中國機械工程學刊第四十三卷第三期第 229~238 頁(民國一百一十一年)
Journal of the Chinese Society of Mechanical Engineers, Vol.43, No.3, pp.229~238 (2022)

Prediction and Evaluation of Fatigue Strength via Mechanical Behavior of Materials

Xin-Tian Liu*, Hai-Jie Wang**, Xiao-Bing Yang** and Jia-Feng Lai***

Keywords：fatigue strength, mechanical behavior, low alloy steel, small samples

ABSTRACT

Besides stress, the fatigue failure of part is also affected by the fatigue strength of materials. Therefore, it is necessary to predict the fatigue strength of materials before analyzing fatigue life. Considering the factors with uncertainty that affect the fatigue strength, the modified factors of fatigue are determined, and the stochastic fatigue strength is predicted. Among them, the distribution type and parameters of roughness factor are analyzed by improving Bootstrap method, which provides a method to analyze small samples. Moreover, the S-N curve cluster that characterizes the fatigue property is obtained, which provides a basis for fatigue life prediction.

INTRODUCTION

Recently the fatigue failure of mechanical components has been gradually analyzed (Li, et al., 2019; Zhu, et al., 2019, Wang, et al., 2017 and Liu et al., 2020). Based on life distribution of multifailure modes was described by Bayes clustering, the relationship between distribution parameters and stress was established, and the ultimately unified probabilistic S-N curve was modeled (Wang, et al., 2018). The fatigue failure is not only related to the stress, but also depends on the fatigue strength of material (Tasi, et al., 2015, Wang et al., 2021 and Liu, et al., 2022). Therefore, only by predicting the fatigue strength of parts be better predicted (Zeng, et al., 2015). Leuders et al. (2015) predicted the fatigue strength for Titanium alloy and optimized the prediction accuracy. Instead of the common fatigue test, Wu et al. (2013) performed the HV test to predict the fatigue strength of FCC metal. Considering the effect of residual stress (Park, et al., 2014), surface treatment (Guagliano, et al., 2004; Sakamoto, et al., 2015 and Luo, et al., 2018) and harness on fatigue performance, Yuan and Li (2017) proposed a new model for fatigue strength prediction. In addition, Wu et al. (2016) proposed an indirect probability model to evaluate reliability of multi-body mechanisms. And Liu et al. (2021) proposed a dynamic reliability model with mixed uncertain parameters based on the uncertainty of parameters in structural dynamic reliability analysis. Other researches of fatigue can be studied by Wang et al. (2019) and Mossaab et al. (2019).

Though the goodness-of-fit Chi-square test (Temme, et al., 2015), moment estimation (Zhao, et al., 2016) and maximize likelihood estimation (Coretto, et al., 2011) have been widely used in statistical inference, they are not applicable for that of small sample data. Due to the limitation of data observation, parameter estimation of small sample has been increasingly studied in many fields, such as national defense, environment and healthy (Hansen, et al., 2007) by methods of Bayes (Cumming, et al., 2009), Bootstrap (Dwivedi, et al., 2017) and Monte Carlo (Vořechovský, et al., 2009 and Vořechovský, et al., 2012). For Bootstrap proposed by Efron (1979), repeated sampling is continuously operated to convert small sample into large sample. Bootstrap is relatively efficient for parameter estimation of small sample, but inevitably there exists an error because the sample parameter is used to replace the original parameter for each sampling (Fu, et al., 2005 and Dwivedi, et al., 2017). To reduce the error, the original sample size needs to be expanded (Wang, et al., 2019 and Zhang, et al., 2018). Ge et al. (2021) introduced the error circle to evaluate fisheye size, and discussed the influencing factors of fatigue strength. Therefore, this paper proposes that the original sample is expanded using normal distribution before the estimation by Bootstrap.

Paper Received September, 2019. Revised J July, 2021. Accepted July, 2021. Author for Correspondence: Xin-Tian Liu.

