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Performance Prediction of a Series-parallel and Multi-Product Production Line with Unreliable Machines, Finite Buffers and Nonconforming Products

Changjun Li* and Bo Li**

Keywords: performance prediction, series-parallel and multi-product production line, queueing model, approximation method

ABSTRACT

It is well known that performance prediction is extremely important to improve the flexibility of a series-parallel and multi-product production line with unreliable machines, finite buffers, and nonconforming products, but how to model and predict its performance is facing great challenges. A novel approximation iteration method based on a discrete state Markov chain and queueing model is developed. The equivalent machines processing rates in isolate state are determined by applying variability transition parameters via Markov chain firstly; Then the throughput of whole production line is determined according to the last workstations effective processing rate in steady-state, which is approximately predicted via queueing modelling; Thirdly, the unknown parameter throughput $X^{(k)}$ is computed by developing an approximate iterative algorithm procedure, then a modified queueing model is used to compute the performance. Finally, to assess the effectiveness of the proposed method, extensive numerical experimental results from the predictive approximation are compared to simulation models, which proves it accurate and believable.

INTRODUCTION

The series-parallel and multi-product production line (SPMPPL) is among the most popular production system being studied (Li and Peng, 2014). As manufacturing systems become more and more complex, their structures, models and analyses become equally complicated. Structurally, a serial queueing system consists of a set of machines (or servers) arranged consecutively with buffers separating each two adjacent machines (see Fig. 1). Although the structure of the system is simple, its behavior is quite complex (Li et al., 2015). Because in this SPMPPL, failures can occur at any given time, and the machines in the production line are highly coupled and as soon as any machines break down, all other machines in the SPMPPL may be forced down at the next workstation soon. In a SPMPPL, buffers are used to attenuate the impact of these random factors. Due to the unreliable machines, finite buffers, and nonconforming products, performance of the SPMPPL become more complicated to be predicted. Most of the existing studies in the area of manufacturing performance prediction have been focused on classic production lines that are traditionally assumed to produce only single product type (Farshad and Walid, 2015). Few of published studies about SPMPPL have considered unreliable machines, finite buffer capacity, and nonconforming products simultaneously, that are extremely common and significant in modern manufacturing factory. This fact motivates us to present a method for the throughput prediction in a SPMPPL, which will aid in obtaining more performance indicators, i. e., starvation probabilities, blocking probabilities, queue length, waiting time, productivity loss, load intensity of workstation, and total production cycle time. The proposed approximate model extends the work of Farshad and Walid (2015) from serial queues multi-product to serials-parallel queues multi-product and nonconforming products scenario. It is different from previous approaches in three aspects. Firstly, the proposed approach focuses on the performance prediction rather than evaluation. Secondly, the approximately predictive model is derived from a $M/M/1/m$ queueing system. Hence, the nonconforming products, which commonly exist in practical production lines, can be considered in our model. Thirdly, existing methods are either highly inaccurate, or are exact but not simple to implement. Our approach studies general line and develops a simple iterative procedure that can be easily understood and conducted.

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* *Doctoral student, School of Aeronautics and Astronautics, University of Electronic Science and Technology of China, Chengdu 611731, P. R. China*

** *Professor, School of Aeronautics and Astronautics, University of Electronic Science and Technology of China, Chengdu 611731, P. R. China*

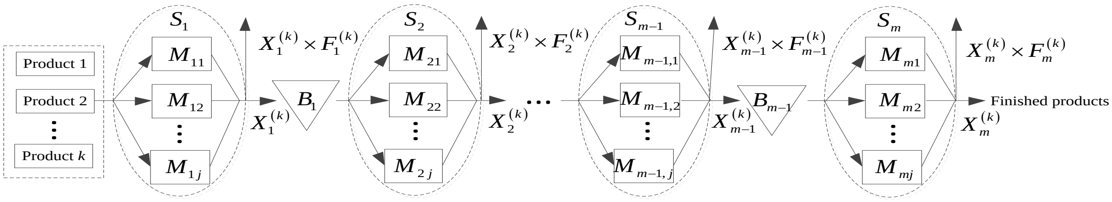

Fig.1 SPMPPL with finite buffers and nonconforming products

The paper is organized as follows. Section LITERATURE REVIEW presents a review of previous works based on prediction models. Section PRELIMINARIES presents some preliminaries including the research problems, assumptions, and notations. Section SOLUTION METHODOLOGY develops to approximately predict the throughput, which tackles the upstream and downstream workstation coupling problem by employing *M/M/*1*/m* queueing model and decomposition method to approximate general processing rate. Then other performance indicators are computed by the help of the general processing rate. Section NUMERICAL EXPERIMENT provides a numerical experiment for proposed approximation method on several scenarios to demonstrate its accuracy, strength and limitations. The proposed method is compared with the results obtained by ARENA simulation. Finally, Section CONCLUSIONS contains some concluding remarks and gives suggestions for further research.

LITERATURE REVIEW

In the existing literature, most of the models have been developed to predict the performance of production line. Although many important results have been achieved, there are still some limitations in the literature and practice.

Queueing models

Queueing models developed so far have proved to be efficient in predicting the performance of traditional serial production line (SPL). Meester and Shanthikumar (1990) proposed using a sample path recursion for the departure processes from the multi-stage of the tandem queueing system to predict the concavity of the throughput of tandem queueing systems with finite buffer storage space. Robinson and Giglio (1999) developed an approximate model based on *M/M/c* queues to predict the rework rate of a two-operation time-constrained system. Buzacott, Liu, and Shanthakumar (1995) proposed an approximation for a two station flow line with *GI/GI/*1*/N* stopped arrival queues to predict throughput. Smith (2013) presented a two-moment approaches to calculate the state probability distribution and predict the performance by *M/G/c/K* queue. Damodaran and Hulett (2012) developed analytical approximations of *G/G/c* queueing model to predict the performance of a manufacturing systems with general distributions, job failures, and parallel processing. Farshad and Walid (2015) present a hybrid model based on a finite Markov chain and $M^X/G^{0,\,C}/1/K$ queueing model to compute the performance, including the queue length, waiting time, and departing batch size, for a non-homogenous asynchronous line with finite buffer capacity. Tempelmeier and Bürger (2001) proposed a *GI/G/*1*/N* max queueing model with finite buffers to predict the performance of SPL. In their developed model, stochastic processing times as well as machine breakdowns, and nonconforming products are considered.

Based on the reviewed literature above, most of them focus on a series-parallel production line using queueing models, but ignore the multi-product scenario, which needs simultaneously consider random failure time, random repair time, finite buffers, and nonconforming products which are extremely common in factory. In particular, most of them are mainly applied to performance evaluation not prediction.

