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# Optimal Process Parameter Selection of Underwater Nd:YAG Laser Micro-channeling on PMMA by Firefly Algorithm and Flower Pollination Algorithm

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**Keywords**： laser micro-channeling, metaheuristic algorithm, firefly algorithm, flower pollination algorithm, optimization

## ABSTRACT

In present research, firefly algorithm and flower pollination algorithm, two novel bio-inspired metaheuristic algorithms, are used to select the optimal parametric combinations for underwater laser micro-channeling process to achieve desired objectives. Firefly algorithm is inspired by the social flashing behavior of fireflies, whereas, the flower pollination algorithm is inspired by the pollination behavior of flowers. Single objective and multi objective optimizations are carried out using these algorithms, in which the objective functions are developed using response surface method. The solution of the optimization problems show that the algorithms are capable to find the feasible optimal parametric combination with high degree of accuracy. Both the algorisms are compared for their accuracy, repeatability, convergence rate and computational time. These algorithms are also found to be capable of predicting accurate trends of the parametric effects.

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## INTRODUCTION

Laser micro-machining has been proved as an effective and low cost fabrication process for micro-fluidic applications. Micro-channels are an integral part of many such micro-fluidic devices. Fabrication of micro-channels on different substrate material is a cumbersome process and may involve several steps of processing including the requirement of clean room facilities and skilled labor. However, laser micro-machining process can be utilized for micro-channel fabrication in one step without the need for any post processing step (Prakash et al., 2013). Laser micro-machining is a contactless, energized beam based material removal process in which material is removed by the process of heating, melting and vaporization. Underwater laser processing has been successfully applied to fabricate micro-channels on different substrate materials (Prakash et al., 2013; Tangwarodomnukun and Chen, 2015). In underwater processing, debris is carried away by thermal convection and bubble movement for lasers with short pulse duration (Prakash and Kumar, 2014). Underwater processing also minimizes the heat affected zone due to higher heat transfer rate resulting in cleaner and smoother microchannel edge. The underwater laser processing was first utilized in 1975 to study material ablation during emission spectroscopy (Ageev, 1975). Underwater laser processing is a novel technique to produce clean, clog free micro-features on materials by utilizing local cooling effect as well as reducing the re-deposition of ablated material on the surface. The material ablation phenomenon underwater is entirely different from open air condition. During underwater ablation, the optical breakdown of molecules and limited expansion of plasma are taking place (Wang, 2006). Due to limited expansion of laser formed plasma, the recoil pressure, generated due to plasma shock waves, increases manifold (Chen et al., 2004; Li et al., 2005). This form of ablation increases the part of cold ablation in contrast to only thermal ablation in open air condition. This also results in increase of material ablation rate in underwater condition. The re-depository materials are generally lighter than water and do not get re-deposited on the work-piece surface, and instead float in the water. Underwater laser processing has also been proved to be very beneficial in through cutting. The quality of the micro-channels is controlled by the process parameters like lamp current, pulse frequency, pulse width, cutting speed, etc (Prakash et al., 2014). These process parameters are required to be optimized for producing the micro-channels of desired quality.

Firefly algorithm (FA) and flower pollination algorithm (FPA) are two newly developed algorithms used for searching global optima within a design space. Firefly algorithm is a bio-inspired swarm-intelligence based algorithm, whereas, flower pollination algorithm is a bio-inspired algorithm but not a swarm-intelligence based algorithm. Firefly algorithm depends upon social flashing behavior of fireflies, whereas, the flower pollination algorithm is inspired by pollination behavior of flowers.

The usual advantage of firefly algorithm is that FA can automatically subdivide its population into subgroups, due to the fact that local attraction is stronger than long-distance attraction. As a result, FA can deal with highly non-linear, multi-modal optimization problems efficiently. Pal et al. (2012) made a comparative study of firefly algorithm and particle swarm optimization for noisy non-linear optimization problems. The study obtained that firefly algorithm can outperform particle swarm optimization for higher level of noises. Galvez and Iglesias (2013) applied firefly algorithm for polynomial bezier surface parameterization. Hashmi et al. (2013) employed firefly algorithm for unconstrained optimization to verify six unimodal engineering optimization problems and gave a detailed formulation and explanation of the algorithm. Fister et al. (2013) made a comprehensive review of firefly algorithms. Yang and He (2013) repoted recent advances and applications of firefly algorithm. Sayadi et al. (2013) applied firefly algorithm to the application of manufacturing cell formation for discrete optimization problem. Johari et al. (2013) applied firefly algorithm on various domains of optimization problem. Several categories of optimization problems such as discrete, chaotic, multi objective and many more were addressed by inspiring the behavior of fireflies. Talatahari et al. (2014) analyzed optimum design of tower structure using firefly algorithm.

Flower pollination algorithm is also an efficient algorithm in the field of metaheuristic optimization inherited from the natural inspiration of pollination process. Pollinators such as insects can travel long distance, and thus they introduce the ability (into the algorithm) that they can escape any local landscape and subsequently explore larger search space. Yang et al. (2012) investigated flower pollination algorithm for global optimization of ten test functions. Ten test functions were used to validate flower pollination algorithm and it was found from the simulation results that FPA is more efficient than both genetic algorithm (GA) and particle swarm optimization (PSO) algorithm. Wang and Jhou (2014) investigated the flower pollination algorithm with dimension by dimension improvement to improve the convergence speed and quality of solutions effectively. In this algorithm a dimension by dimension based update and evaluation strategy on solutions was used with the progress of iterations. Sharawi et al. (2014) applied flower pollination optimization algorithm for wireless sensor network lifetime optimization. A wireless sensor network energy aware clustering formation model was proposed based upon intra cluster distances using FPA. It was found from the performance analysis that applying FPA on WSN (wireless sensor network) clustering was more efficient compared to classical LEACH approach. Balasubramani et al. (2014) made a study on flower pollination algorithm and its application. FPA has been extensively researched to solve integer programming problems (El-Henawy and Ismail, 2014), sudoku puzzles (Raouf et al., 2014). Yang et al. (2014) applied flower pollination algorithm for a novel approach for multi-objective optimization to solve a set of multi objective test functions and two bi objective design benchmarks. A comparison of the algorithm with other stochastic algorithms showed that FPA is an efficient algorithm in terms of fast convergence rate. Prathiba et al. (2014) proposed FPA for optimizing economic load dispatch in power system operation. The objective was to minimizing the fuel cost by effectively setting the real power outputs from generators.

In this paper, laser micro-channeling parameters are optimized, using firefly algorithm and flower pollination algorithm. Response surface models, which correlate the quality characteristics or responses with process parameters, are used as objective functions in both the algorithms. Single as well as and multi objective optimizations are performed. Statistical analyses are performed with the results obtained to measure the accuracy of the predicted results. Results are further compared with the optimum results found with conventional optimization techniques, which shows greater improvement. Comparison is also made between both the algorithms in terms of accuracy and repeatability of the results, convergence rate and computational time. In addition to that parametric trends are also analyzed.

## METHODOLOGY

### Firefly algorithm (FA)

The firefly algorithm is a nature-inspired metaheuristic algorithm introduced in 2008 by Yang to solve optimization problems (Yang, 2009; Yang, 2010). The algorithm is based on the social flashing behavior of fireflies in nature. Although the algorithm has many similarities with other swarm based algorithms such as particle swarm optimization (PSO), artificial bee colony optimization (ABC) and ant colony optimization (ACO), the firefly algorithm has proved to be much simpler both in concept and implementation. The superiority of FA over PSO is already established for solving noisy nonlinear optimization problems (Pal et al., 2012).

### *Flashing Behavior of Fireflies*

Fireflies or lightning bugs are member of a family of insects that can produce natural light to attract a mate or prey. There are near to two thousand firefly species, and most of them produce short and rhythmic flashes (Hashmi et al., 2013). The intensity ($I$) of flashes has a negative effect with increased distance ($r$) increases conforms to the inverse square law and thus most fireflies can communicate only up to several hundred meters. That is the intensity of the light, $I$, goes on decreasing in terms of $I \alpha 1/r^2$, as the distance, $r$, will increase. Additionally, light is absorbed by air continuously, thus with increased distance light becomes weaker. The combination of these two above mentioned factors make most fireflies visible upto a limited distance, which is quite enough for fireflies to communicate with each other.

In the firefly algorithm, there are three particular idealized rules, which are based on some of the major flashing characteristics of real fireflies. They are (Hashmi et al., 2013):

1. Attraction of one firefly to other fireflies does not depend upon their sex.
2. Attractiveness is proportional to brightness and both have a negative effect with increase distance.
3. The landscape of the objective function determines the brightness of a firefly.

### *Attractiveness and Light Intensity*

In the Firefly algorithm, there are two important issues: the variation of the light intensity and the formulation of the attractiveness. It has been mentioned above that the light intensity follows the inverse square law (Hashmi et al., 2013) i.e.:

$$I(r) = \frac{I}{r} \tag{1}$$

where $I(r)$ denotes light intensity at a distance $r$ and $I_s$ denotes the intensity at the source.