** Associate Professor, School of Mechanical and Automotive Engineering, Shanghai University of Engineering Science, Shanghai 201620, China.*

*** Gradnated Student, School of Mechanical and Automotive Engineering, Shanghai University of Engineering Science, Shanghai 201620, China.*

**** Professor, College of Petroleum and Chemical Engineering, Beibu Gulf University, Qinzhou, Guangxi 535011, China*

This paper mainly discusses the prediction of stochastic fatigue strength for low alloy steel AISI 8630M, and that is organized as follows. In section 2, quantitative model of fatigue strength affected by mechanical behavior of materials is established. In section 3, various fatigue modified factors that affect the fatigue strength are determined considering the different working conditions for wheel hub. In section 4, the distribution characteristic of fatigue strength is analyzed, and then the S-N curve cluster that describe the fatigue properties are obtained. Finally, in section 5, the conclusions are summarized.

THE QUANTITATIVE MODEL OF FATIGUE STRENGTH

In practical engineering, the fatigue strength of structures is affected by many uncertain factors, such as geometric size, material properties, load and temperature. So the fatigue strength is also an uncertain variable. Currently, fatigue test is used to study the fatigue characteristics of low alloy steel, and fatigue test data of AISI 8630M low alloy steel are selected to predict fatigue strength of wheel hub (Wormsen, et al., 2015).

Mischke (1987) and Nisbett-Budynas (2006) proposed a method for the prediction of stochastic fatigue strength. Then the relationships between the fatigue limits of sample, actual part and the tensile strength are shown as:

$$S'_e = \varphi_{d_0} S_{ut} \tag{1}$$

$$S_e = \varphi S_{ut} \tag{2}$$

$$\varphi_{d_0} = 0.440 d_0^{-0.1133}(1, 0.146) \tag{3}$$

where S'_e is the fatigue limit of sample, S_e is the actual part, S_{ut} is the tensile strength and φ_{d_0} is the proportional coefficient for the sample diameter of d_0.

In this paper, the sample diameter is taken as d_0 =8mm=0.315in and S_{ut} =771MPa=110.143kpsi. According to Eq. (3) and (1), φ_{d_0} and S'_e can be obtained as φ_{d_0} =0.502(1, 0.146)=(0.502, 0.073), S'_e =55.292(1, 0.146) = (55.292, 8.073)kpsi. Then the mean and standard deviation of φ_{d_0} and S'_e are expanded to the intervals under the interval level of γ_1 =5% and γ_2 =0.5%, respectively. Therefore, $\mu_{\varphi_{d_0}}$ =[0.477, 0.527], $\sigma_{\varphi_{d_0}}$ =[0.069, 0.077], $\mu_{S'_e}$ =[55.016, 55.568], $\sigma_{S'_e}$ =[8.033, 8.113].

The factors of affecting fatigue strength was introduced by Marin (1962), that mainly includes the chemical composition, dimension, heat treatment, stress concentration, machined surface, temperature and loading. Shigley et al. (1983) put forward a method to quantify the factors that affect the fatigue strength, and formulated the relationship between the fatigue limit of sample and actual part as follows:

$$S_e = k_a k_b k_c k_d k_e k_r S'_e \tag{4}$$

where k_r, k_a, k_b, k_c, k_d and k_e are modified factors that characterize the effect of roughness, surface, dimension, loading, temperature and miscellaneous effect roughness on the fatigue strength.

DETERMINATION OF FATIGUE MODIFIED FACTORS

Roughness Factor

In mathematical statistics and engineering research, the type and parameter of the overall distribution can be deduced by statistics inference, and includes hypothesis testing and parameter estimation. Then small sample fatigue test data of AISI 8630M is analyzed to obtain the distribution type and parameters.

Non-parametrical Hypothesis Testing of Small Sample Data

At present, the fatigue properties of different materials were studied by many researchers via fatigue test (Chen, et al., 2018 and Liu, et al., 2010). And the fatigue test of low alloy steel was introduced by Wormsen et al. (2015), which included the operation process, sample selection and loading stress, as well as obtained the test data of different low alloy steel. In this paper, the fatigue test data of AISI 8630M under pulsating cyclic load shown in Table 1 is selected to predict the fatigue strength of wheel hub.