Markovian models

Previous studies (Alexandros and Chrissoleon, 2009, Anwarul, Farshad, and Walid, 2010, Feng, Zheng, and Li, 2011, Wu et al., 2016, Yang, 2009) related Markovian models have mainly focused on system with serial machines and most of the published models were limited to short lines with a maximum of three workstations in series. Anwarul, Farshad, and Walid (2010) developed a general Markovian method to predict the throughput for continuous material production system with two processing workstations and an intermediate finite buffer capacity. Tan and Gershwin (2009) proposed an approximation method based on discrete state Markov chain to predict the throughput of a series-parallel line with finite buffer capacity. Perlman, Elalouf, and Bodinger (2014) constructed a continuous-time Markov chain model to predict the throughput in a line consisting of two unreliable machines and one finite buffer. Kim and Morrison (2014) proposed exact Markovian models to predict and analyse the throughput in SPL with reliable machines, deterministic processing time, and random setup time.

Unfortunately, most of the methods in the area of Markovian models that aim at parallel processing machines or focus on single station not series, are very difficult to obtain relatively exact solutions of serial lines with more than two machines. The reason is that the number of production states in the Markov chain, increases exponentially with the increasing number of machines and inter-workstation buffer sizes (Schwarz and Epp, 2015). For example, a production line with four machines and inter-workstation of capacity 3 gives rise to a Markov chain with 19402 states (Hillier and So, 1991). Therefore, applying exact Markovian models to address the complexity of exact mathematical solutions is extremely challenging.

Data models

Tan and Gershwin (2009) proposed a prediction model base on simulation data to predict and analyse the throughput of five balanced and serial stations line. Baker, Powell, and Pyke (1994) developed the procedures based data distribution to approximately predict the throughput of unbalanced, unbuffered, and three-station serial lines. Tirkel (2011) proposed to apply machine learning and data mining methods to predict the cycle time of a single lot in wafer fabrication line. Yang (2013) presented a data-driven continuous fluid flow approach, founded on nonlinear system dynamics principles modelling line to predict instantaneous throughput.

In spite of the capability of solving complex problems of mathematical modelling and solution, most of these methods applied to performance prediction can obtain no more than one indicator, such as total production cycle time, waiting time, or machine load intensity. Furthermore, the data models need large history data, and the original data must be accurate and believable. Therefore, the data modes still have some limitations in exact performance prediction of the new design production lines at present.

Simulation models

Sawomir and Justyna (2015) applied PLANT simulation software to predict throughput and provide a product life span analysis of automatic production lines with different capacities of intermediate buffers. McNamara, Shaaban, and Hudson (2013) used PROMODEL simulation software to predict the throughput, idle (force down) time, and average buffer capacities in unreliable unpaced lines. Walid (2003) presented a simulation model to predict the maximum contribution of buffers on the overall performance considered setup time, multi-product, unreliable four workstations, and three intermediate buffers line. Said and Ismail (2013) developed a simulation model using ARENA simulation software to predict and analyse the production line layout of the manufacturing system design to identify the key performance.

However, the simulation often fails to provide a comprehensive and quantitative understanding of the relationships between performance measures and system parameters (Li et al., 2015, Yang, 2013). For long production lines, simulation model development and experimentation are usually time-consuming, and a long execution time is needed to obtain statistically valid results. Especially, the use of simulation is not suggested when a problem can be solved analytically (Banks et al., 2010, Schwarz and Epp, 2015).

Other models

To approximate prediction for throughput in the production lines with parallel-machine workstations and blocking, Shin and Moon (2016) developed a decomposition modelling method using the subsystems with three workstations including virtual station and two buffers between workstations. Veeger et al. (2011) proposed an aggregate modelling method to predict the cycle time distribution of integrated processing workstations. Hulett and Damodaran (2011) proposed a hybrid model using *M/M/*c queueing network model combining with parametric decomposition approach without considering the unreliable machines. Dhouib, Gharbi, and Landolsi (2009) presented a homogenization and an aggregation approach converting the discrete operating behavior of each original machine into a continuous and steady-state mode to predict the throughput of multi-product and unreliable lines without considering buffer capacities.

Many studies, as reviewed above, are no feasible solutions for the SPMPPL with more than two workstations in series and one buffer in the middle (Farshad and Walid, 2015). Most existing several methods in the literature predict the performance only for a single-product line. The published approaches either only consider the series-parallel line, or only consider the multi-product. Besides, to the best of our knowledge, there is no published literature to predict this performance in a SPMPPL considering unreliable machines, finite buffers, and nonconforming products simultaneously. This fact motivates us to present a prediction method for the throughput in a SPMPPL with above properties. It will ultimately enable us to obtain the key performance indicators, including starvation probabilities, blocking probabilities, queue length, waiting time, productivity loss, machine load intensity, and total production cycle time.

PRELIMINARIES

The SPMPPL in the processing is highly coupled, because an asynchronous line with finite buffers, upon processing completion at a workstation, departing products join the WIP buffer at the next workstation, if that space is available. Otherwise, such products are typically held at the current workstation until space becomes available in the next buffer. During that time, the workstation can not process any other products. The schematic diagram of the SPMPPL with nonconforming products is presented in Fig. 1 illustrating the SPMPPL to be investigated, which is a production line composed of *M* workstations and *M*-1 buffers. Besides, for various factors, a SPMPPL is unavoidable to produce nonconforming products at every workstation. These nonconforming products will be found in time through a special inspection station, and immediately depart the SPMPPL. The total number of these conforming products will be gradually decreased with the time of passing through the workstations. Due to these complex coupling properties, how to model and predict its performance is facing great challenges. Therefore, a method of performance prediction for a SPMPPL, as mentioned above, is thus needed.

Assumptions

1) The last workstation of the line will never be blocked before all types of the product batches processing are complete.
2) This processing sequence is assumed to be the same for each type of products.
3) The mean failure rate λ_{ij} and mean repair rate μ_{ij} are exponentially distributed.
4) It follows a first-come-first service (FCFS) rule.
5) The setup time of the whole SPMPPL is deterministic.
6) Products arrive as Poisson flow.
7) The time to failure and repair of each machine are assumed to exponentially distributed.
8) Failures happen only when a machine is in processing.
9) The first setup is required in the whole production line, when all of a product type are completed and before another type of product is started (see Fig. 2).
10) Do not consider the products transfer time between workstations and included it in the processing time.
11) All machines are statistically independent in failures, repairs, and processing time from each other.
12) The product inter-arrival time is given.