When air is used as a medium, the light intensity can be determined as follows (Hashmi et al., 2013):

$$I(r) = I_0 e^{-\gamma} \tag{2}$$

To avoid the singularity at $r$ = 0 in Eq. (1), the equations can be approximated in the following Gaussian form:

$$I(r) = I_0 e^{-\gamma r^2} \tag{3}$$

Since a firefly's attractiveness is proportional to the light intensity seen by adjacent fireflies, we can now define the attractiveness $\beta$ of a firefly as:

$$\beta = \beta_0 e^{-\gamma r^2} \tag{4}$$

where $\beta_0$ is the attractiveness at $r = 0$. Since it is often faster to calculate$1/(1+r^2)$ than an exponential function, the above function, if necessary, can be approximated as (Yang, 2010):

$$\beta = \frac{\beta_0}{1+\gamma r^2} \tag{5}$$

Both, Eqs. (4) and (5) define a characteristic distance $\Gamma = 1/\sqrt{\gamma}$ depending upon which the attractiveness is changing in a significant manner from $\beta_0$ to $\beta_0 e^{-1}$ for Eq. (4) or $\beta_0/2$ for Eq. (5). In the real time implementation, the attractiveness function $\beta(r)$ can be any monotonically decreasing functions such as the following generalized form (Yang, 2010):

$$\beta(r) = \beta_0 e^{-\gamma r^n} \quad (m \geq 1) \tag{6}$$

For a fixed $\gamma$, the characteristic length becomes (Yang, 2010):

$$\Gamma = \gamma^{-1/m} \rightarrow 1, m \rightarrow \infty \tag{7}$$

Conversely, for a specific length scale $\Gamma$ in an optimization problem, the parameter $\gamma$ can be used as a typical initial value. That is (Yang, 2010):

$$\gamma = 1/ r^{m} \tag{8}$$

### *Firefly Distance*

The distance between any two fireflies $i$ and $j$ at $x_i$ and $x_j$ respectively, the Cartesian distance is determined by Eq. (9) where $x_{i,k}$ is the $k_{th}$ component of the spatial coordinate $x_i$ of the $i_{th}$ firefly and $d$ is the number of dimensions (Pal et al., 2012).

$$r_{ij} = \left\| x_i - x_j \right\| = \sqrt{\sum_{k=1}^{d} \left( x_{i,k} - x_{j,k} \right)^2} \tag{9}$$

In 2-dimensional case, we have (Pal et al., 2012):

$$r_{ij} = \sqrt{\left( (x_i - x_j)^2 - (y_i - y_j)^2 \right)} \tag{10}$$

### *Firefly Movement*

The movement of a firefly $i$ for the attraction to another more attractive (brighter) firefly $j$ is determined by (Yang, 2010):

$$x_i = x_i + \beta_0 e^{-\gamma r_{ij}^4} (x_j - x_i) + \alpha \varepsilon_i \tag{11}$$

where the second term is due to the attraction while the third term is randomization with $\alpha$ being the randomization parameter and $\varepsilon_i$ being the vector of random numbers drawn from a Gaussian distribution or uniform distribution.

It is very important to point out that Eq. (11) is a random-walk partial towards the brighter fireflies. If $\beta_0$ = 0, it becomes a simple random walk. Furthermore, the randomization term can easily be prolonged to other distributions such as Lévy flights.

The parameter $\gamma$ now characterizes the contrast of the attractiveness, and its value is crucially important in determining the speed of the convergence and how the firefly algorithm behaves. In theory, $\gamma \in [0, \infty)$, but in actual practice, $\gamma$ = O (1) is determined by the characteristic length $\Gamma$ of the system to be optimized. Thus, for most applications, it typically varies from 0.1 to 10 (Yang, 2010).

***Convergence***

For any large number of fireflies ($n$), if $n$>>$m$, where $m$ is the number of local optima of an optimization problem, the convergence of the algorithm can be achieved. Here, the initial location of $n$ fireflies is distributed uniformly in the entire search space, and as the iterations of the algorithm continue fireflies converge into all the local optimum. By comparing the best solutions among all these optima, the global optima are achieved (Galvez and Iglesias, 2013).

**Flower pollination algorithm (FPA)**

Flower pollination algorithm is a metaheuristic and bio-inspired intelligence based algorithm developed by Xin-She Yang, 2012, based on the pollination process of the flower plants. It does not fall in the category of swarm intelligence based algorithms. In the flower pollination algorithm, there are four particular idealized rules, which are based on pollination characteristics of flower plants. These are as follows:

1. Global pollination process can be achieved by pollen- carrying pollinators performing Levy flights for both biotic and cross pollination.
2. Local pollination can take place for both abiotic and self pollination.
3. The new generation reproduction probability depends on the flower consistency and proportional to flowers' similarities / differences (Sharawi et al., 2014).
4. The transformation between local pollination and global pollination is controlled by a switch probability $p \in [0, 1]$.

For abiotic, flowers do not need any pollinators for the pollens transferring process (Beverly, 2007). In general, biotic pollination form is considered to follow for most of the flowers. Each plant can have multiple flowers, and each flower patch often release millions, and even billions of pollen gametes. For simplicity, we assume that each plant has only one flower, and each flower produce only one pollen gamete (Sharawi et al., 20104). For simplicity, we assume that each plant has only one flower, and each flower produce only one pollen gamete. According to the rules stated above, the flower pollination algorithm (FPA) can be represented mathematically as follows:

Flower pollens are carried by pollinators such as insects which can often fly and move in a much longer range and thus, pollens can travel over a long distance in the global pollination step. The pollinators intend to achieve the global optimization of reproduction based on flower consistency, this can mathematically achieve by (Sharawi et al., 20104):

$$x_i^{t+1} = x_i^t + L(x_i^t - g_*) \tag{12}$$

where, $x^t{}_i$ denotes solution vector $x_i$ for pollen $i$ at iteration $t$, and $g_*$ denotes current best solution found so far during iterations. The parameter $L$ denotes the strength of the pollination, which essentially is a step size. Since, insects may move over a long distance with various distance steps, we can use a Lévy flight to mimic this characteristic efficiently (Pavlyukevich, 2009). Lévy flight using Lévy step is a powerful random walk because both global and local search capabilities can be carried out at the same time. In contrast with standard Random walks, Lévy flights have occasional long jumps, which enable the algorithm to jump out any local valleys. Lévy steps obey the following approximation (Sharawi et al., 20104):

$$L \approx \frac{\gamma t(\gamma)\sin\left(\frac{\pi\gamma}{2}\right)}{\pi}\frac{1}{s^1+\gamma} \qquad s >> s_0 > 0 \tag{13}$$

where, $\tau(\gamma)$ is the standard gamma function, and this distribution is valid for large steps $s > 0$. For case of local pollination, achieved by abiotic and self-pollination depending upon flower constancy, the mathematical representation is as follows (Sharawi et al., 20104):

$$x_i^{t+1} = x_i^t + \in (x_j^t - x_k^t) \tag{14}$$

Where, $x^t{}_j$ and $x^t{}_k$ are pollens from the different flowers of the same plant species. These essentially mimic the flower constancy in a limited neighborhood (Yang, 2012). $\epsilon$ is local random walk drawn from a uniform distribution in [0, 1].

Though Flower pollination activities can occur at all scales, both local and global, in practice, adjacent flower patches or flowers in the not-so-far-away neighborhood are more likely to be pollinated by local flower pollens than those are far away. For this, the switch probability $p \in [0, 1]$ is used to control the exchange of the pollination process from local to global and vice versa.

# DEVELOPMENT OF EMPIRICAL MODELS USING RSM

Central composite design of response surface methodology is used for planning the experimental work. Experiments are conducted on a 3 mm thick PMMA (Poly-methyl-meth-acrylate) sheet using a Nd:YAG laser having 1.06 µm wavelength. All the experiments are conducted in submerged water condition having water level 1mm above the workpiece surface (Prakash et al., 2013). The 1mm of water level is decided based on various pilot experiments giving the best output results. Fig. 1 shows the laser micromachining system used for the experimental work. Four key process parameters, which affect the process significantly, are taken as process parameters namely, lamp current, pulse frequency, pulse width, and cutting speed. Lamp current directly corresponds to total laser fluence or energy consumed by laser to emit desired pulses. Pulse frequency is the number of pulses emitted by laser per unit time. Pulse width denotes the percentage of "ON" time duration per cycle time.

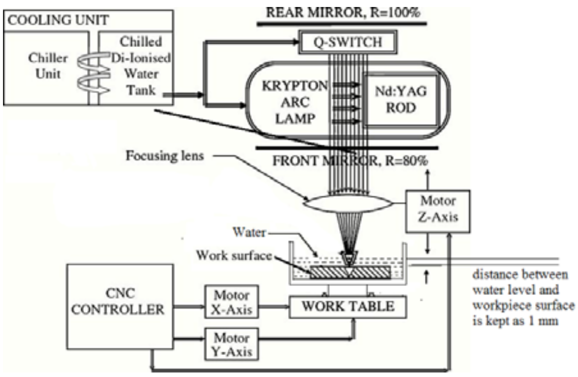


Fig. 1 (a) Photographic view, and (b) schematic view of underwater Nd:YAG laser micro-machining system

Table 1. Laser micro-channeling process parameters and their coded levels (Prakash et al., 2013)

| Parameter | Unit | Symbol | Level | | | | |
|---|---|---|---|---|---|---|---|
| | | | -2 | -1 | 0 | +1 | +2 |
| Lamp current | A | *A* | 13 | 14 | 15 | 16 | 17 |
| Pulse frequency | kHz | *B* | 1 | 2 | 3 | 4 | 5 |
| Pulse width | % | *C* | 3 | 6 | 9 | 12 | 15 |
| Cutting speed | mm/s | *D* | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |

Cutting speed represents the speed of movement of laser head with respect to the workpiece or vice-versa. The different parameters considered and their RSM-coded levels are given in Table 1. Thus, each parameter is constrained within the values corresponding to -2 and +2 coded levels. The two responses recorded are average channel depth (CD) and average burr width (BW).The responses are measured by using Olympus-STM-6, a 3-dimensional optical measuring microscope. The experimental results are presented previously (Prakash et al., 2013). Fig. 2 shows photographic view of micro-channel fabricated on PMMA using Nd:YAG pulsed laser in underwater condition. RSM in combination with multiple regression analysis is used to develop the empirical models to correlate the responses with process parameters. The models developed in terms of coded factors using RSM are given below:

$$Y_{(CD)}\ (\mu m) = 91.554 + 7.479\,A + 3.985\,B + 7.461\,C - 4.909\,D - 10.121\,A\,C + 8.086\,A\,D + 8.382\,B\,C + 4.658\,B\,D - 9.916\,C\,D - 2.166\,A^2 - 4.241\,C^2 - 4.385\,D^2 \qquad (15)$$

$$Y_{(BW)}\ (\mu m) = 54.667 + 2.640\,A + 3.089\,B - 4.737\,C + 6.140\,D + 2.789\,A\,C - 5.006\,B\,C - 5.238\,B\,D + 7.078\,C\,D + 2.748\,A^2 + 3.634\,B^2 + 7.420\,D^2 \qquad (16)$$

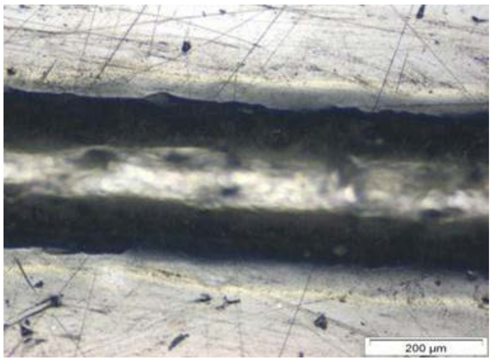


Fig. 2 Photographic view of micro-channel fabricated on PMMA (Prakash et al., 2014)

## OPTIMIZATION USING FA AND FPA: RESULTS AND DISCUSSIONS

To achieve superior micro-channel quality, it is desired that channel depth should be maximum and burr width should be minimum. Thus, the process parameters are to be selected in such a way that both the conditions should satisfy, simultaneously. Two newly developed bio-inspired metaheuristic algorithms, namely firefly algorithm (FA) and flower pollination algorithm (FPA) are utilized to find the solution of single-objective optimization, as well as, multi objective optimization of laser micro-channeling process parameters, to tackle such optimization problems.