Table 1. The fatigue test data of AISI 8630M (Wormsen, et al., 2015)

Sample No.	Roughness R_a	Max. Stress S_{max} (MPa)	Min. Stress S_{max} (MPa)	Sycles
U1	3.4	400	-400	17763
U2	3.3	375	-375	472044
U3	3.2	375	-375	282400
U39	3.1	375	-375	253600
U40	3.1	400	-400	55958
U41	3.2	375	-375	223224
U42	3.2	375	-375	199337
U43	3.3	363	-363	151781
U44	3.3	363	-363	232133
U45	3.4	363	-363	166092
U46	3.4	350	-350	187233
U47	3.5	350	-350	200719
U48	3.5	350	-350	281634

The Goodness-of-fit Chi-square test is widely used in the non-parametrical hypothesis testing. And due to the deviation level between the actual and theoretical distribution is increased, it cannot be well applied to small sample data. Fisher (1922) proposed the exact probability method to well solve the hypothesis testing of small sample data. The test can be mainly carried out as: (1) The test statistic χ^2 is firstly calculated by the Goodness-of-fit Chi-square test. (2) The all combinations that satisfy $A_1 + A_2 + ... + A_m = n$ are listed by exhaustive method and then the chi-square χ_i^2 of each combination is calculated, where the combination number is K=(n+m+1)!/n!/(m-1)! and m is the divided interval number. (3) The probability of each combination P_i is also calculated as $P_i = n!\prod_{j=1}^{m}\pi_j^{A_j} / \prod_{j=1}^{m} A_j!$, and $\sum_{i=1}^{K} P_i = 1$. (4) Finally, the exact probability P_e can be obtained by summing up the probability P_i that satisfies $\chi_i^2 \ge \chi^2$. If P_e>0.05, there is no significant difference and the original hypothesis H_0 is acceptable. That is, the variable X follows a certain hypothetical distribution.

The exact probability method can be used to test whether the sample roughness R_a follows the normal distribution. And the original hypothesis can be shown as follows:

$$H_0 : F(R_a) = \varphi(\frac{R_a - \mu}{\sigma}); \quad H_1 : F(R_a) \ne \varphi(\frac{R_a - \mu}{\sigma})$$

The calculation results of χ^2 sample value are shown in Table 2.

Table 2. The calculation results of χ^2 sample value

i	Interval	Actual frequency v_i	Probability estimation $\hat{p}_i$	Excepted frequency $n\hat{p}_i$
1	(-∞, 3.3)	5	0.5	6.5
2	[3.3, 3.4]	3	0.2794	3.6322
3	(3.4, +∞)	5	0.2206	2.8678

And the test statistic $\chi^2 = \sum_{i=1}^{3} \frac{(v_i - n\hat{p}_i)^2}{n\hat{p}_i} = 2.4015$.

The sample number n=13 and the number of divided interval m=3, so the combination number that satisfies $\sum_{i=1}^{3} v_i = 13$ is K=(n+m-1)!/n!/(m-1)!=105. Then the chi-square χ_i^2 and probability P_i of each combination can be calculated as follows:

$$\chi_i^2 = \frac{(v_{i1} - 6.5)^2}{6.5} + \frac{(v_{i2} - 3.6322)^2}{3.6322} + \frac{(v_{i3} - 2.8678)^2}{2.8678} \tag{5}$$

$$P_i = n!(0.5^{v_{i1}} 0.2794^{v_{i2}} 0.2206^{v_{i3}}) / v_{i1}! v_{i2}! v_{i3}! \tag{6}$$

After the chi-square χ_i^2 and probability P_i of 105 combinations are calculated, the probability P_i that satisfies $\chi_i^2 \ge \chi^2$ are summed up and then the exact probability P_e =0.36238>0.05. Therefore, H_0 is acceptable. And the roughness R_a follows the normal distribution.

Parameter Estimation of Small Sample Data

To analyze the distribution characteristic of R_a, the distribution type and the parameters need to be determined. For the large and small sample data, there are corresponding method to estimate the parameter. In this paper, the point estimation and interval estimation of μ_{R_a} are carried out and compared by maximum likelihood estimation (MLE) method and modified Bootstrap method, respectively.