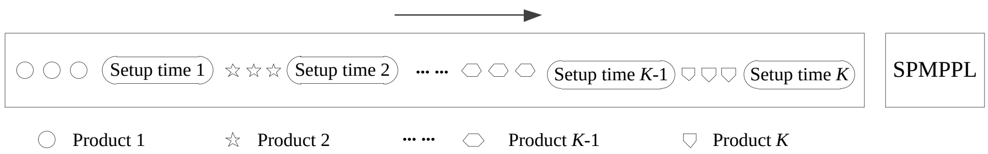

Fig. 2 Entered sequence of multi-product products with setup time

Notations

Throughout this paper, the notations used in the model are presented in Table 1.

Table 1. Notations

Notations	Description
k	A subscript k indicates a parameter that describes product type, k=1, 2, …, K (k is the number of all types of product).
i	A subscript i indicates workstation in series, where i=1, 2, …, m.
j	A subscript j indicates number of machines in parallel at workstation i where j=1, 2, .., n.
S_i	Workstation i, that is consisted one or several parallel machines.
λ_{ij}	Failure rate of machine M_{ij}.
μ_{ij}	Repair rate of machine M_{ij}.
e_{ij}	Efficiency of M_{ij} processing independent.
$v_{ij}^{(k)}$	Product type k average processing rate of M_{ij}.
$P_{ij}^{(p)}$	Probability that machine M_{ij} is processing.
$P_{ij}^{(p)}$	Probability that machine M_{ij} is idle.
$P_{ij}^{(u)}$	Probability that machine M_{ij} is up.
$P_{ij}^{(d)}$	Probability that machine M_{ij} is down.
$P_{ij}^{(s)}$	Probability that machine M_{ij} is starved.
$P_i^{(ks)}$	Probability that S_i is starved when it is processing product type k.
$P_i^{(kb)}$	Probability that S_i is blocked when it is processing product type k.
$x_i^{(k)}$	Equivalent processing rate of product k at workstation S_i in isolate.
$P_i^{(s)}$	Probability that S_i is starved.
$P_i^{(b)}$	Probability that S_i is blocked.
$X_i^{(k)}$	Equivalent processing rate of product type k at S_i in line.
$X^{(k)}$	Generate processing rate (i.e., throughput) of product type k in line.
THR_{1ki}	Average throughput of product type k at S_i.
$BP^{(k)}$	Processing time of product type k.
$SU^{(k)}$	Setup time, that is required to process another product type k.
$BC^{(k)}$	Processing time of product type k pluses setup time.
B_i	Maximum capacities of buffer size before S_i. (see Fig. 3).
$f_i^{(k)}$	Nonconforming ration of product type k produced at S_i.
ζ	Index set of quality inspection workstations.
H	An element in the set of ζ.
b_i	Capacity of buffer size before S_i.
$\rho_i^{(k)}$	Load intensity of S_i when it is processing the product type k.
$\bar{\rho}_i^{(k)}$	Average load intensity of S_i when it is processing the product type k..
$W_i^{(k)}$	Average waiting time of product type k at S_i.
$L_i^{(k)}$	Queue length of product type k before S_i.
$P_i^{(loss)}$	Productivity loss ratio of S_i.

SOLUTION METHODOLOGY

In this section, an approximation iteration method based on Markov chain and queueing model is presented to solve the problems above mentioned. The entered sequence of multi-product products with setup time is shown in Fig. 2. Note that, different products need a setup time to prepare the workstations before they enter the line, but the same type of products does not need the setup time. A set of parallel machines at a workstation is replaced by an equivalent workstation. The equivalent SPMPPL (ESPMPPL) considering nonconforming products is shown in Fig. 3. There are m-workstation (S_i) and m-1-buffer (B_i) in the middle. The buffer capacities are finite in ESPMPPL. The Poisson flow is also Poisson flow after the processing of fork-join, or filter, according to the well-known limit theorems for Poisson (or general) stochastic processes (Dallery, Liu, and Towsley, 1997). Then an iteration algorithm procedure with $M/M/1/m$ queueing model is developed to compute $X^{(k)}$ by estimation with inline. Finally, the queueing model is applied to compute the performance once more, and the key performance indicators of the ESPMPPL are obtained eventually.

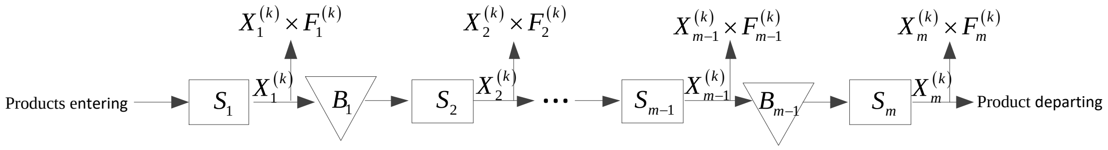

Fig. 3 ESPMPPL with nonconforming products

Machines in isolate

A SPMPPL can be converted into ESPMPPL (see Fig. 3), if the equivalent processing rates of each workstation in isolate are obtained. The general and deterministic processing times and operation dependent failures of the machines are assumed with exponentially distributed failures and repairs. To obtain the equivalent processing rate $x_i^{(k)}$ of the equivalent machines, at steady state, Gershwin (1994) and Anwarul, F., and Walid (2010) presented the following equations:

Probability that machine M_{ij} is down

$$P_{ij}^{(d)} = \frac{\lambda_{ij}}{\mu_{ij} + \lambda_{ij}} \tag{1}$$

Probability that machine M_{ij} is up

$$P_{ij}^{(u)} = \frac{\mu_{ij}}{\mu_{ij} + \lambda_{ij}} \tag{2}$$

Balance equation

$$P_{ij}^{(d)} \times \mu_{ij} = P_{ij}^{(u)} \times \lambda_{ij} \tag{3}$$

Consequently, efficiency of machine M_{ij} in isolation

$$e_{ij} = \frac{\mu_{ij}}{\lambda_{ij} + \mu_{ij}} \tag{4}$$

The processing rate of product type k at single machine M_{ij} be defined as:

$$x_{ij}^{(k)} = v_{ij}^{(k)} \times e_{ij} \tag{5}$$

Thus, $x_i^{(k)}$ is given as:

$$x_i^{(k)} = \sum_{j=1}^{n} x_{ij}^{(k)} = \sum_{j=1}^{n} (v_{ij}^{(k)} \times e_{ij}) \tag{6}$$

Machines in ESPMPPL

Unlike a machine in isolate, the machines may be idle, when they are in the SPMPPL. An idle machine is in working order but unable to operate because it has no products to process (starved) or cannot transfer its completed products to the downstream machine or buffer (blocked). The status of machine M_{ij} performing in a SPL can also be described by a discrete state Markov chain where the states of M_{ij} are busy, idle, and down. It is shown in Fig. 4.

