate rotor tooth surface is a pair of conjugate surface. When the screw rotates, the screw tooth surface can be formed by the enveloping of the gear rotor tooth surface. The motion relationship between the gate rotor and screw rotor in coordinate system is shown in figure 2.

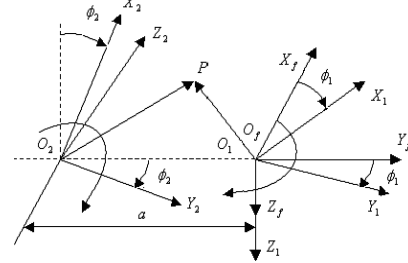


Fig. 2. Diagram of meshing pair motion system of single screw compressor

The moving coordinate systems  $S_1(X_1, Y_1, Z_1)$  and  $S_2(X_2, Y_2, Z_2)$  are fixed on the gate rotor tooth surface and screw rotor tooth surface respectively. Coordinate system  $S_1$  and  $S_2$  can rotate. The fixed coordinate system  $S_f(X_f, Y_f, Z_f)$  is fixed on the compressor body. Every coordinate system satisfies with the right-hand screw rule.  $\phi_1$  and  $\phi_2$  are the angles around the each circle center of gate rotor surface and screw rotor surface, when the coordinate  $S_1$  rotates  $\phi_1$  around  $Z_1$  axis, the system  $S_2$  rotates  $\phi_2$  around  $Z_2$  axis, the following equation is obtained (Wu et al., 2013)

$$\frac{\phi_2}{\phi_1} = \frac{\omega_b}{\omega_a} = \frac{n_b}{n_a} \quad (5)$$

where  $n_a$  and  $n_b$  are number of teeth of gate rotor and screw rotor respectively,  $\omega_a$  and  $\omega_b$  are rotational angular velocity of gate rotor and screw rotor respectively,  $a$  in figure 1 are the center distance between the gate rotor and screw rotor,  $P$  is the meshing point between the gate rotor and screw rotor.

The gate rotor tooth surface equation in coordinate system  $S_1$  can be denoted by  $R_{1i}(\lambda, \delta) = [X_{1i}(\lambda, \delta) \ Y_{1i}(\lambda, \delta) \ Z_{1i}(\lambda, \delta)]$ ,  $i = AB, BC, CD, AI$ . Based on coordinate transformation, the tooth surface equation of gate rotor in coordinate system  $S_2$  can be described, and the corresponding transformation matrix is expressed as follows:

$$M_{21} = \begin{bmatrix} \sin \phi_2 \sin \phi_1 & \sin \phi_2 \cos \phi_1 & -\cos \phi_2 & a \sin \phi_2 \\ \cos \phi_2 \sin \phi_1 & \cos \phi_2 \cos \phi_1 & \sin \phi_2 & a \cos \phi_2 \\ \cos \phi_1 & -\sin \phi_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)$$

Based on meshing equation  $N_f^1 V_f^{12} = 0$ ,  $N_f^1$  is the normal line equation of gate rotor tooth surface,  $V_f^{12}$  is the relative speed of meshing point between the gate rotor and screw rotor, then the meshing motion equation between screw rotor and gate rotor of single screw compressor is expressed as follows:

$$\begin{aligned} & [x_{1i}(x_{1i\delta} y_{1i\delta} - x_{1i\delta} y_{1i\delta}) + z_{1i}(z_{1i\delta} y_{1i\delta} - y_{1i\delta} z_{1i\delta})] \sin \phi_1 \\ & + [z_{1i}(z_{1i\delta} x_{1i\delta} - x_{1i\delta} z_{1i\delta}) + y_{1i}(y_{1i\delta} x_{1i\delta} - x_{1i\delta} y_{1i\delta})] \cos \phi_1 \\ & + [x_{1i}(x_{1i\delta} z_{1i\delta} - x_{1i\delta} z_{1i\delta}) + y_{1i}(y_{1i\delta} z_{1i\delta} - y_{1i\delta} z_{1i\delta})] \frac{\partial \phi_1}{\partial \phi_2} \\ & - a(x_{1i\delta} y_{1i\delta} - y_{1i\delta} x_{1i\delta}) = 0 \end{aligned} \quad (7)$$