For this, computer programs are developed in MATLAB® on an Intel® Core™ i3-380M CPU @ 2.53 GHz, 3.00GB RAM operating platform using firefly algorithm and flower pollination algorithm. The algorithm specific parameters used are given in Table 2. It is observed that both the algorithms have obtained optimum values for all the test functions.

### Single Objective Optimization

At first, single objective optimization is considered, and the two responses, channel depth and burr width are optimized, independently. The maximum value of channel depth obtained by FA and FPA, both is 223.44 μm. The minimum value of burr width achieved is 25.17 μm. The results, along with the optimal parameter settings for the responses, are summarized in Table 3 which is same for both, FA and FPA, which ensures effectiveness and accuracy of both the algorithms. The potentiality of FA over PSO in nonlinear optimization problems is demonstrated previously (Pal et al., 2012). Table 3 also presents the optimal micro-channeling parameters obtained by using RSM (Prakash et al., 2013) and the corresponding response values. It is observed from Table 3 that both FA and FPA outperform the existing optimal value, and gives effective optimal parametric setting.

Table 2. Algorithm specific parameters considered for FA and FPA

| | | |
|---|---|---|
| Firefly algorithm | Maximum iterations | 500 |
| | Function evaluations | 10000 |
| | Number of fireflies ($n$) | 20 |
| | Light absorption coefficient ($\gamma$) | 1 |
| | Attractiveness ($\beta$) | 0.2 |
| Flower pollination algorithm | Maximum iterations | 500 |
| | Function evaluations | 10000 |
| | Population size ($n$) | 20 |
| | Probability switch ($p$) | 0.8 |

Table 3: Results of single objective optimization

| Response | Optimization Technique | Optimal value | Optimal parameter setting | | | |
|---|---|---|---|---|---|---|
| | | | *A* | *B* | *C* | *D* |
| $Y_{(CD)}$ (μm) | FA | 223.44 | 13.00 | 5.00 | 15.00 | 0.10 |
| | FPA | 223.44 | 13.00 | 5.00 | 15.00 | 0.10 |
| | RSM | 156.51 | 13.00 | 1.00 | 15.00 | 0.10 |
| $Y_{(BW)}$ (μm) | FA | 25.17 | 13.51 | 2.96 | 15.00 | 0.16 |
| | FPA | 25.17 | 13.51 | 2.96 | 15.00 | 0.16 |
| | RSM | 32.91 | 13.00 | 1.00 | 15.00 | 0.10 |

Fig. 3 shows the comparisons of functional evaluations for FA and FPA with respect to channel depth by means of (a) algorithm convergence plot, (b) histogram of functional evaluations and (c) pie chart of the functional evaluations. It is evident from Fig. 3 (a) that both the algorithms show a fast convergence to its global optima, whereas FA shows faster convergence than FPA. During optimization of channel depth, the FA converges to the optima after only 502 evaluations, whereas 566 evaluations have been taken by FPA. It is seen from Fig. 3 (b) that the mean value of the results obtained by FA is 215.8 μm, which is closer to the optimum result, compared to the mean value obtained by FPA which is 210.1μm. It is further seen that the results are more scattered and dispersed about their mean values for FPA. As the population mean value of the functional evaluations for FA is much closer to its optima than that for FPA, it can be concluded that more number of evaluations have converged to its optima for FA as compared to FPA. Again it has been clearly observed during optimization using FPA that the evaluated responses are spread out over a wide range of values compared to FA, which ensures that FA is more effective than FPA for obtaining optima. The mean absolute deviation value (MAD), i.e. the deviation of the functional evaluations about their mean value is also a measure of variation in the evaluated results. MAD value obtained for FA during CD optimization is 7.255 μm where for FPA it is 21.417 μm. From obtained MAD, it is quite obvious that the variation in the process about the mean value is more for FPA which ensures that the convergence quality of FA is far superior compared to FPA. From the pie charts in Fig. 3 (c) it is observe that the quality of convergence is better in case of FA than FPA, for the case of optimization of channel depth. For FA almost 7996 evaluations have been found converged to its optimal value indicated by the orange region, whereas, for FPA it is 7283 evaluations indicated by the yellow region.

Fig. 4 shows the comparisons of functional evaluations for FA and FPA with respect to burr width by means of (a) algorithm convergence plot, (b) histogram of functional evaluations, and (c) pie chart of the functional evaluations. It is observed from Fig. 4 (a) that FPA gives faster convergence than FA for the case of optimizing burr width. FA takes 3901 evaluations to reach the optima, whereas FPA takes only 3361 evaluations. However, both the algorithm takes more number of evaluations as compared to the case of optimization of channel depth. It is seen from Fig. 4 (b) that the mean value of the functional evaluations obtained by FA is 27.14 μm, which is closer to the optima compared to the mean value obtained by FPA which is 29.40 μm. The results are more dispersed about the mean value for the case of FPA than FA. MAD value obtained for FA and FPA are 3.438 μm and 8.066 μm, respectively. However, from Fig. 4 (c), it is observed that FPA gives more optimized evaluations than FA indicated by the orange region for both the cases. For FA it is found only 767 evaluations where for FPA 1067 evaluations have been converged to its optimal value.

The computational time recorded for the optimization of channel depth is 0.76 seconds for FA and 0.59 seconds for FPA. The computational time recorded for the optimization of burr width is 1.03 seconds for FA and 0.49 seconds for FPA. Therefore, the average computational time observed for single objective optimization in this study is 0.90 seconds for FA and only 0.54 seconds for FPA. Though the average computational times for both the algorithms are less than 1second, FPA exhibits faster computation than FA for single objective optimization. An earlier investigation shows that FA performs better than PSO, in terms of the time taken to reach the optimum or near optimum value (Pal et al., 2012). The variations of channel depth with process parameters are displayed in Fig. 5. The parametric trends predicted by FA and FPA are in close agreement with the results published in earlier experimental research work (Parakash et al., 2013). FA and FPA are more advantageous than RSM, in prediction of parametric trend, because no parameter is held constant during analysis, and hence, the true overall trend can be predicted. It is seen from Fig. 5 that FA and FPA give exactly same results for channel depth optimization i.e. channel depth has an optimized value for the coded level of lamp current and cutting speed at -2 i.e. 13A and 0.1 mm/s, respectively, and for the coded level of pulse frequency and pulse width at +2 i.e. 5 kHz and 15%, respectively.

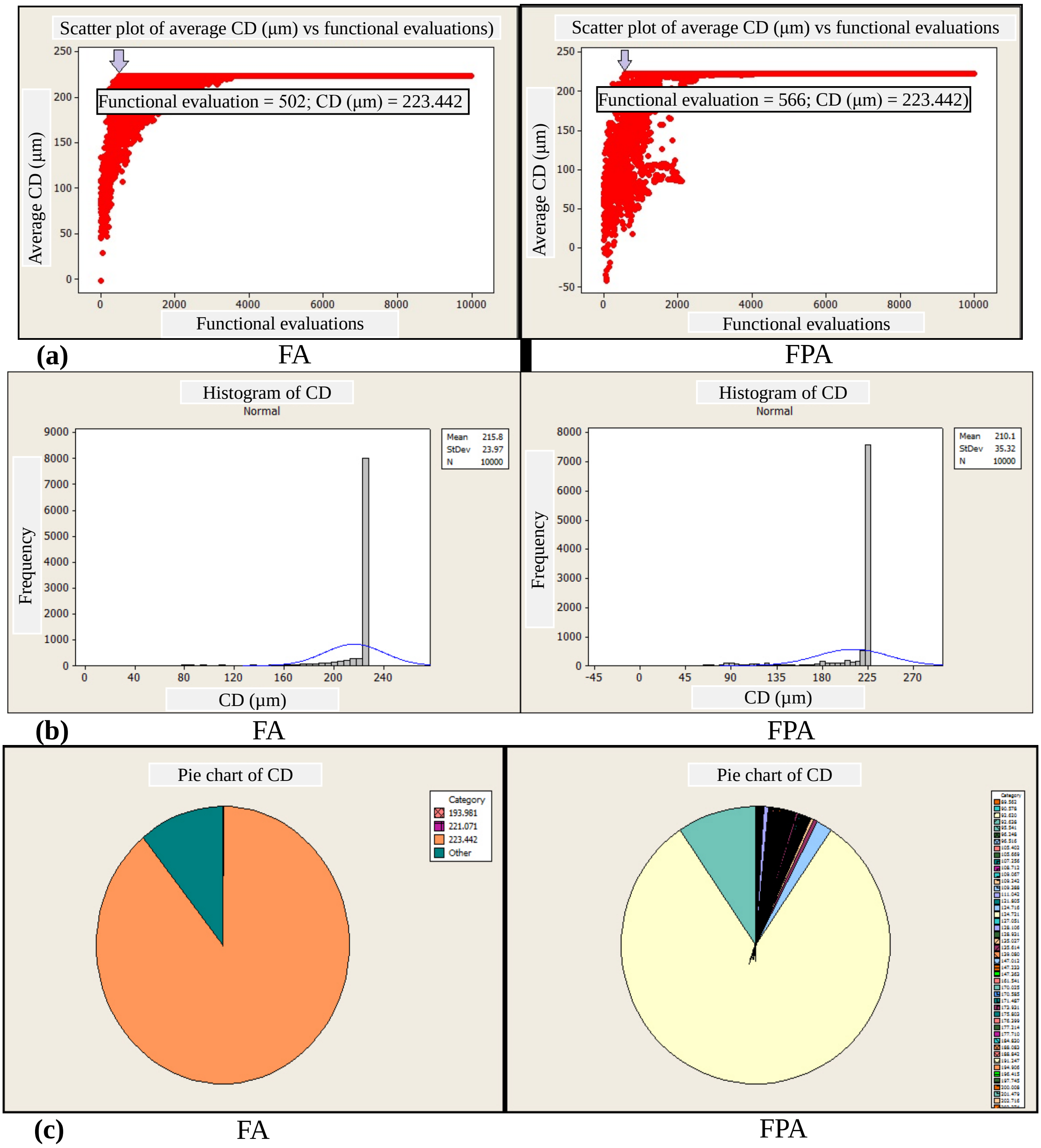


Fig. 3 Comparisons of functional evaluations for FA and FPA with respect to channel depth by means of (a) algorithm convergence plot, (b) histogram of functional evaluations, and (c) pie chart of the functional