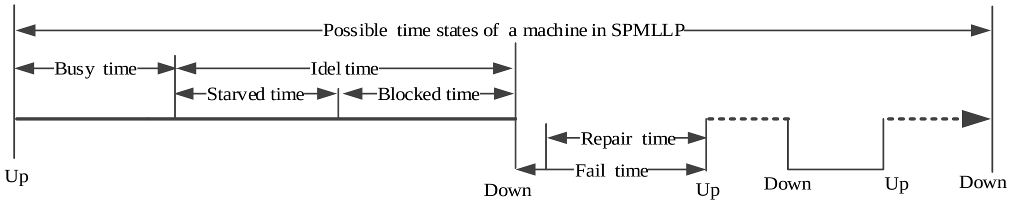

Fig. 4 Possible time states of a machine in SPMPPL

As observed, equivalent processing rate of S_i should not only consider the workstation own states but also the coupling properties of its upstream and downstream workstations, both of them can limit the processing rate of each workstation in the ESPMPPL. The state of the single machine can be up or down time, and the up time can be divided to two states: processing and idle, namely:

$$P_{ij}^{(u)} = P_{ij}^{(p)} + P_{ij}^{(i)} \tag{7}$$

The state probabilities of processing, down, and idle for single machine M_{ij} is given by:

$$P_{ij}^{(p)} + P_{ij}^{(d)} + P_{ij}^{(i)} = 1 \tag{8}$$

The steady-state equation for machine M_{ij} at down state for this Markov chain is:

$$P_{ij}^{(p)} \times \lambda_{ij} = P_{ij}^{(d)} \times \mu_{ij} \tag{9}$$

From Eq. (7), (8), and (9), $P_{ij}^{(p)}$ can be written as:

$$P_{ij}^{(p)} = e_{ij} \times \left(1 - P_{ij}^{(i)}\right) \tag{10}$$

Therefore, to improve prediction accuracy, the $x_{ij}^{(k)}$ in Eq. (5) is redefined by:

$$x_{ij}^{(k)} = v_{ij}^{(k)} \times e_{ij} \times \left(1 - P_{ij}^{(ki)}\right) \tag{11}$$

Consequently, the equivalent processing rate $x_i^{(k)}$ of S_i is written:

$$x_i^{(k)} = \sum_{j=1}^{n} x_{ij}^{(k)} = \sum_{j=1}^{n} \left(v_{ij}^{(k)} \times e_{ij} \times \left(1 - P_{ij}^{(ki)}\right)\right) \tag{12}$$

Considering the starvation and blocking state of the upstream and downstream workstations in the ESPMPPL, it should be modified as follow:

$$X_i^{(k)} = x_i^{(k)} \times \left(1 - P_i^{(ks)}\right) \times \left(1 - P_i^{(kb)}\right) \tag{13}$$

Nonconforming products in ESPMPPL

When considering the nonconforming product scenario, based on Tempelmeier and Bürger (2001), the following equations are considered in this paper. The processing sequence of product type k is from station S_h to station S_H (with $h<H$). $f_h^{(k)}$ is the proportion of nonconforming product type k produced at the workstation S_h , so the proportion of conforming products produced at S_h is $1 - f_h^{(k)}$. The proportion of conforming products at S_H is $\prod_h^H \left(1 - f_h^{(k)}\right)$. The proportion of nonconforming products produced at workstation S_H is $1 - \prod_h^H \left(1 - f_h^{(k)}\right)$. So, $F_H^{(k)}$ can be computed by:

$$F_H^{(k)} = \begin{cases} 1 - \prod_h^H \left(1 - f_h^{(k)}\right) & \forall h, H \in \zeta \wedge h < H \le m \\ 0 & \forall h, H \notin \zeta \end{cases} \tag{14}$$

Thus, $X_i^{(k)}$ can be written:

$$X_i^{(k)} = x_i^{(k)} \times \left(1 - P_i^{(ks)}\right) \times \left(1 - P_i^{(kb)}\right) \times \left(1 - F_H^{(k)}\right) \tag{15}$$

Note that the unknown parameters $P_i^{(ks)}$ and $P_i^{(kb)}$ can be calculated by Eqs. (19) and (21), respectively.

In order to compute the generate processing rate $X^{(k)}$ easily, approximately define $X^{(k)}$ as the processing rate of the last workstation S_m. As assuming the last workstation S_m is never blocked, so $P_m^{(kb)} = 0$. Then the generate processing rate is defined by:

$$X^{(k)} = X_m^{(k)} \times \left(1 - P_m^{(ks)}\right) \times \left(1 - F_m^{(k)}\right) \quad (16)$$

Approximate iteration algorithm

In this subsection, an approximate iteration algorithm is provided to obtain the generate processing rate. According to the queueing model, the production system will eventually enter the steady state, a tolerance value ε is given as a stopping rule between two successive iterations. The unknown $X^{(k)}$ is solved by developing an approximate iteration algorithm procedure (see Table 2).

Table 2. The proposed algorithm procedure

Algorithm procedure	
1	Procedure 1 (Initialization)
1.1	Given initial processing rate $X_{old} = (x_1, x_2, ..., x_m)$, buffer capacity B_i, ratio of nonconforming product F_i;
1.2	Set $P_m^{(b)} = 0$, $P_1^{(s)} = 0$, $X_{new} = (0,0,...,0)$
1.3	Define tolerance ε =0.0001;
1.4	For $i = 1, 2, ..., m$;
1.5	{
1.6	Calculate $P_i^{(s)}$ approximately with *M/M/1/m* queueing model (see Eq. (19);
1.7	Calculate $P_i^{(b)}$ approximately with *M/M/1/m* queueing model (see Eq. (21));
1.8	Calculate $X_{new(i)}$ by Eq. (16);
1.9	}
1.10	End (for)
1.11	End Procedure 1
2	Procedure 2 (Iteration)
2.1	While abs ($X_{new(m)}$ - $X_{old(m)}$) <= ε ; (Stopping rule)
2.2	{
2.3	Set $X_{old} \leftarrow X_{new}$;
2.4	For $i = 1, 2, ..., m-1$;
2.5	Calculate $P_i^{(s)}$ approximately with *M/M/1/m* queueing model (see Eq. (19));
2.6	Calculate $P_i^{(b)}$ approximately with *M/M/1/m* queueing model (see Eq. (21));
2.7	If $i \in h$
2.8	$X_{new(i)} = X_{old(i)} \times \left(1 - P_i^{(b)}\right) \times \left(1 - P_i^{(s)}\right) \times \left(1 - F_i\right)$
2.9	Else
2.10	$X_{new(i)} = X_{old(i)} \times \left(1 - P_i^{(b)}\right) \times \left(1 - P_i^{(s)}\right)$
2.11	End if
2.12	End for
2.13	}
2.14	End (repeat)
2.15	Output X_{new}
2.16	End Procedure 2