The screw groove surface equation meshing with region AB is expressed as follows:

$$\begin{aligned} x_{2AB} &= \lambda \sin \phi_2 \cos(\alpha_r - \phi_1) + v \sin \phi_2 \sin \phi_1 - w \sin \phi_2 \cos \phi_1 + a \sin \phi_2 \\ y_{2AB} &= \lambda \cos \phi_2 \cos(\alpha_r - \phi_1) + v \cos \phi_2 \sin \phi_1 - w \cos \phi_2 \cos \phi_1 + a \cos \phi_2 \\ z_{2AB} &= \lambda \sin(\alpha_r - \phi_1) + v \cos \phi_1 + w \cos \phi_1 \end{aligned} \quad (8)$$

$$\begin{aligned} \dot{x}_{2CD} &= \lambda \sin \phi_2 \cos(-\alpha_r + \phi_1) - v \sin \phi_2 \sin \phi_1 + w \sin \phi_2 \cos \phi_1 + a \sin \phi_2 \\ \dot{y}_{2CD} &= \lambda \cos \phi_2 \cos(-\alpha_r + \phi_1) - v \cos \phi_2 \sin \phi_1 + w \cos \phi_2 \cos \phi_1 + a \cos \phi_2 \\ \dot{z}_{2CD} &= -\lambda \sin(-\alpha_r + \phi_1) - v \cos \phi_1 - w \cos \phi_1 \end{aligned} \quad (9)$$

The contact line equation of screw rotor is expressed as follows (Li et al., 2013):

$$\begin{cases} R_{2i}(\lambda, \delta) = 0 \\ N_f^1 V_f^{12} = 0 \end{cases} \quad (10)$$

The contact line equation of gate rotor is expressed as follows:

$$\begin{cases} R_{1i}(\lambda, \delta) = 0 \\ N_f^1 V_f^{12} = 0 \end{cases} \quad (11)$$

During the procession of meshing motion between the screw rotor and gate rotor, the normal positive direction of induced curvature along the contact line is set as the direction of the curvature of the screw body, and the calculating formulation is expressed as follows (Wang et al., 2014):

$$k^{(12)} = \frac{\left\{ \frac{1}{D^2} [\Phi_v(Fr_v^{(1)} - Gr_w^{(1)}) - \Phi_w(Er_v^{(1)} - Fr_w^{(1)})] \right\}^2}{\Psi} \quad (12)$$

where  $r_v^{(1)}$  and  $r_w^{(1)}$  are derivatives of radius vector of meshing point is in the coordinate system  $S_1$  to  $v$  and  $w$  respectively,  $\Phi_v$  and  $\Phi_w$  are the derivatives of meshing function to  $v$  and  $w$  respectively,  $\Psi$  is the undercutting limit function of screw and the star wheel gear transmission,  $E$ ,  $F$ ,  $G$  are first basic volume of surface,

$$E = (r_v^{(1)})^2, \quad F = r_v^{(1)} \cdot r_w^{(1)}, \quad G = (r_w^{(1)})^2, \quad D^2 = EG - F^2 \neq 0.$$

## ANALYSIS OF MESHING PAIR OG SINGLE SCREW COMPRESSOR

The main parameters of meshing pair for single screw compressor are listed as follows: the number of screw head for the single screw compressor is six, the number of gate rotor teeth is 14, the diameters of screw rotor and gate rotor are 180mm and 185mm respectively, the thickness of gate rotor is 4.5mm, the tooth width of gate rotor is 35mm, the center distance is 124mm, and the meshing status of screw rotor and gate rotor is good. The contacting line distribution of meshing pair for single screw compressor is shown in figure 3. X-Z plane of contacting contour of meshing pair is shown in figure 4.