evaluations

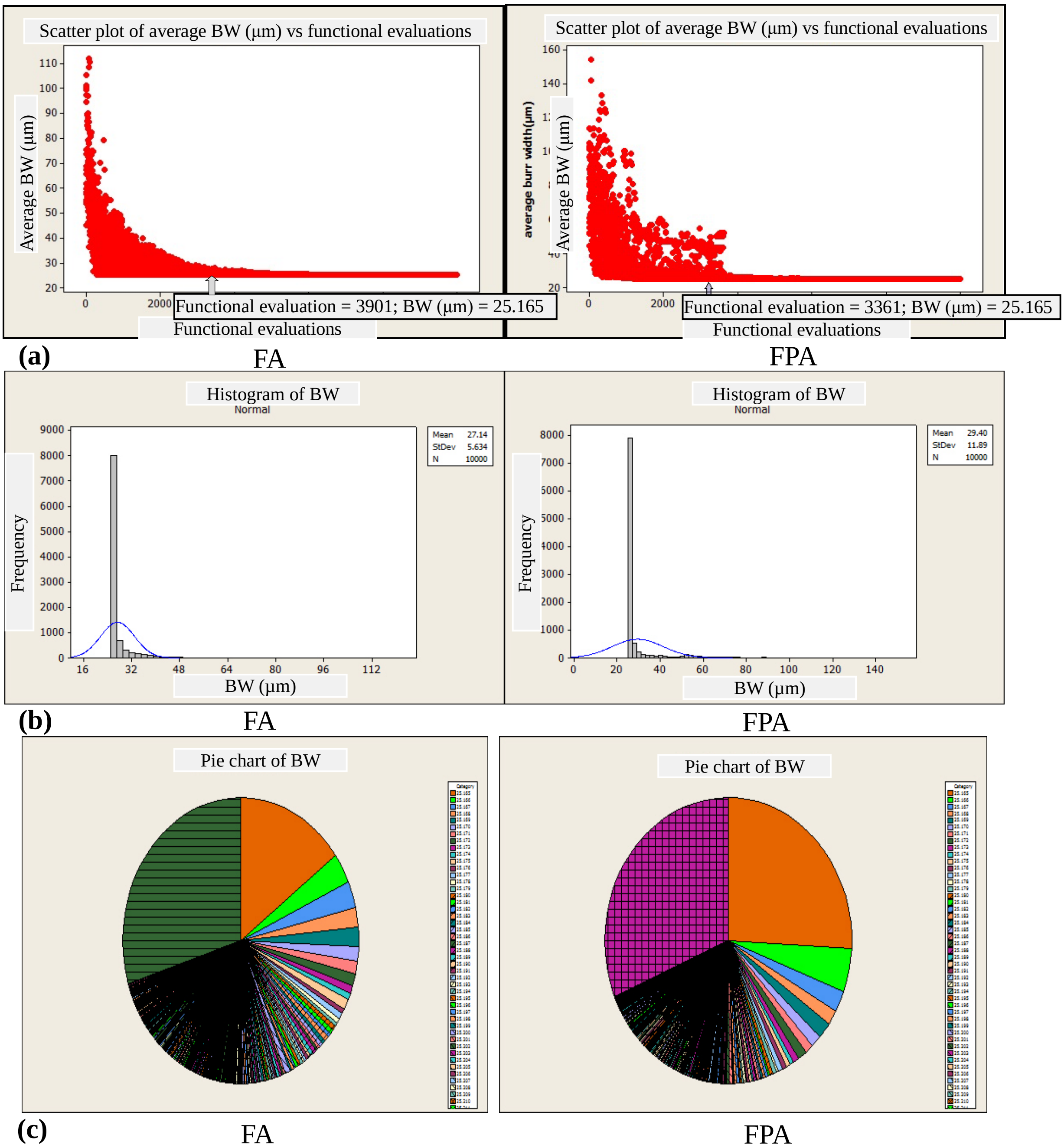


Fig. 4: Comparisons of functional evaluations for FA and FPA with respect to burr width by means of (a) algorithm convergence plot, (b) histogram of functional evaluations, and (c) pie chart of the functional evaluations

It is also seen from Fig. 5 that the mean values of the functional evaluations are closer to its optima for FA than FPA, as the results are more dispersed about their mean value for FPA, which is obtained by plotting of $y_{mean}$ and $y_{std}$. Thus it can be concluded that the FA shows superior quality of the convergence to its optima than FPA, with respect to all the process parameters.

The variations of channel depth with process parameters are displayed in Fig. 5. The parametric trends predicted by FA and FPA are in close agreement with the results published in earlier experimental research work (Parakash et al., 2013). FA and FPA are more advantageous than RSM, in prediction of parametric trend, because no parameter is held constant during analysis, and hence, the true overall trend can be predicted. It is seen from Fig. 5 that FA and FPA give exactly same results for channel depth optimization i.e. channel depth has an optimized value for the coded level of lamp current and cutting speed at -2 i.e. 13A and 0.1 mm/s, respectively, and for the coded level of pulse frequency and pulse width at +2 i.e. 5 kHz and 15%, respectively. It is also seen from Fig. 5 that the mean values of the functional evaluations are closer to its optima for FA than FPA, as the results are more dispersed about their mean value for FPA, which is obtained by plotting of $y_{mean}$ and $y_{std}$. Thus it can be

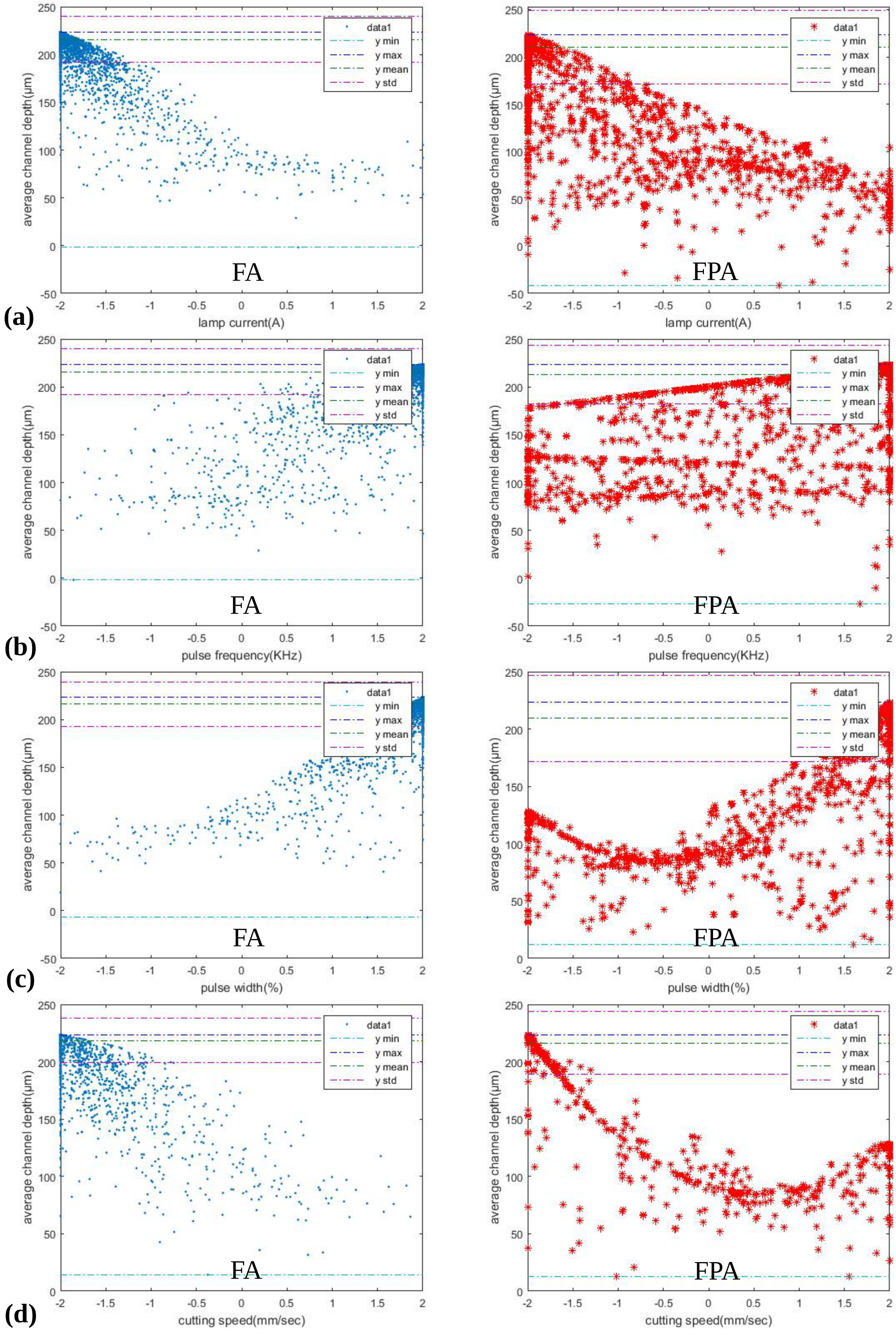


Fig. 5 Scatter diagrams of channel depth for (a) lamp current, (b) pulse frequency, (c) pulse width, and (d) cutting speed