Performance calculation

Computing starvation and blocking probabilities: Consider two stations and one buffer at a machine: For state 0, it has $X_i p_i^{(0)} = X_{i+1} p_i^{(1)}$, therefore $p_i^{(1)} = \frac{X_i}{X_{i+1}} p_i^{(1)}$; For state 1, it has $X_i p_i^{(1)} = X_{i+1} p_i^{(2)}$, therefore $p_i^{(2)} = \frac{X_i}{X_{i+1}} p_i^{(1)} = \rho_i^2 p_i^{(0)}$. For state B_{i-1}, it has $X_i p_i^{(B_i - 1)} = X_i p_i^{(0)}$, therefore $p_i^{(B_i)} = \frac{X_i}{X_{i+1}} p_i^{(B_i - 1)} = \rho_i^{B_i} p_i^{(0)}$. According to regularity, $1 = \sum_{b_i=0}^{B_i} p_i^{(b)} = \sum_{b_i=0}^{B_i} \rho_i^{b_i} p_i^{(0)}$, therefor $p_i^{(b_i)} = \frac{1-\rho}{1-\rho^{B_i+1}} \times \rho^{b_i}$, where b_i=1, …, B_i.

If $\rho_i^{(k)} = 1$, it has:

$$P_i^{(0)} = \left(\sum_{b_i=0}^{B_i} \rho_i^{b_i}\right) = \frac{1}{B_i + 1} \quad (17)$$

Otherwise, (namely $\rho_i^{(k)} \neq 1$)

For: $1 = \sum_{b_i=0}^{B_i} P_i^{(b)} = \sum_{b_i=0}^{B_i} \rho_i^{b_i} P_i^{(0)} = \frac{1-\rho_i^{B_i+1}}{1-\rho_i} P_i^{(0)}$, The starvation probabilities can be obtained:

$$P_i^{(0)} = \frac{1-\rho_i}{1-\rho_i^{B_i+1}} \quad (18)$$

Note that the ρ_i which is defined by $\rho_i = X / x_i$. Then, $P_i^{(0)}$ can be written:

$$P_i^{(0)} = \begin{cases} \frac{1}{B_i + 1}, & \rho_i = 1 \\ \frac{1-\rho_i}{1-\rho_i^{B_i+1}}, & \rho_i \neq 1 \end{cases} \quad (19)$$

Because,

$$P_i^{(b_i)} = \frac{1-\rho_i}{1-\rho_i^{b_i+1}} \rho_i^{b_i}, \quad b_i = 1, 2, ..., B_i \quad (20)$$

When the arrival products quantities reach to the maximum buffer capacity B_i, define blocking probability as $P_i^{(B_i)}$ at workstation S_i. Thus, $P_i^{(B_i)}$ can be written as follows:

$$P_i^{(B_i)} = \frac{1-\rho_i}{1-\rho_i^{B_i+1}} \rho_i^{B_i} \quad (21)$$

Computing queue length: The queue length $L_i^{(k)}$ at workstation S_i is a key indicator to predict the ESPMPPL, which can be used to design the buffer capacity in serials and machines number in parallelism, it is computed as follows:

$$L_i^{(k)} = \frac{\rho_i^{(k)}}{1-\rho_i^{(k)}} - \frac{B_i \times \rho_i^{(k)B_i+1} + \rho_i^{(k)}}{1-\rho_i^{(k)B_i+1}} \quad (22)$$

A detailed derivation of $L_i^{(k)}$ is given in Appendix.

Computing waiting time: when the ESPMPPL is processing product type of k, because of the effect arrival rate $X_i^{(k)} = x_i^{(k)} \times \left(1 - P_i^{(kB_i)}\right)$, according to Eq. (21) and the Little's law $W_i^{(k)} = L_i^{(k)} / X_i^{(k)}$, the average waiting time $W_i^{(k)}$ at S_i can be obtained as below:

$$W_i^{(k)} = \frac{\rho_i^{(k)}}{X_{i+1}^{(k)} \times \left(1-\rho_i^{(k)}\right)} - \frac{B_i \times \rho_i^{(k)B_i}}{X_{i+1}^{(k)} \times \left(1-\rho_i^{(k)B_i}\right)} \quad (23)$$

A detailed derivation of $W_i^{(k)}$ is given in Appendix.

Computing productivity loss: Productivity loss are not inevitable in practice manufacturing facility, they are potentially incurred whenever machines are idle (starved or blocked), due to machine failures or bottlenecks blocking from excessive accumulation of inventories between workstations. The randomness is main due to random processing time, as well as random failure rate λ_{ij} and random repair rate μ_{ij}. To obtain the productivity loss ratio at workstation S_i, when machines are at production line, the loss ratio includes two parts: starvation and blocking. Define: $P_i^{(loss)} = P_i^{(B_i)} + P_i^{(0)}$, according to Eq. (19) and (21), the final productivity loss ratio is obtained.

$$P_i^{(loss)} = \begin{cases} \dfrac{\rho_i^{B_i} - \rho_i^{B_i+1}}{1-\rho_i^{B_i+1}} + \dfrac{1}{B_i+1}, \rho_i = 1 \\ \dfrac{1-\rho_i+\rho_i^{B_i} - \rho_i^{B_i+1}}{1-\rho_i^{B_i+1}}, \rho_i \neq 1 \end{cases} \tag{24}$$

Computing load intensity of workstation: Load intensity, namely average processing load intensity $\bar{\rho}_i^{(k)}$, which is a significant indicator in analysing the machines working status. For $\bar{\rho}_i^{(k)} = X_i^{(k)} / x_{i+1}^{(k)}$, the machine load intensity $\bar{\rho}_i^{(k)}$ can be calculated according to queueing model of *M/M/1/m*.

$$\bar{\rho}_i^{(k)} = \frac{\rho_i^k - \rho_i^{B_i+1}}{1-\rho_i^{B_i+1}} \tag{25}$$

To simplify Eq. (25), it can obtain

$$\bar{\rho}_i^{(k)} = 1 - p_i^{(k0)} \tag{26}$$

A detailed derivation of $\bar{\rho}_i^{(k)}$ is given in Appendix.