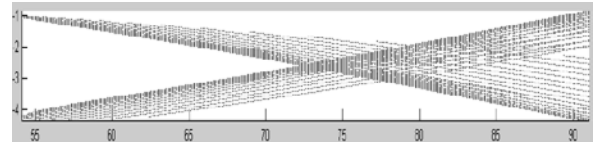


Fig. 3. Diagram of contact line distribution of meshing pair

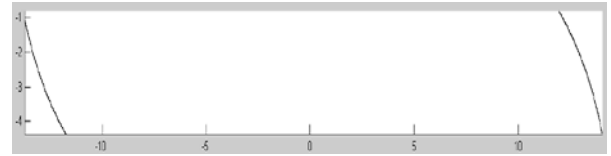


Fig. 4. Diagram of X-Z plane of contacting contour of meshing pair

## Confirmation of Center Distance

The center distance  $a$  is decided by the depth of the tooth of gate rotor engaging into the screw groove, and the depth is also relates to meshing angle and screw groove volume. The distance is not big is better, because the width of screw groove will decrease with increasing of meshing depth. Some scientists refers that when diameter of the screw rotor is equal to that of gate rotor, and the optimal center distance is 0.8 times the diameter of the screw rotor, and meshing effect is best, the meshing pair has good bearing ability and heat dissipation condition. When the diameter of gate rotor is larger than that of the screw rotor, the optimal center distance of the meshing pair is 1-1.1 times the diameter of the screw rotor. According to the size of meshing pair in this research, the center distance of meshing ranges from 180mm to 198mm, Therefore the Study when the center distance of 185mm and 190mm, star front tooth surface contact angle between the line direction and the direction of relative velocity, the star wheel

tooth surface contact angle between the line direction and the direction of relative velocity and the star wheel top angle between the contact line direction and the direction of the relative velocity. When the center distance of meshing pair is 185mm and 190mm, the induced angle  $\beta_1$  between the contact line direction and the direction of relative velocity for the former tooth surface of gate rotor, the induced angle  $\beta_2$  between the contact line direction and the direction of relative velocity for the back tooth surface of gate rotor and the induced angle  $\beta_3$  between the contact line direction and the direction of relative velocity for the top tooth surface of gate rotor are calculated.

Table 1 and table 2 show that angle between contact line and relative velocity for side surface of gate rotor when the gate rotates from  $0^\circ$  to  $180^\circ$ , the marks (1)-(5) of points chosen from tooth root to tooth top of contact line are corresponding to  $\beta_1$  and  $\beta_2$ , the marks (1)-(5) of points chosen from contact line along tooth thickness direction are corresponding to  $\beta_3$ .

**Table 1.** Font Induced angle between the contact line direction and relative velocity when the center distance is 185mm

| Order of line | Rotating angle $\phi_1 / (^\circ)$ | Induced angle/ $(^\circ)$ | Point number |       |       |       |       |
|---------------|------------------------------------|---------------------------|--------------|-------|-------|-------|-------|
|               |                                    |                           | (1)          | (2)   | (3)   | (4)   | (5)   |
| 1             | 0                                  | $\beta_1$                 | 69.84        | 69.04 | 66.42 | —     | —     |
|               |                                    | $\beta_2$                 | 77.43        | 69.45 | 69.42 | 68.43 | —     |
|               |                                    | $\beta_3$                 | 66.42        | 64.52 | 69.43 | 64.35 | —     |
| 2             | 55                                 | $\beta_1$                 | 68.41        | 69.36 | 72.55 | —     | —     |
|               |                                    | $\beta_2$                 | 66.54        | 77.49 | 77.94 | 77.32 | —     |
|               |                                    | $\beta_3$                 | 76.95        | 77.39 | 72.69 | —     | —     |
| 3             | 95                                 | $\beta_1$                 | 65.43        | 76.43 | 79.41 | —     | —     |
|               |                                    | $\beta_2$                 | 77.54        | 72.58 | 70.54 | 72.53 | 70.75 |
|               |                                    | $\beta_3$                 | 73.58        | 70.32 | 73.49 | 74.52 | —     |
| 4             | 145                                | $\beta_1$                 | 69.53        | 72.15 | 70.26 | 68.38 | —     |
|               |                                    | $\beta_2$                 | 73.46        | 73.55 | 74.83 | —     | —     |
|               |                                    | $\beta_3$                 | 71.85        | 74.03 | —     | —     | —     |
| 5             | 180                                | $\beta_1$                 | 72.36        | 71.84 | 70.34 | —     | —     |
|               |                                    | $\beta_2$                 | 69.43        | 70.46 | 76.42 | 70.42 | —     |
|               |                                    | $\beta_3$                 | 70.41        | 69.43 | 69.89 | —     | —     |