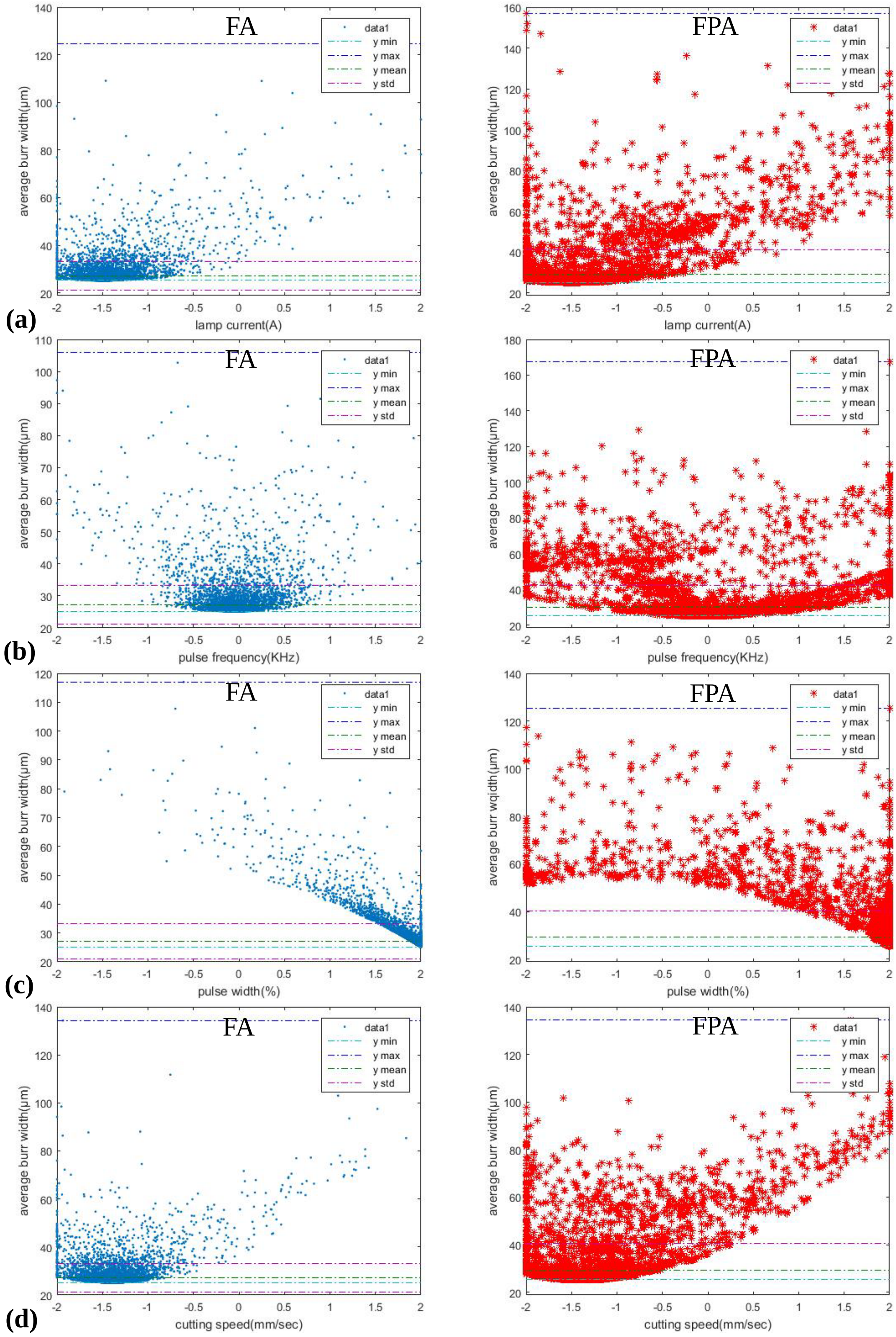


Fig. 6 Scatter diagrams of burr width for (a) lamp current, (b) pulse frequency, (c) pulse width, and (d) cutting speed

concluded that the FA shows superior quality of the convergence to its optima than FPA, with respect to all the process parameters.

Fig. 6 shows the effects of all process parameters on burr width, which depict same trend as presented earlier (Parakash et al., 2013). FA and FPA, both provide same results i.e. the burr width has an optimized value at the coded level of lamp current at -1.50 i.e. 13.51 A, at coded level of pulse frequency at -0.04 i.e. 2.96 kHz, at coded level of pulse width at +2 i.e. 15%, and at coded level of cutting speed at -1.38 i.e. 0.16 mm/s, respectively. It is also observed from the same figure that FA performs better than FPA in terms of convergence, as evaluated by plotting $y_{mean}$ and $y_{std}$.

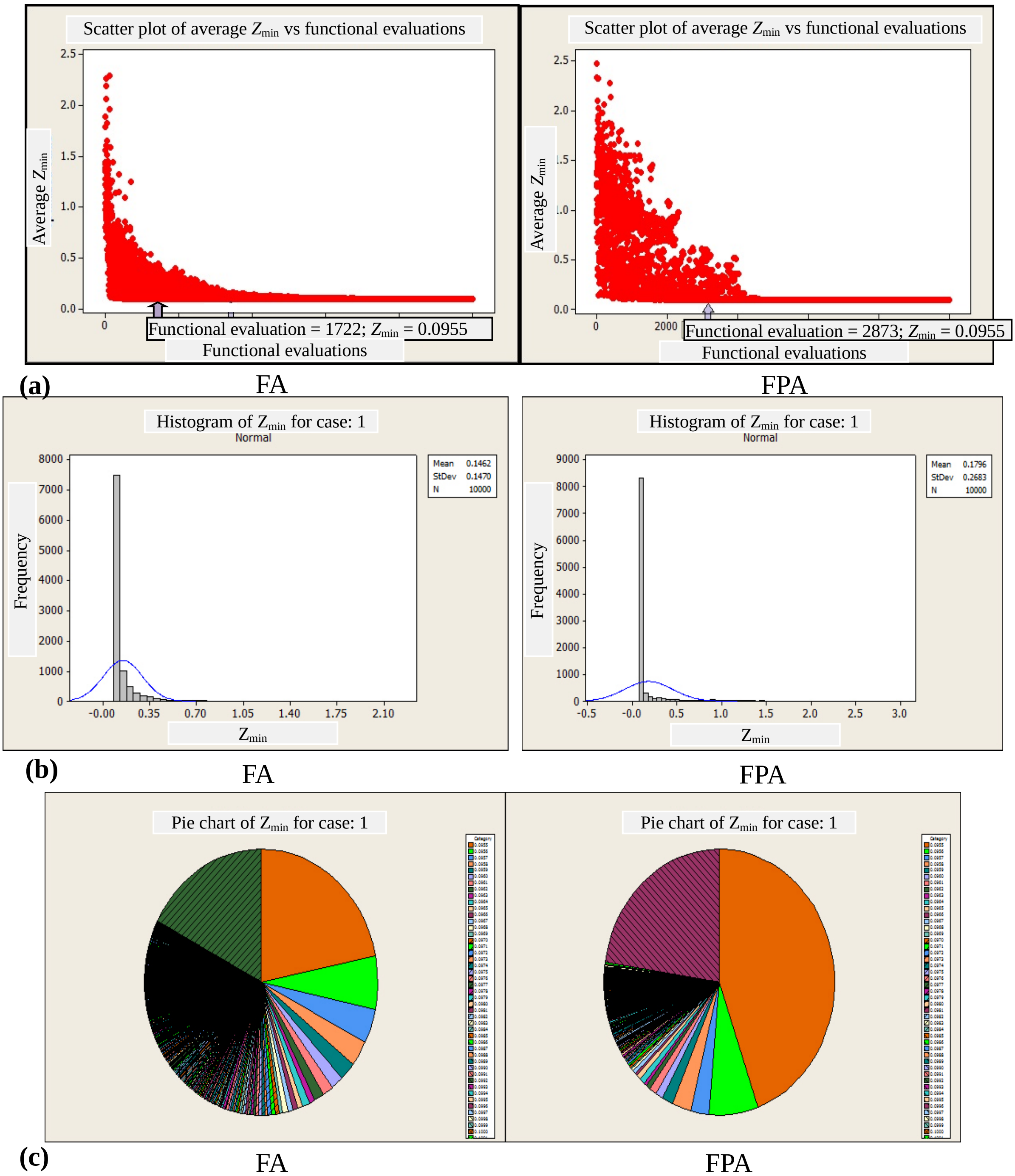


Fig. 7: Comparisons of functional evaluations for FA and FPA with respect to case: 1 by means of (a) algorithm convergence plot, (b) histogram of functional evaluations, and (c) pie chart of the functional evaluations

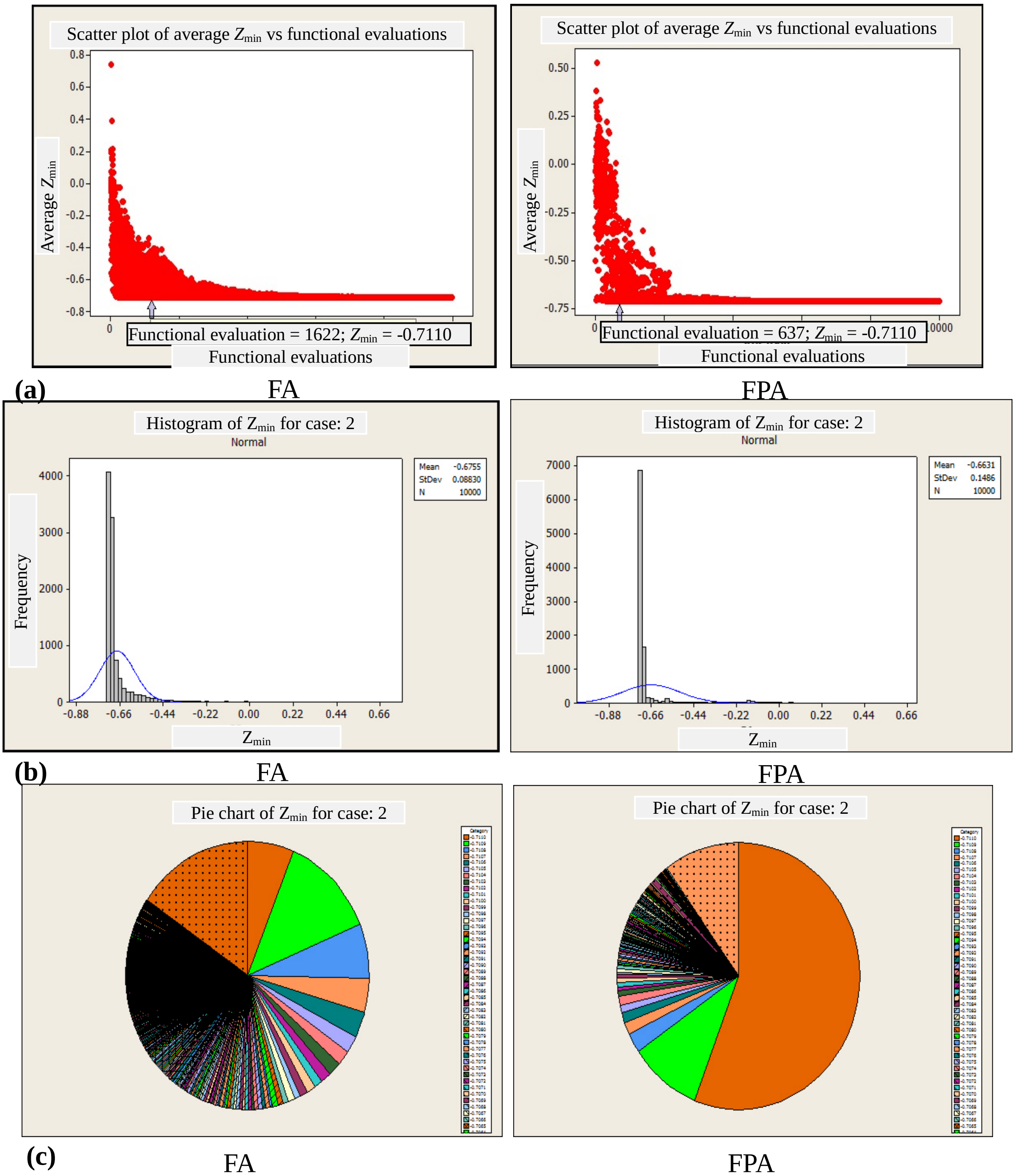


Fig. 8 Comparisons of functional evaluations for FA and FPA with respect to case: 2 by means of (a) algorithm convergence plot, (b) histogram of functional evaluations, and (c) pie chart of the functional evaluations