Computing total product cycle time: A model based on the summation of total cycle time *TC* method proposed by Farshad and Walid (2015), is modified appropriately according to requirements.

$$TC = \sum_{k=1}^{K} BC^{(k)} \tag{27}$$

And,

$$BC^{(k)} = SU^{(k)} + BP^{(k)} \tag{28}$$

By definition,

$$BP^{(k)} = T_{k\_out} - T_{k\_in} \tag{29}$$

$$BP^{(k)} = \sum_{i=1}^{m} THR_{1i}^{(k)} + \left(J^{(k)} - 1\right) / X^{(k)} \tag{30}$$

Then, $BC^{(k)}$ can be written:

$$BC^{(k)} = SU^{(k)} + \sum_{i=1}^{m} THR_{1i}^{(k)} + \left(J^{(k)} - 1\right) / X^{(k)} \tag{31}$$

Therefor,

$$TC = \sum_{k=1}^{K} SU^{(k)} + \sum_{k=1}^{K}\sum_{i=1}^{m} THR_{1i}^{(k)} + \sum_{k=1}^{K}\left(J^{(k)} - 1\right) / X^{(k)} \tag{32}$$

Now, in order to obtain $THR_{1i}^{(k)}$, which is calculated as follows:

$$THR_{1i}^{(k)} = \sum_{i=1}^{m} \frac{1}{X_i^{(k)}} \tag{33}$$

NUMERICAL EXPERIMENT

In this section, a numerical example will be performed to validate the accuracy of the proposed method. Due to the novelty of our research object and prediction model, no existing method is found to compare to the proposed method. Therefore, a popular computer simulation using ARENA software in the area of manufacturing is used to provide a comparison. It is useful in verifying model assumptions and propositions in capacity planning and scheduling controls. Above all, it can be modelled and adjusted precisely to meet various experimental purposes (Shanthikumar, Ding, and Zhang, 2007). In this paper, ARENA is used to simulate the SPMPPL, and MATLAB is used to compute the proposed method.

Design of experiments

The SPMPPL is composed of 5 workstations with intervening buffers. The first, third, and fifth workstation consist of one machine each, the second and fourth workstation contain two and three machines in parallel, respectively. The processing rate of the machine in different workstations may be not the same. The quantity of products waiting to be processed: type 1 is 600, type 2 is 550, type 3 is 700, the setup time is 20 seconds, 15 seconds, 30 seconds, respectively. The default buffer capacities used before every workstation in the experiments are 5, unless otherwise specified. In setting options of ARENA, the simulation is terminated until all products are completed from first workstation enter the line and depart from the last workstation finally. ‘Hours per day’ is 24 hours, ‘Base time units’ is seconds. The time between arrivals of three types of product at the first workstation obeys Poisson distribution. Before running the simulation model, the initial system is empty, namely there is no product in the model and all machines are idle. This is not in harmony with real condition. Therefore, it is necessary to consider the results after the moment that system reaches the steady state. After completing an initial run by ARENA, it can be observed that the graph of warm-up time shows the stability at about 6, 000 seconds. To lower the risk, a 9, 000 seconds warm-up period is considered. To calculate the sufficient number of replications, the below formula (Ahmed, 1999, Kamrani et al., 2014, Toledo et al., 2003) was used.

$$N_\psi = \left(\frac{s(\psi) \times t_{\psi-1,1-\alpha/2}}{\bar{\chi}(\psi)\varepsilon}\right)^2 \tag{34}$$

where N_ψ indicates the number of replications, $s(\psi)$ indicates the data standard deviation, t is the test statistic obtained from t-table, ψ is the number of initial replications that is assumed to be 5, α is the confidence interval as 90%, $\bar{\chi}(\psi)$ is the allowable percentage error. The allowable error percentage of 10% with $t_{4,0.95}$ equals 2.132. Therefore, an initial sample consisting of 10 replications is run.

All machines in the three types of the SPMPPL have the same λ_{ij} and μ_{ij}, but different $x_{ij}^{(k)}$ in each same workstation. The nonconforming ration of three types of product in inspecting workstation

$f^{(1)}$=(0.0112, 0, 0.0128, 0, 0.0152), $f^{(2)}$=(0.0212, 0, 0.0150, 0, 0.0120), $f^{(3)}$=(0.0212, 0, 0.0150, 0, 0.0120). The condition field 'Hold module' dialog box is set to 'scan for condition':

$$NQ(Workstation\ processing\ \ i.\ Queue) < B_i$$

It is used to test whether buffer space is available in the ESPMPPL. To validate the accuracy of the proposed method, Scenario 1: workstation parameters of the ESPMPPL are predicted at each single station. Scenario 2: load intensity of workstation change trend of the ESPMPPL is predicted with increasing buffer capacities. Scenario 3: the whole performance parameters of the ESPMPPL are predicted for the whole line. Following sets of the data (see Table 3) are used for the experiment.

Table 3. Parameters for numerical experiment.

Parameters	Machine (M_{ij})							
	11	21	22	31	41	42	43	51
μ_{ij}	0.0250	0.0200	0.0200	0.0189	0.0220	0.0220	0.0220	0.0250
λ_{ij}	0.0010	0.0010	0.0010	0.0008	0.0012	0.0012	0.0012	0.0015
$v_{ij}^{(1)}$	1.2500	0.6500	0.6500	1.3250	0.5580	0.5580	0.5580	1.2050
$v_{ij}^{(2)}$	1.7500	0.8745	0.8745	1.8000	0.5805	0.5805	0.5805	1.8050
$v_{ij}^{(3)}$	1.5500	0.8005	0.8005	1.6000	0.5050	0.5050	0.5050	1.7088

The default time unit used in the experiments is seconds, unless otherwise specified. The percentage difference in prediction results are calculated as follows in this paper.

$$Error(\varepsilon') = \frac{|approximation - simulation|}{simulation} \times 100\% \qquad (35)$$

To study the accuracy of the prediction results obtained in Section 4, the indicators of performance of production line are compared, such as the throughput (using Eq. (16)), starvation probabilities (using Eq. (19), blocking probabilities (using Eq. (21)), queue length (using Eq. (22), waiting time (using Eq. (23), productivity loss ratio (using Eq. (24), load intensity of workstation (using Eq. (26), and total cycle time (using Eq. (32) in three types of products. At last, the performance of whole ESPMPPL are also compared as well.

Experimental results

The performance prediction results of each workstation are shown from Table 4 to Table 7, and the performance prediction results of the whole SPMPPL are shown in Table 8.

Performance of each workstation: the proposed procedure meets the accuracy as defined previously. while the generate processing rate is predicted, its iterative times of three types of product stopped at 12, 13, 13, respectively. The results from the proposed procedure are shown in Fig. 5.

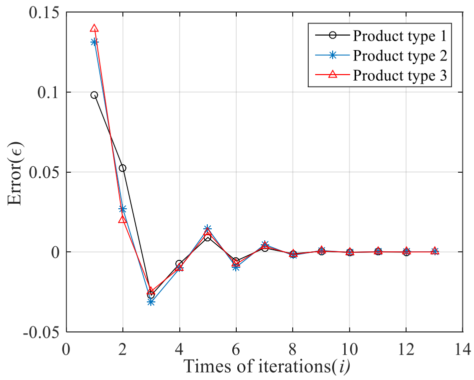

Fig. 5 Results in every iterative times.