**Table 2.** Font Induced angle between the contact line direction and relative velocity when the center distance is 190mm

| Order of line | Rotating angle $\phi_1 / (^\circ)$ | Induced angle/ $(^\circ)$ | Point number |       |        |       |       |
|---------------|------------------------------------|---------------------------|--------------|-------|--------|-------|-------|
|               |                                    |                           | (1)          | (2)   | (3)    | (4)   | (5)   |
| 1             | 0                                  | $\beta_1$                 | 88.32        | 87.93 | 86.42  | —     | —     |
|               |                                    | $\beta_2$                 | 89.62        | 88.12 | 87.53  | 86.45 | —     |
|               |                                    | $\beta_3$                 | 88.36        | 87.10 | 86.43  | 88.05 | —     |
| 2             | 45                                 | $\beta_1$                 | 87.32        | 89.46 | 87.43  | —     | —     |
|               |                                    | $\beta_2$                 | 87.54        | 88.44 | 88.384 | 88.21 | —     |
|               |                                    | $\beta_3$                 | 87.59        | 88.49 | 87.33  | —     | —     |
| 3             | 90                                 | $\beta_1$                 | 87.44        | 88.53 | 89.03  | —     | —     |
|               |                                    | $\beta_2$                 | 86.64        | 86.73 | 87.65  | 88.28 | 84.32 |
|               |                                    | $\beta_3$                 | 88.73        | 89.54 | 88.48  | 87.28 | —     |
| 4             | 135                                | $\beta_1$                 | 88.65        | 89.06 | 87.04  | 87.93 | —     |
|               |                                    | $\beta_2$                 | 88.95        | 87.93 | 88.52  | —     | —     |
|               |                                    | $\beta_3$                 | 88.13        | 89.27 | —      | —     | —     |
| 5             | 180                                | $\beta_1$                 | 88.58        | 87.83 | 89.06  | —     | —     |
|               |                                    | $\beta_2$                 | 89.05        | 86.96 | 89.43  | 87.75 | —     |
|               |                                    | $\beta_3$                 | 88.93        | 88.49 | 88.36  | —     | —     |

## EFFECT OF MESHING ANGLE ON MESHING PERFORMANCE OF MESHINGPAIR

The meshing angle is calculated by the following expression:

$$\alpha_1 = 6\delta + 2.5\varphi \quad (13)$$

where  $\delta$  denotes the half of tooth width of gate rotor,  $\varphi$  denotes the angle of screw rotor slot.

The meshing angle  $\alpha_1$  ranges from  $80^\circ$  to  $95^\circ$ , the meshing angle is  $80^\circ$ ,  $90^\circ$ , and  $95^\circ$  respectively in this research, the meshing performance of transient contacting line is calculated when the rotation angle of gate rotor changes from  $0^\circ$  to  $180^\circ$ , and the corresponding calculation results are shown in table 3 to table 5.