**Multi objective optimization**

In this section, channel depth and burr width, both are optimized simultaneously, using firefly algorithm and flower pollination algorithm. The following objective function is developed for carrying out multi objective optimization:

$$\min Z = \frac{w_1 \times BW}{BW_{\min}} - \frac{w_2 \times CD}{CD_{\max}} \tag{17}$$

where, $w_1$ and $w_2$ are the weights (relative importance) assigned to burr width and channel depth, respectively (such that $w_1$+$w_2$ = 1), and the min and max values in the denominator of the expression (Eq. 17) are those obtained from single response optimization results of LTW using FA and FPA. The choice of weights depends entirely on the preference of the process engineer, or can be determined by analytic hierarchy process (AHP). $Z$ is minimized in

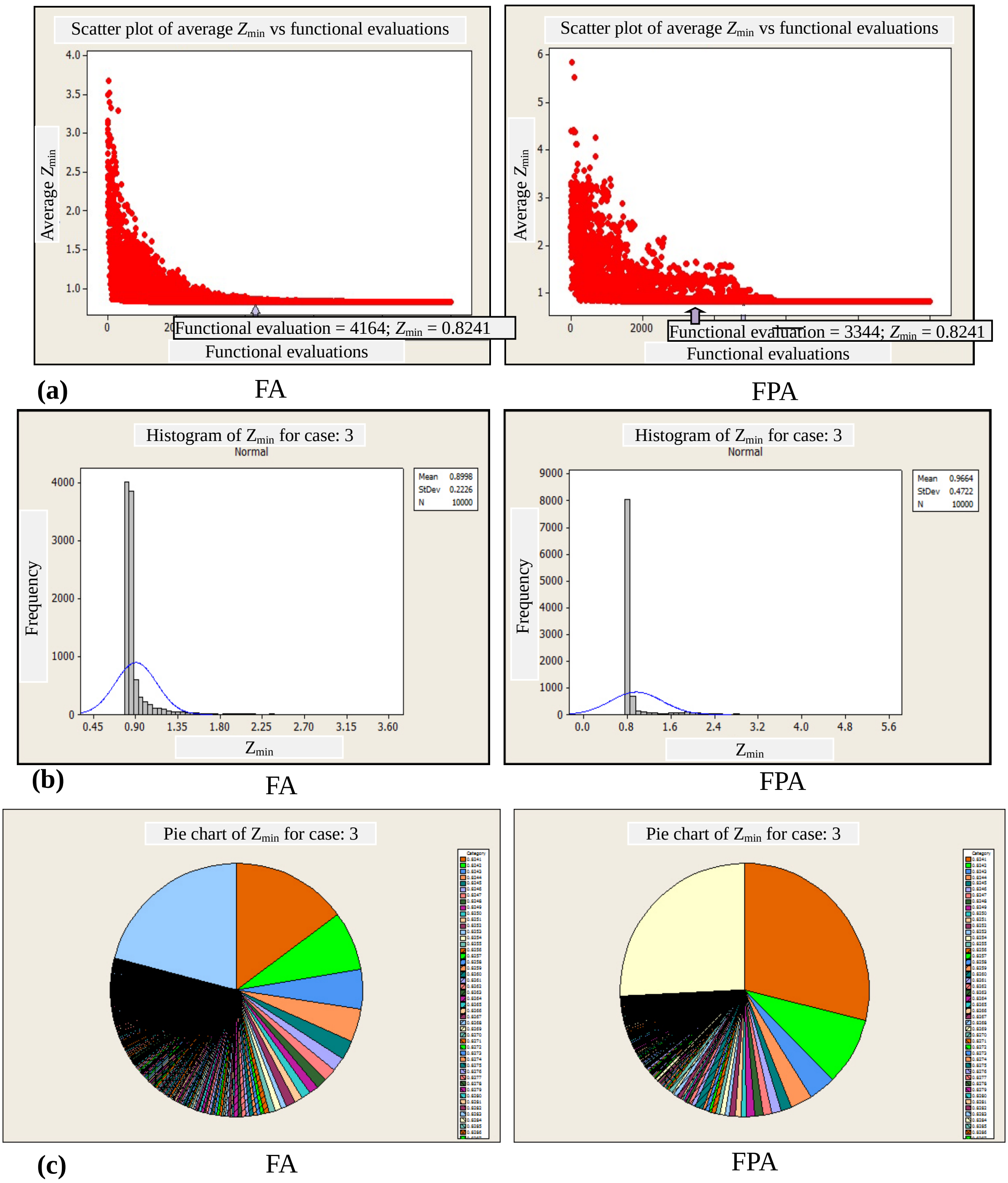


Fig. 9 Comparisons of functional evaluations for FA and FPA with respect to case: 3 by means of (a) algorithm convergence plot, (b) histogram of functional evaluations, and (c) pie chart of the functional evaluations

all the cases. Table 4 shows the results of multi response optimization according to the selected criteria. It is seen from Table 4 that, both, FA and FPA outperform the existing optimal value (Parakash et al., 2013), and gives effective optimal parametric setting.

Figs. 7-9 show the comparisons of functional evaluations for FA and FPA with respect to selected criteria by means of (a) algorithm convergence plot, (b) histogram of functional evaluations, and (c) pie chart of the functional evaluations. It is observed from Figs. 7-9 that FPA exhibits more convergence speed than FA, during multi objective optimization, except for the case 1. For case 1, FA converges to its optimized value after 1722 evaluations, whereas, FPA converges after 2873 evaluations. For case 2, FA converges to its optimized value after 1622 evaluations, whereas FPA converges only after 637 evaluations. For case 3, FA converges to its optimized

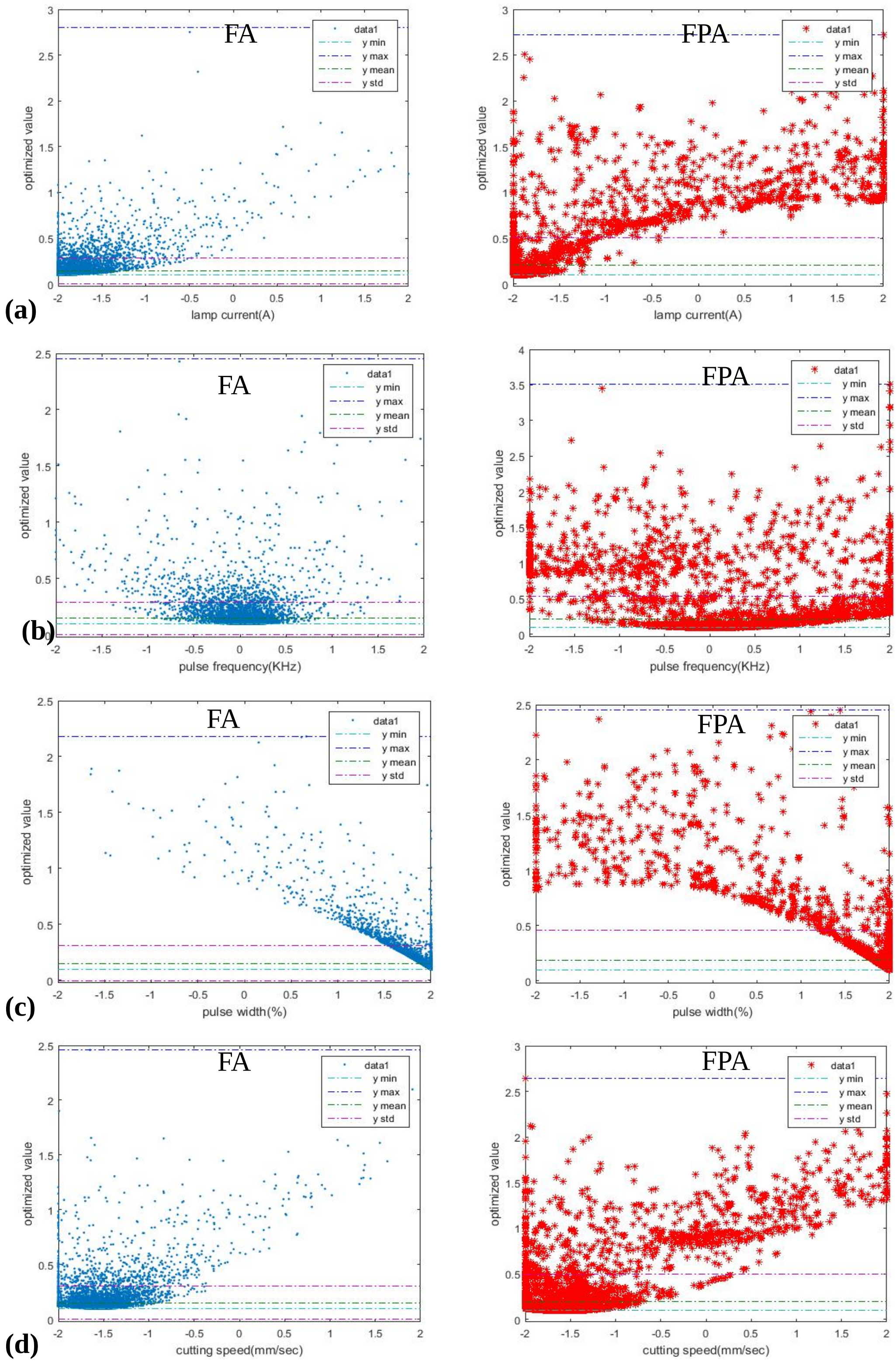


Fig. 10 Scatter diagrams of multi objective function (Z) with respect to (a) lamp current, (b) pulse frequency, (c) pulse width, and (d) cutting speed, for case: 1