In addition, Table 4, Table 5, and Table 6 show the performance indicators $P_i^{(kb)}$, $P_i^{(ks)}$, $W_i^{(k)}$, $L_i^{(k)}$, $P_i^{(loss)}$, and $\bar{\rho}_i^{(k)}$. Their maximum errors in three types of product, between prediction and simulation results are 3.88%, 4.20%, 3.61%, 4.69%, 4.57%, and 3.18%, respectively. The load intensity of workstation varies over the buffer capacities is shown in Table 7.

Table 4. Performance of workstations: scenario 1 for k=1.

Indicators	S_1	S_2	S_3	S_4	S_5
$P_i^{(kb)}(\varepsilon)$	0.1066 (2.83)	0.0564 (3.09)	0.0174 (2.96)	0.0483 (1.47)	*n/a*
$P_i^{(ks)}(\varepsilon)$	*n/a*	0.2412 (4.05)	0.3420 (1.21)	0.4977 (0.84)	0.3648 (4.20)
$W_i^{(k)}(\varepsilon)$	*n/a*	1.3547 (1.12)	1.0359 (3.61)	0.5449 (2.68)	1.0941 (2.24)
$L_i^{(k)}(\varepsilon)$	*n/a*	1.2727 (2.03)	0.8665 (3.10)	0.4345 (3.68)	0.7901 (4.69)
$P_i^{(loss)}(\varepsilon)$	0.1066 (2.83)	0.2976 (3.72)	0.3593 (3.87)	0.5460 (2.96)	0.3648 (4.29)
$\bar{\rho}_i^{(k)}(\varepsilon)$	0.8934 (0.71)	0.7024 (1.78)	0.6407 (2.09)	0.4540 (3.18)	0.6352 (0.09)

Table 5. Performance of workstations: scenario 1 for k=2.

Indicators	S_1	S_2	S_3	S_4	S_5
$P_i^{(kb)}(\varepsilon)$	0.1173 (2.09)	0.0599 (3.54)	0.0447 (2.61)	0.0300 (0.33)	*n/a*
$P_i^{(ks)}(\varepsilon)$	*n/a*	0.2254 (2.83)	0.3329 (3.90)	0.3762 (1.49)	0.4307 (0.30)
$W_i^{(k)}(\varepsilon)$	*n/a*	1.0449 (2.54)	0.7876 (0.06)	0.7316 (2.96)	0.6145 (2.45)
$L_i^{(k)}(\varepsilon)$	*n/a*	1.3482 (2.26)	0.8982 (3.46)	0.7536 (2.91)	0.5956 (3.22)
$P_i^{(loss)}(\varepsilon)$	0.1173 (1.76)	0.2853 (2.04)	0.3776 (4.57)	0.4062 (1.22)	0.4307 (0.14)

$\overline{\rho}_i^{(k)}(\varepsilon)$	0.8827 (1.03)	0.7147 (2.26)	0.6224 (0.56)	0.5938 (2.22)	0.5693 (2.36)

Table 6. Performance of workstations: scenario 1 for k=3.

Indicators	S_1	S_2	S_3	S_4	S_5
$P_i^{(kb)}(\varepsilon)$	0.1081(1.91)	0.0619 (3.69)	0.0455 (3.88)	0.0241 (2.55)	*n/a*
$P_i^{(ks)}(\varepsilon)$	*n/a*	0.2389 (2.65)	0.3279 (2.98)	0.3736 (2.83)	0.4585 (0.13)
$W_i^{(k)}(\varepsilon)$	*n/a*	1.1058 (1.38)	0.8970 (0.79)	0.8311 (3.13)	0.6010 (0.56)
$L_i^{(k)}(\varepsilon)$	*n/a*	1.2833 (4.53)	0.9161 (3.59)	0.7617 (2.48)	0.5246 (3.10)
$P_i^{(loss)}(\varepsilon)$	0.1081 (3.25)	0.3008 (4.20)	0.3734 (3.13)	0.3799 (0.45)	0.4585 (1.93)
$\overline{\rho}_i^{(k)}(\varepsilon)$	0.8919 (0.01)	0.6992 (1.67)	0.6266 (2.65)	0.6023 (2.19)	0.5415 (1.56)

Table 7. Load intensity of workstation processing product type 1: scenario 2 with increasing buffer capacity: 5-120.

Station	B_i=5	B_i =10	B_i =15	B_i =20	B_i =30	B_i =50	B_i =80	B_i =100	B_i =120
S_1	0.8934	0.9485	0.9686	0.9788	0.9889	0.9963	0.9992	0.9997	0.9999
S_2	0.7024	0.8295	0.8769	0.9012	0.9253	0.9425	0.9491	0.9502	0.9506
S_3	0.6407	0.7872	0.8419	0.8698	0.8974	0.9170	0.9242	0.9254	0.9258
S_4	0.4540	0.5883	0.6387	0.6643	0.6895	0.7078	0.7156	0.7176	0.7187
S_5	0.6352	0.8115	0.8804	0.9160	0.9514	0.9777	0.9900	0.9933	0.9953

Performance of the whole ESPMPPL: as one facet of simulation model verification, the performance indicators of the whole line are next verified, which include $X^{(k)}$, $BC^{(k)}$, and $TC^{(k)}$. It can be found that the maximum average error value is 4.66%, which is close to the theoretical and practice expected value. The iterative processing results from the proposed procedure and the ARENA simulation are compared in Table 8.

Table 8. Performance of whole production line: scenario 3 for three types of products.

Indicators	Avg. Approx.			Avg. Simul.			Avg. Err. (%)		
	k=1	k=2	k=3	k=1	k=2	k=3	k=1	k=2	k=3
$X^{(k)}$	0.7112	0.9494	0.8624	0.6903	0.9886	0.8506	3.0277	3.9652	1.3873
$BC^{(k)}$	846.2527	581.1956	813.7727	873.4375	609.6192	838.1054	3.1124	4.6625	2.9033
$TC^{(k)}$	866.2527	596.1956	843.7727	893.4375	621.6192	871.1058	3.0427	4.0899	3.1377

Result discussions

In this section, some representative discussions are expounded for the above results.