**Table 3.** Meshing performance parameters of contacting line when the meshing angle is  $80^\circ$

| Order of line | Rotating angle           | Point number                              |           |       |       |       |       |       |       |  |
|---------------|--------------------------|-------------------------------------------|-----------|-------|-------|-------|-------|-------|-------|--|
|               | $\phi_1$<br>/ $^{\circ}$ | Meshing performance                       |           |       | (1)   | (2)   | (3)   | (4)   | (5)   |  |
|               |                          |                                           |           |       |       |       |       |       |       |  |
| 1             | 0                        | Induced curvature/ $10^{-4}\text{m}^{-1}$ | $k_1$     | 10.34 | 9.86  | 9.83  | —     | —     |       |  |
|               |                          |                                           | $k_2$     | —     | 11.56 | 11.32 | —     | 10.45 | 9.60  |  |
|               |                          |                                           | $k_3$     | —     | —     | —     | —     | 11.29 | 10.48 |  |
|               |                          | Included angle/ $^{\circ}$                | $\beta_1$ | 70.45 | 69.98 | 68.54 | —     | —     | —     |  |
|               |                          |                                           | $\beta_2$ | 70.53 | 70.25 | 69.97 | 68.55 | —     | —     |  |
|               |                          |                                           | $\beta_3$ | 71.45 | 71.03 | 70.92 | 70.35 | —     | —     |  |
| 2             | 45                       | Induced curvature/ $10^{-4}\text{m}^{-1}$ | $k_1$     | 9.64  | 9.53  | 9.24  | —     | —     |       |  |
|               |                          |                                           | $k_2$     | —     | —     | 10.46 | —     | 10.03 | 9.46  |  |
|               |                          |                                           | $k_3$     | —     | —     | —     | —     | 10.68 | 10.37 |  |
|               |                          | Included angle/ $^{\circ}$                | $\beta_1$ | 72.04 | 71.84 | 70.74 | —     | —     | —     |  |
|               |                          |                                           | $\beta_2$ | 74.36 | 74.03 | 73.56 | 73.12 | —     | —     |  |
|               |                          |                                           | $\beta_3$ | 76.84 | 76.25 | 75.03 | —     | —     | —     |  |
| 3             | 90                       | Induced curvature/ $10^{-4}\text{m}^{-1}$ | $k_1$     | 8.84  | 8.16  | 8.03  | —     | —     |       |  |
|               |                          |                                           | $k_2$     | —     | —     | —     | 9.53  | —     | 8.48  |  |
|               |                          |                                           | $k_3$     | —     | —     | 8.53  | 7.46  | —     | 6.98  |  |
|               |                          | Included angle/ $^{\circ}$                | $\beta_1$ | 73.85 | 72.46 | 71.35 | —     | —     | —     |  |
|               |                          |                                           | $\beta_2$ | 75.64 | 74.48 | 74.13 | 73.74 | —     | 88.32 |  |
|               |                          |                                           | $\beta_3$ | 77.48 | 76.36 | 75.26 | 74.48 | —     | —     |  |
| 4             | 135                      | Induced curvature/ $10^{-4}\text{m}^{-1}$ | $k_1$     | —     | 7.39  | 7.14  | 7.34  | —     | 7.12  |  |
|               |                          |                                           | $k_2$     | —     | 8.23  | 8.19  | 7.95  | —     | —     |  |
|               |                          |                                           | $k_3$     | 7.25  | 7.06  | —     | —     | —     | —     |  |
|               |                          | Included angle/ $^{\circ}$                | $\beta_1$ | 74.65 | 73.83 | —     | —     | —     | —     |  |
|               |                          |                                           | $\beta_2$ | 75.72 | 74.68 | —     | —     | —     | —     |  |
|               |                          |                                           | $\beta_3$ | 76.56 | 75.84 | 74.13 | —     | —     | —     |  |
| 5             | 180                      | Induced curvature/ $10^{-4}\text{m}^{-1}$ | $k_1$     | —     | —     | 6.47  | 6.26  | —     | 6.12  |  |
|               |                          |                                           | $k_2$     | 7.2   | 7.10  | 6.92  | —     | —     | —     |  |
|               |                          |                                           | $k_3$     | 6.63  | 6.13  | —     | —     | —     | —     |  |
|               |                          | Included angle/ $^{\circ}$                | $\beta_1$ | 75.64 | 74.83 | 73.69 | —     | —     | —     |  |
|               |                          |                                           | $\beta_2$ | 76.53 | 75.72 | 74.38 | 73.64 | —     | —     |  |
|               |                          |                                           | $\beta_3$ | 77.84 | 76.23 | 75.52 | —     | —     | —     |  |