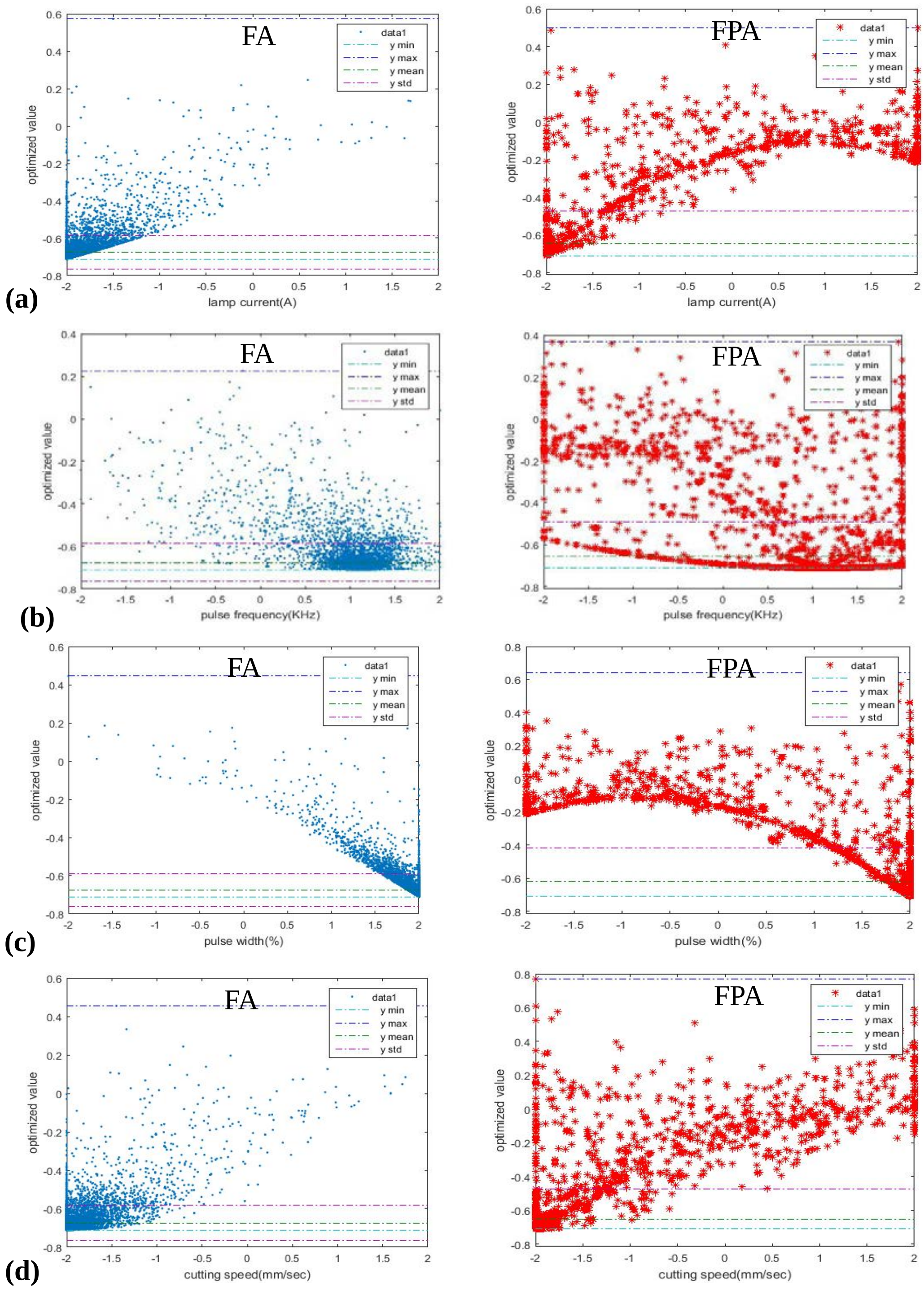


Fig. 11 Scatter diagrams of multi objective function (Z) with respect to (a) lamp current, (b) pulse frequency, (c) pulse width, and (d) cutting speed, for case: 2

value after 4164 evaluations, whereas FPA takes 3344 evaluations to converge. It is evident from Figs. 7-9 that the mean values of the multi objective optimization results obtained by FA are closer to their optima, as compared to the mean values obtained by FPA, which is quantified in terms standard deviations in the histograms. MAD values obtained for FA are less than FPA for all the cases. From the pie charts of Figs. 7-9, it is observed that FPA provides more optimized evaluations indicated by the orange region than FA, for all the cases, for same number of evaluations i.e. 10000. The computational times recorded for multi objective optimization using FA are 0.99 seconds, 1.01 seconds, and 1.02 seconds for case: 1, case: 2, and case: 3, respectively. Times taken by FPA are 0.49 seconds, 0.52 seconds, and 0.50 seconds for performing multi objective optimization according to case: 1, case: 2, and case: 3, respectively. Therefore, the average computational time observed for multi objective optimization in this study is 1.01 seconds for FA and only 0.50 seconds for FPA. The computational time of FPA is almost half of the computational time exhibited by FA. Figs. 10-12 are the scatter plots of multi objective function (*Z*) with

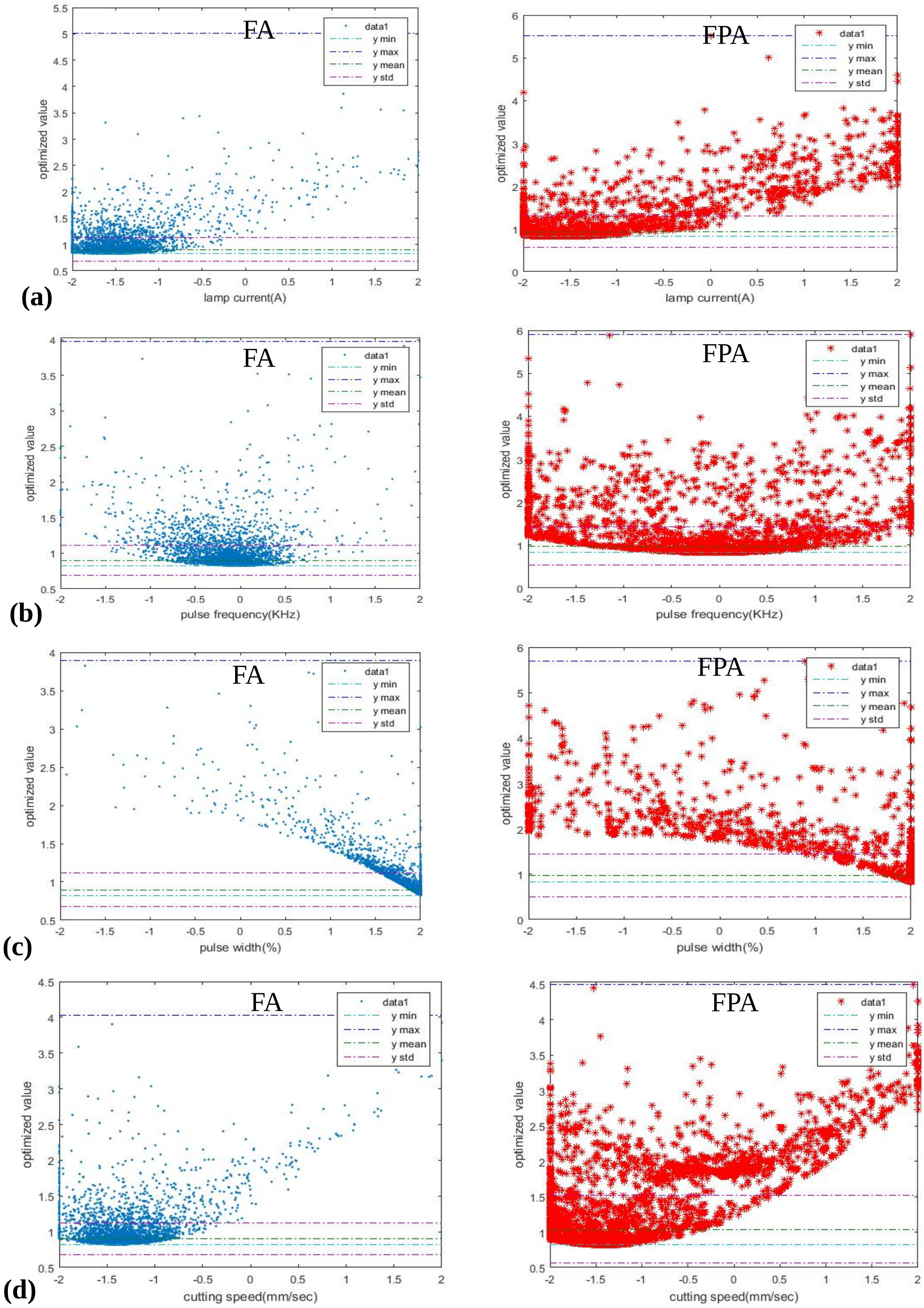


Fig. 12 Scatter diagrams of multi objective function (Z) with respect to (a) lamp current, (b) pulse frequency, (c) pulse width, and (d) cutting speed, for case: 3

respect to process parameters for case: 1, case: 2, and case: 3, respectively. It is obvious from the Figs. 10-12, that for all of the three cases FA exhibit better accuracy than FPA, as evaluated by plotting $y_{mean}$ and $y_{std}$. The optimal setting of the process parameters for all the cases can be easily determined from these figures, which are already furnished in Table 4.