- As observed in Table 5, the average queue length $L_i^{(k)}$ in the ESPMPPL from Station 2 to Station 5 is 1.3482, 0.8982, 0.7536, 0.5956, respectively. The maximum relative error value is 3.46%. This can be an explanation for the increasing accuracy as MTTF increases and MTTR decreases. According to the experiment results, the fifth station has the shortest queue length, this is because the fifth station has a faster processing rate and lower failure probability than the fourth station in isolate.
- As observed in Table 6, the load intensities of stations have the maximum occupying at the first station, which is approximate to 89.19%, and the minimum occupying appeared at the fifth station is 54.15%, namely almost half of the time the fifth station is in state of idleness. In this situation, improving the performance of the fifth workstation does not have significant impact on the whole line as long as the bottleneck before the fifth station does not change.
- As observed in Table 7, on the one hand, the relative error gradually increases with the increase of buffer capacities can be found. Namely, the buffer capacities are smaller, the accuracy of the proposed method is higher. On the other hand, it also can be found that the occupying time is increasing as well, but when the buffer capacity reaches to 100, the occupying time increases extremely slowly, that is to say, increasing the buffer capacities is not an effective way to decrease occupying time. The design of the ESPMPPL buffer capacity contributes to the ESPMPPL for most of the cases, which is in conformity to the finding of Farshad and Walid (2015). More generally, buffer capacity limitations in stations give rise to a bottleneck phenomenon, involving starvation and blocking. Accordingly, increasing the buffer capacity from 5 to 10, the increasing value of occupy probability is 0.0551, but when buffer capacity is from 100 to 120, the occupy probabilities improved by around 0.0002. With the buffer capacities increasing, the promotion's effect of buffer capacities become progressively smaller.
- Comparing the results generated in Table 8 shows that when the number of products increases, the proposed approximation method demonstrates higher accuracy. Therefore, the number of products cannot be ignored, otherwise the approximate error can still be large. Based on the proposed approximate iteration algorithm procedure in Table 2, the prediction model is built upon the assumption that the $M/M/1/m$ queueing model can always reach steady state conditions. But the

queueing of simulation may not reach steady state conditions in this situation. It may cause a large difference between the results of the proposed method and simulation.

CONCLUSIONS

This paper aims at the specific problems of performance prediction for a SPMPPL with unreliable machines, finite buffers, and nonconforming products. An approximation approach is proposed to model and predict its performance using discrete state Markov chain and $M/M/1/m$ queueing model. The main idea is to assume products arrival follow a Poisson distribution, and consider parallel machines in production line as an equation machine. Then the SPMPPL is treated as common serial queueing system, namely ESPMPPL. Because each equation machine has forward input and backward input, the unknown parameter $X^{(k)}$ solved by developing an approximate iterative algorithm procedure. Then extra indicators of performance including starvation probabilities, blocking probabilities, waiting time, queue length, productivity loss, load intensity of workstation, and total cycle time are computed by a $M/M/1/m$ queueing model again. Finally, numerical experimental is conducted to prove the effectiveness and limitations of the proposed method by three types of products. Most of the performance prediction results derived by the proposed method are at the 95% confidence level. The computed results are in good agreement with the simulated results.

The proposed method can provide an effective guidance for the production decision-makers, and help the product manufacturing manager to respond quickly, so as to support more reasonable production planning decision-making. It also can be used to design a new production line or predict the performance of an existing production line based on actual data collected. It should be noted that there are still some limitations, although the proposed method focuses on a SPMPPL with unreliable machines, finite buffers, and nonconforming processing problems, it involves substantial simplifications. To confirm the accuracy and robustness of this simplified method, a more complete model and core-algorithm of prediction considering the coefficient of variation and products arrival under mixed entering sequence should be investigated, which will be addressed in our next research.

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APPENDIX: EQUATION DERIVATION

1. Queue length ($L_i^{(k)}$) in Eq. (22) .

$$
\begin{aligned}
L_i^{(k)} &= \sum_{b_i=0}^{B_i-1} b_i P_i^{(k)b_i+1} \\
&= \rho_i^{(k)2} P_i^{(k0)} \sum_{b_i=1}^{B_i-1} b_i P_i^{(k)b_i-1} \\
&= \rho_i^{(k)2} P_i^{(k0)} \left(\frac{1-\rho_i^{(k)B_i}}{1-\rho_i^{(k)}} \right)' \\
&= \rho_i^{(k)2} P_i^{(k0)} \frac{1-\rho_i^{(k)B_i-1}\left(B_i-\left(B_i-1\right)\rho_i^{(k)}\right)}{\left(1-\rho_i^{(k)}\right)^2} \\
&= \frac{\rho_i^2\left(1-B_i\rho_i^{(k)B_i}+\left(B_i-1\right)\rho_i^{(k)B_i}\right)}{\left(1-\rho_i^{(k)}\right)\left(1-\rho_i^{(k)B_i+1}\right)} \\
&= \frac{\rho_i^{(k)}}{1-\rho_i^{(k)}} - \frac{B_i\rho_i^{(k)B_i+1}+\rho_i^{(k)B_i}}{1-\rho_i^{(k)B_i+1}}
\end{aligned}
$$

2. Waiting time $W_i^{(k)}$ in Eq. (23).

$$
\begin{aligned}
W_i^{(k)} &= \frac{L_i^{(k)}}{\lambda_i^{(ek)}} \\
&= \frac{\rho_i^{(k)}\left(1-m\rho^{m-1}+\left(m+1\right)\rho_i^{(k)m}\right)}{X_{i+1}^{(k)}\left(1-\rho_i^{(k)}\right)\left(1-\rho_i^{(k)m}\right)} \\
&= \frac{\rho_i^{(k)}}{X_{i+1}^{(k)}\left(1-\rho_i^{(k)}\right)} - \frac{m\rho_i^{(k)m}}{X_{i+1}^{(k)}\left(1-\rho_i^{(k)m}\right)}
\end{aligned}
$$

3. Machine load intensity ($\bar{\rho}_i^{(k)}$) in Eq. (26).

$$
\begin{aligned}
\bar{\rho}_i^{(k)} &= \frac{X_i^{(ke)}}{X_{i+1}^{(k)}} \\
&= \frac{X_i^{(k)}\left(1-P_i^m\right)}{X_{i+1}^{(k)}} \\
&= \frac{\rho_i^k-\rho_i^{B_i+1}}{1-\rho_i^{B_i+1}} \\
&= 1-P_i^{(k0)}
\end{aligned}
$$

考慮不可靠設備有限緩衝區和不良品之串並聯多產品生產線性能預測

李長軍，李波
電子科技大學航空航天學院

摘要

性能預測對于提升不可靠機器、有限緩衝區和不良品的串幷聯多產品生產綫柔性極其重要，但如何建模幷預測其性能面臨巨大挑戰。本文提出壹種新的基于離散馬爾科夫鏈和排隊論的近似迭代方法。首先，等效幷聯機器加工速率由單台機器速率的狀態馬爾科夫性合成;然後整條生產綫的產出$X^{(k)}$由最末工站在平穩狀態下的有效加工速率決定;未知參數$X^{(k)}$由近似迭代算法程序獲得後再利用排隊模型計算性能。最後采用數值仿真方法驗證了所提方法之有效性。