**Table 4.** Meshing performance parameters of contacting line when the meshing angle is  $90^\circ$

| Order of line | Rotating angle    | Point number                                            |                |       |       |       |       |       |
|---------------|-------------------|---------------------------------------------------------|----------------|-------|-------|-------|-------|-------|
|               | $\phi_1$<br>/ (°) | Meshing performance                                     | (1)            | (2)   | (3)   | (4)   | (5)   |       |
|               |                   |                                                         |                |       |       |       |       |       |
| 1             | 0                 | Induced curvature/<br>10 <sup>-4</sup> mm <sup>-1</sup> | k <sub>1</sub> | 9.95  | 9.37  | 9.04  | —     | —     |
|               |                   |                                                         | k <sub>2</sub> | —     | 10.26 | 10.04 | 9.73  | 9.17  |
|               |                   |                                                         | k <sub>3</sub> | —     | —     | —     | 10.01 | 9.85  |
|               |                   | Included angle <sup>c</sup>                             | β <sub>1</sub> | 88.03 | 87.79 | 87.25 | —     | —     |
|               |                   |                                                         | β <sub>2</sub> | 88.32 | 88.16 | 87.95 | 87.21 | —     |
|               |                   |                                                         | β <sub>3</sub> | 88.93 | 88.57 | 88.07 | 87.82 | —     |
| 2             | 45                | Induced curvature/<br>10 <sup>-4</sup> mm <sup>-1</sup> | k <sub>1</sub> | 8.52  | 8.27  | 8.01  | —     | —     |
|               |                   |                                                         | k <sub>2</sub> | —     | —     | 9.65  | 9.14  | 8.97  |
|               |                   |                                                         | k <sub>3</sub> | —     | —     | —     | 9.52  | 9.22  |
|               |                   | Included angle <sup>c</sup>                             | β <sub>1</sub> | 88.36 | 87.47 | 87.08 | —     | —     |
|               |                   |                                                         | β <sub>2</sub> | 88.63 | 88.32 | 87.95 | 87.53 | —     |
|               |                   |                                                         | β <sub>3</sub> | 89.05 | 88.72 | 88.20 | —     | —     |
| 3             | 90                | Induced curvature/<br>10 <sup>-4</sup> mm <sup>-1</sup> | k <sub>1</sub> | 7.72  | 7.38  | 6.82  | —     | —     |
|               |                   |                                                         | k <sub>2</sub> | —     | —     | —     | 8.77  | 8.14  |
|               |                   |                                                         | k <sub>3</sub> | —     | —     | 7.66  | 7.03  | 6.84  |
|               |                   | Included angle <sup>c</sup>                             | β <sub>1</sub> | 88.83 | 88.29 | 87.84 | —     | —     |
|               |                   |                                                         | β <sub>2</sub> | 88.96 | 88.07 | 87.89 | 87.51 | 87.21 |
|               |                   |                                                         | β <sub>3</sub> | 89.17 | 88.22 | 88.02 | 87.82 | —     |
| 4             | 135               | Induced curvature/<br>10 <sup>-4</sup> mm <sup>-1</sup> | k <sub>1</sub> | —     | 6.83  | 6.59  | 6.02  | 5.94  |
|               |                   |                                                         | k <sub>2</sub> | —     | 7.69  | 7.22  | 6.86  | —     |
|               |                   |                                                         | k <sub>3</sub> | 6.72  | 6.28  | —     | —     | —     |
|               |                   | Included angle <sup>c</sup>                             | β <sub>1</sub> | 88.03 | 87.44 | —     | —     | —     |
|               |                   |                                                         | β <sub>2</sub> | 88.43 | 87.74 | —     | —     | —     |
|               |                   |                                                         |                |       |       |       |       |       |

|   |     |                                               |           |       |       |       |       |      |
|---|-----|-----------------------------------------------|-----------|-------|-------|-------|-------|------|
| 5 | 180 | Induced curvature/<br>$10^{-6}\text{mm}^{-1}$ | $\beta_1$ | 89.34 | 88.46 | 88.06 | —     | —    |
|   |     |                                               | $k_1$     | 6.43  | 6.01  | 5.88  | 5.39  | 5.04 |
|   |     |                                               | $k_2$     | 5.47  | 5.21  | —     | —     | —    |
|   |     | Included angle $^\circ$                       | $\beta_1$ | 88.49 | 87.73 | 87.36 | —     | —    |
|   |     |                                               | $\beta_2$ | 88.78 | 87.96 | 87.69 | 87.72 | —    |
|   |     |                                               | $\beta_3$ | 89.45 | 88.65 | 88.21 | —     | —    |
|   |     | Induced curvature/<br>$10^{-6}\text{mm}^{-1}$ | $k_1$     | —     | —     | —     | —     | —    |
|   |     |                                               | $k_2$     | —     | —     | —     | —     | —    |
|   |     |                                               | $k_3$     | —     | —     | —     | —     | —    |