Table 4. Results of multi objective optimization according to the selected criteria

| Conditions | Optimization Technique | $Z_{min}$ | Optimal value | | Optimal parameter setting | | | |
|---|---|---|---|---|---|---|---|---|
| | | | Channel depth | Burr width | *A* | *B* | *C* | *D* |
| Case 1: | FA | 0.096 | 189.93 | 26.20 | 13.00 | 3.04 | 15.00 | 0.14 |
| $w_1 = w_2 = 0.5$ | FPA | 0.096 | 189.93 | 26.20 | 13.00 | 3.04 | 15.00 | 0.14 |
| | RSM | - | 155.19 | 36.53 | 14.21 | 5.00 | 15.00 | 0.22 |
| Case 2: | FA | -0.711 | 213.18 | 37.21 | 13.00 | 4.11 | 15.00 | 0.10 |
| $w_1 = 0.1$, $w_2 = 0.9$ | FPA | -0.711 | 213.22 | 37.21 | 13.00 | 4.11 | 15.00 | 0.10 |
| | RSM | - | - | - | - | - | - | - |
| Case 3: | FA | 0.824 | 171.04 | 25.19 | 13.43 | 2.97 | 15.00 | 0.16 |
| $w_1 = 0.9$, $w_2 = 0.1$ | FPA | 0.824 | 171.08 | 25.18 | 13.43 | 2.97 | 15.00 | 0.16 |
| | RSM | - | - | - | - | - | - | - |

## CONCLUSION

The following conclusions can be drawn from this study:

1. Firefly algorithm and flower pollination algorithm, both, can be successfully implemented for single objective and multi objective optimization of laser micro-channeling parameters and also for prediction of parametric trends.
2. FA shows faster convergence rate for the maximization of channel depth, whereas, FPA exhibits faster convergence rate in case of minimization of burr width, during single objective optimization of the process. FPA exhibits faster convergence rate than FA during multi objective optimization for all the cases, except for the case: 1.
3. FA gives more optimized iterations i.e. more effective for the case of maximization of channel depth, whereas, FPA exhibits higher efficiency during the case of minimization of burr width, even for multi objective optimization, where the objective value has been minimized for all the mentioned cases.
4. FPA exhibits much lower computational time than FA for both, single and multi objective optimization.
5. Both the algorithms, FA and FPA, exhibit exceptionally low statistical variability and can be used efficiently for overall response trend prediction in case of responses dependent on several process parameters.
6. Both the algorithms give superior optimal results, both, in case of single and multi objective optimization, than the results obtained by response surface methodology.

## REFERENCES

Ageev, V.A., “Investigation of optical erosion of metals in liquids”, *J. Appl. Sprectosc.*, Vol. 23, No. 1, pp. 903-906 (1975).

Balasubramani, K., Marcus, K., “A study on flower pollination algorithm and its applications”, *Int. J. Appl. Innov. Eng. Mgmt.*, Vol. 3, pp. 230-235 (2014).

Beverly J. G., *Understanding flowers and flowering: An integrated approach*, Oxford: Oxford University Press (2007).

Chen, X., Xu, R.Q., Chen, J.P., Lu, J., Ni, X.W., “Shock-wave propagation and cavitation bubble oscillation by Nd:YAG laser ablation of a metal in water”, *Appl. Opt.*, Vol. 43, No. 16, pp. 3251–3257 (2004).

El-Henawy, I., Ismail, M., “An improved chaotic flower pollination algorithm for solving large integer programming problems”, *Int. J. Digi. Cnt. Technol. Appl.*, Vol. No. 3, pp. 72–81 (2014).

Fister I., Fister I. Jr., Yang X.S., Brest J., “A comprehensive review of firefly algorithms”, *Swarm Evol. Comp.*, Vol. 13, pp. 34-46 (2013).

Galvez A., Iglesias A., “Firefly algorithm for polynomial bezier surface parameterization”, *J. Appl. Math.* Article ID 237984 (2013).

Hashmi, A., Goel, N., Goel, S., and Gupta, D., “Firefly algorithm for unconstrained optimization”, *IOSR J. Comp. Eng.*, Vol. 11, pp. 75-78 (2013).

Johari N.F., Zain A.M., Mustaffa N.H., Udin A., “Firefly Algorithm for Optimization Problem”,. *Appl. Mech. Mater.*, Vol. 421, pp. 512-517 (2013).

Li M., Zhang, H., Shen, Z., Lu, J., Ni, X., “Physical analyses of optical breakdown and plasma formation in water induced by laser", *Acta Photonica Sinica*, Vol. 34, No. 11, pp. 1610–1614 (2005).

Pal S., Rai C.S., Singh A., "Comparative study of firefly algorithm and particle swarm optimization for noisy non-linear optimization problems", *Int. J. Intel. Sys. Appl.*, Vol. 10, pp. 50-57 (2012).

Pavlyukevich I., "Levy flights, non-local search and simulated annealing", *J. Comp. Phy.*, Vol. 226, pp. 1830-1844 (2009).

Prakash, S., Acherjee, B., Kuar, A.S., Mitra, S., "An experimental investigation on Nd:YAG laser microchanneling on polymethyl methacrylate submerged in water", *Proc. IMechE Part B: J. Eng. Manuf.*, Vol. 227, No. 4, pp. 508-519 (2013).

Prakash, S., Acherjee, B., Kuar, A.S., Mitra, S., "Grey relational analysis based optimization of underwater Nd:YAG laser micro-channeling on PMMA", *Proc. Eng.*, Vol. 97, pp. 1406-1415 (2014).

Prakash, S., Kumar, S., "Fabrication of microchannels: A review". *Proc. IMechE Part B: J. Eng. Manuf.*, Vol. 229, No. 8, pp. 1273-1288 (2014).

Prathiba, R., Moses M.B., Sakthivel, S., "Flower pollination algorithm applied for different economic load dispatch problems", *Int. J. Eng. Technol.*, Vol. 6, No. 2, pp. 1009-1016 (2014).

Raouf, O.A., El-henawy, I., Baset, M.A., "A novel hybrid flower pollination algorithm with chaotic harmony search for solving sudoku puzzles", *Int. J. Mod. Edu. Comp. Sci.*, Vol. 3, pp. 38–44 (2014).

Sharawi, M., Emary, E., Saroit, I.A., Mahdy, H.E., "Flower pollination optimization algorithm for wireless sensor network lifetime global optimization", *Int. J. Soft Comp. Eng.*, Vol. 4, pp. 54-59 (2014).

Sayadi M.K., Hafezalkotob A., Naini S.G.J., "Firefly-inspired algorithm for discrete optimization problems: An application to manufacturing cell formation", *J. Manuf. Sys.*, Vol. 32, No. 1, pp. 78-84 (2013).

Talatahari S., Gandomi A.H., Yun G.J., "Optimum design of tower structures using firefly algorithm", *Struct. Des. Tall Spec.*, Vol. 23, No. 5, pp. 350-361 (2014).

Tangwarodomnukun, V., Chen, H.Y., "Laser ablation of PMMA in air, water, and ethanol environments". *Mater. Manuf. Process.*, Vol. 30, No. 5, pp. 685-691 (2015).

Wang, S.C., Lee, C.Y., Chen, H.P. Thermoplastic microchannel fabrication using carbon dioxide laser ablation. *Journal of Chromatogr.* 2006; A 1111 (2): 252 – 257.

Wang, R., Jhou, Y., "Flower pollination algorithm with dimension by dimension improvement", *Math. Problems Eng.*, Vol. 2014, 9 pages (2014), http://dx.doi.org/10.1155/2014/481791.

Yang X.S., "Firefly algorithms for multimodal optimization", in stochastic algorithms: Foundations and applications, *Lect. Notes Comp. Sci.*, Vol. 5792, pp. 169-178 (2009).

Yang X.S., *Nature Inspired Metaheuristic Algorithm*, Second ed., in: Firefly algorithm, University of Cambridge, United Kingdom, Luniver Press, pp. 81-89 (2010).

Yang, X. S., "Flower pollination algorithm for global optimization", in: Unconventional Computation and Natural Computation, *Lect. Notes Comp. Sci.* Vol. 7445, pp. 240–249 (2012).

Yang X.S, He X., "Firefly algorithm recent advances and applications", *Int. J. Swarm Intel.* Vol. 1, No. 1, pp. 36-50 (2013).

Yang, X.S., Karamanoglu, M., He, X., "Flower pollination algorithm: A novel approach for multi objective optimization", *Eng. Opt.*, Vol. 46, pp. 1222-1237 (2014).

## NOMENCLATURE

| | |
|---|---|
| $A$ | Randomization parameter |
| $A$ | Lamp Current |
| $B$ | Attractiveness of a firefly |
| $\beta_0$ | Attractiveness of a firefly at a distance $r = 0$ |
| $B$ | Pulse frequency |
| $\Gamma$ | Light absorption coefficient/ Contrast of attractiveness |
| $\Gamma$ | Characteristic distance |
| $C$ | Pulse width |
| $D$ | Cutting speed |
| $\epsilon$ | Local random walk drawn from a uniform distribution in [0, 1] |
| $\in_i$ | Vector of random numbers drawn from Gaussian or normal distribution |
| $g_*$: | Current best solution found among all solutions at the current generation/iteration |
| $I(r)$ | Light intensity of a firefly at a distance $r$ |
| $I_S$ | Light intensity of a firefly at source |
| $L$ | Strength of pollination |
| $M$ | Number of local optima of an optimization problem |
| $N$ | Number of fireflies or population size |
| $p$ | Switch probability |
| $R$ | Distance between two fireflies |
| $r_{ij}$: | Distance between two fireflies i,j |
| s | Large step |

| | |
|---|---|
| $\tau(\gamma)$ | Standard gamma function |
| $x_i$ | Position of firefly i in $x$ coordinate |
| $x_j$ | Position of firefly j in $x$ coordinate |
| $x_{i,k}$ | $k_{th}$ component of spatial coordinate of $x_i$ |
| $x_{j,k}$ | $k_{th}$ component of spatial coordinate of $x_j$ |
| $x_i^t$ | Solution vector $x_i$ for pollen i at iteration t |
| $x_j^t$, $x_k^t$ | Pollens from the different flowers of the same plant species |
| $y_i$ | Position of firefly i in $y$ coordinate |
| $y_j$ | Position of firefly j in $y$ coordinate |
| $W$ | Weightage |
| $Z$ | Output of objective function for multi objective optimization |
| $Z_{min}$ | Optimized value in case of multi-objective optimization |
| BW | Burr width |
| CD | Channel Depth |
| FA | Firefly Algorithm |
| FPA | Flower pollination algorithm |
| MAD | Mean absolute deviation |
| RSM | Response Surface Methodology |