**Table 5.** Meshing performance parameters of contacting line when the meshing angle is  $95^\circ$

| Order of line | Rotating angle<br>$\phi_1 / (^{\circ})$ | Meshing performance                           | Point number |       |       |       |       |      |
|---------------|-----------------------------------------|-----------------------------------------------|--------------|-------|-------|-------|-------|------|
|               |                                         |                                               | (1)          | (2)   | (3)   | (4)   | (5)   |      |
| 1             | 0                                       | Induced curvature/<br>$10^{-4}\text{mm}^{-1}$ | $k_1$        | 10.14 | 9.94  | 9.52  | —     | —    |
|               |                                         | $k_2$                                         | —            | 10.76 | 10.62 | 9.96  | 9.47  |      |
|               |                                         | $k_3$                                         | —            | —     | —     | 10.47 | 10.03 |      |
|               |                                         | Included angle $^{\circ}$                     | $\beta_1$    | 82.02 | 81.67 | 81.05 | —     | —    |
|               |                                         | $\beta_2$                                     | 82.48        | 82.03 | 81.87 | 81.34 | —     |      |
|               |                                         | $\beta_3$                                     | 82.91        | 82.62 | 82.11 | 81.78 | —     |      |
| 2             | 45                                      | Induced curvature/<br>$10^{-4}\text{mm}^{-1}$ | $k_1$        | 8.95  | 8.62  | 8.26  | —     | —    |
|               |                                         | $k_2$                                         | —            | —     | 10.04 | 9.67  | 9.37  |      |
|               |                                         | $k_3$                                         | —            | —     | —     | 9.83  | 9.67  |      |
|               |                                         | Included angle $^{\circ}$                     | $\beta_1$    | 82.17 | 81.88 | 81.29 | —     | —    |
|               |                                         | $\beta_2$                                     | 82.77        | 82.26 | 81.74 | 81.08 | —     |      |
|               |                                         | $\beta_3$                                     | 83.39        | 82.84 | 82.11 | —     | —     |      |
| 3             | 90                                      | Induced curvature/<br>$10^{-4}\text{mm}^{-1}$ | $k_1$        | 8.15  | 8.02  | 7.94  | —     | —    |
|               |                                         | $k_2$                                         | —            | —     | —     | 8.99  | 8.53  |      |
|               |                                         | $k_3$                                         | —            | —     | 8.15  | 7.65  | 7.21  |      |
|               |                                         | Included angle $^{\circ}$                     | $\beta_1$    | 81.88 | 81.27 | 80.92 | —     | —    |
|               |                                         | $\beta_2$                                     | 82.91        | 82.31 | 81.80 | 80.61 | 80.02 |      |
|               |                                         | $\beta_3$                                     | 83.89        | 83.13 | 82.74 | 81.55 | —     |      |
| 4             | 135                                     | Induced curvature/<br>$10^{-4}\text{mm}^{-1}$ | $k_1$        | —     | 7.24  | 7.01  | 7.73  | 6.37 |
|               |                                         | $k_2$                                         | —            | 8.28  | 7.68  | 7.15  | —     |      |
|               |                                         | $k_3$                                         | 7.27         | 6.94  | —     | —     | —     |      |
|               |                                         | Included angle $^{\circ}$                     | $\beta_1$    | 82.55 | 81.92 | —     | —     | —    |
|               |                                         | $\beta_2$                                     | 83.17        | 82.42 | —     | —     | —     |      |
|               |                                         | $\beta_3$                                     | 84.09        | 83.66 | 82.45 | —     | —     |      |
| 5             | 180                                     | Induced curvature/<br>$10^{-4}\text{mm}^{-1}$ | $k_1$        | —     | —     | 6.27  | 6.03  | 5.73 |
|               |                                         | $k_2$                                         | 7.1          | 6.84  | 6.32  | —     | —     |      |
|               |                                         | $k_3$                                         | 6.31         | 6.04  | —     | —     | —     |      |
|               |                                         | Included angle $^{\circ}$                     | $\beta_1$    | 82.86 | 81.82 | 80.52 | —     | —    |
|               |                                         | $\beta_2$                                     | 83.53        | 82.71 | 82.08 | 81.85 | —     |      |
|               |                                         | $\beta_3$                                     | 84.37        | 83.72 | 82.69 | —     | —     |      |

When the meshing angle changes from  $80^\circ$  to  $95^\circ$ , the induced curvature decreases first and then increases. When the meshing angle is equal to  $90^\circ$  induced curvature is smallest, therefore the meshing pair has highest contact strength. In addition, when the meshing pair changes from  $80^\circ$  to  $95^\circ$ , the induced angle between contact line direction and relative speed increases first and then decreases. When the meshing angle is equal to  $90^\circ$ , the induced angle between contact line direction and relative speed is closest to  $90^\circ$ , therefore the lubrication effect of meshing pair is best, then the optimal meshing angle is  $90^\circ$ .

### EFFECT OF CLOSED ANGLE ON MESHING PERFORMANCE OF MESHING OF MESHING PAIR

The closed angle of the single screw compressor is the included angle between the tooth of the gate rotor and the center line, which can affect the performance of the single screw compressor greatly, the calculation of the closed angle can be carried out through two methods. When the chamfer in the air inlet end of the single screw compressor satisfies the condition  $\beta = \alpha' - \delta$ , the closed angle can be calculated by the following expression:

$$\alpha'' = \alpha' - \delta \quad (14)$$

When the chamfer in the air inlet end of the single screw compressor satisfies the condition  $\beta < \alpha' - \delta$ , the closed angle can be calculated based on expression (14). When the chamfer in the air inlet end of the single screw compressor satisfies the condition  $\beta > \alpha' - \delta$ , the closed angle can be calculated by the expression:

$$\alpha'' = \arctg \frac{l' + tg\beta(a - \sqrt{r_2^2 - l'^2})}{a} - \arcsin \frac{b}{2\sqrt{a^2 + [l' + tg\beta(a - \sqrt{r_2^2 - l'^2})]^2}} \quad (15)$$

Then the effect of closed angle on the meshing performance of the single screw compressor is analyzed based on simulation analysis, the closed angle is taken as  $32^\circ$ ,  $36^\circ$  and  $40^\circ$  respectively, the calculation results of meshing performance of the meshing pair are listed in figures 5 and 6.

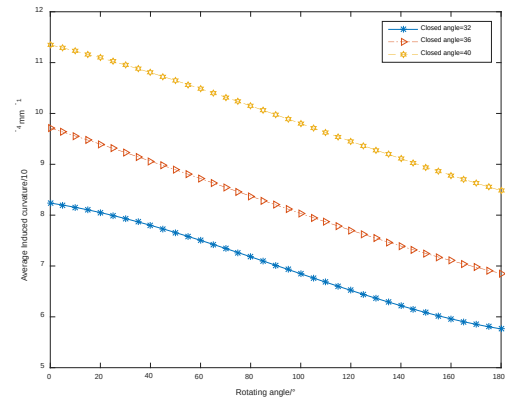


Fig.5. Average induced curvature changing curves with different closed angle

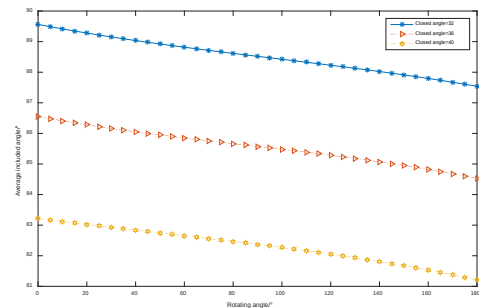


Fig.6. Average induced angle changing curves with different closed angle

As seen from fig.5 and fig.6, the average induced curvature of tooth contact line at closed

angle of  $32^\circ$  is least, and the average induced curvature of tooth contact line at closed angle of  $40^\circ$  is biggest, and average induced curvature of tooth contact line at closed angle of  $36^\circ$  between the two above cases. The induced angle of tooth contact line is negative to closed angle. When the closed angle is  $40^\circ$ , the average induced angle is least. Therefore the average induced angle when the closed angle is  $32^\circ$  is biggest and is closet to  $90^\circ$ , and the lubrication condition is best when the closed angle is  $32^\circ$ . Therefore the optimal closed angle should be confirmed through considering the contact strength and the lubrication condition comprehensively.

A tank vehicle may be regarded as a control system upon which various inputs are imposed. During a turning maneuver, the multiple steering angle induced by the driver can be considered as an input to the system and the motion variables of the tank vehicle, such as lateral acceleration and curvature, may be regarded as outputs.

## CONCLUSIONS

The tooth surface equations of meshing pair for single screw compressor are constructed, and then the contact line equation is obtained through meshing equations. The induced curvature of several points on contact line are calculated when the rotating angle of gate rotor is different, and the contact line distributions of meshing pair are obtained, the meshing performance can be improved, the tooth surface strength and lubrication condition can be also improved. And the optimal center distance of meshing pair is obtained through calculating the induced angle between contact line direction and relative velocity. The effect of meshing angle and closed angle on the induced curvature of tooth contact line and induced angle are obtained based on simulation results. The other parameters of single screw compressor can be optimized based on the same method.

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## 單螺桿壓縮機嚙合副的最佳接觸分析

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## 摘要

為了改善單螺桿壓縮機嚙合副的傳動性能和潤滑狀況，在齒面接觸分析的基礎上，對單螺桿壓縮機進行了接觸分析。建立了嚙合副接觸的數學模型，利用共軛原理得到了接觸線方程。對嚙合副進行了接觸分析。計算了接觸線上各點的誘導曲率，結果表明，接觸面較近，有利於提高齒面接觸強度，並通過計算接觸線方向與相對速度的誘導角，得到了最佳中心距。此外，還得到了嚙合角和閉合角對單螺桿壓縮機嚙合性能的影響，為製造性能更好的單螺桿壓縮機提供了有效的依據。