For the variable R_a that follows the normal distribution, the point estimation of mean μ_{R_a} by MLE can be given as Eq. (7). If the standard deviation σ_{R_a} is unknown, the confidence interval of μ_{R_a} under the confidence level of $1-\beta$ can be obtained as Eq. (8) and (9). The estimation results are shown in Table 3.

$$\hat{\mu}_{R_a} = \overline{R}_a \tag{7}$$

$$(\overline{R}_a - \frac{S}{\sqrt{n}} t_{1-\beta/2}(n-1), \overline{R}_a + \frac{S}{\sqrt{n}} t_{1-\beta/2}(n-1)) \tag{8}$$

$$S = \sqrt{\frac{1}{n-1}(\sum_{i=1}^{n} R_{ai}^2 - n\overline{R}_a^2)} \tag{9}$$

where R_{ai} (i=1, 2,…, 13) is the sample data, $\overline{R}_a$ is the mean of sample data and S is the sample standard deviation.

Different with the MLE method, the Bootstrap method is more suitable for parameter estimation of small sample data, which was proposed by Efron (1979). Essentially the Bootstrap is to convert the small sample into large sample by continuously sampling from the sample data, and the size of small sample data is $n \le 30$ (Sheng, et al., 2008). After each sampling, the parameter of sample data can be replaced by the parameter estimation of sub sample data. In this paper, the sampling number N is selected as 10000. The point estimation and confidence interval of μ_{R_a} are shown in Table 3.

Unfortunately, the parameter replacing with repeated sampling could inevitably lead to errors by Bootstrap. To reduce the errors, the original sample data is expanded using normal distribution before the parameter estimation by Bootstrap.

The each sample data R_{ai} (i=1, 2,…, 13) is firstly expanded to the interval $[R_{ai}(1-\alpha), R_{ai}(1+\alpha)]$ under different significance level of α =0.1%, 0.2%, 0.5%, 1%, 2%, 5%, and then R_{ai} is taken as the center to generate some data according to normal distribution $N(\mu_i, \sigma_i^2)$ in this interval. Assuming that the each sample data R_{ai} is expanded in the confidence interval under the confidence level of 1- β =95%, the Eq. (10) is established as follows:

$$[R_{ai}(1-\alpha), R_{ai}(1+\alpha)] = [\mu_i - u_{1-\beta/2}\sigma_i, \mu_i + u_{1-\beta/2}\sigma_i] \tag{10}$$

And if $\mu_i = R_{ai}$, then there exists that $R_{ai}\alpha = u_{1-\beta/2}\sigma_i$, so $\sigma_i = R_{ai}\alpha / u_{1-\beta/2} = R_{ai}\alpha / 0.96$.

After expanding the each sample data, the

Bootstrap method is adopted to determine the point estimation and confidence interval of μR_a. The parameter distribution diagrams of μR_a at significance level α=0.1% is shown in Fig.1. And the parameter distribution diagrams of μR_a at other significance level are similar to Fig. 1. Then the point estimation and confidence interval are shown in Table 3.

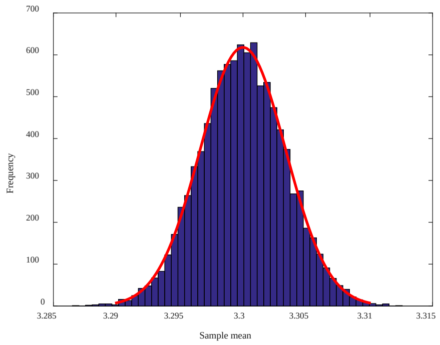

Fig. 1. Parameter distribution diagrams of μR_a at α=0.1%

Table 3. Point estimation and confidence interval of μR_a at significance level α

α	Point Estimation			Confidence Interval	Interval Length
	Estimati-on	Expec-tation	Error		
MLE Method	3.3	3.3	0	(3.218200, 3.381800)	0.163600
Bootstrap Method	3.3007	3.3	0.0007	(3.230800, 3.369200)	0.138400
α=0.1%	3.299939	3.3	0.000061	(3.293157, 3.306800)	0.013643
α=0.2%	3.300378	3.3	0.000378	(3.293406, 3.307305)	0.013899
α=0.5%	3.299273	3.3	0.000727	(3.292274, 3.306394)	0.014120
α=1.0%	3.301152	3.3	0.001152	(3.293976, 3.308172)	0.014196
α=2.0%	3.298713	3.3	0.001287	(3.291517, 3.305812)	0.014295
α=5.0%	3.297075	3.3	0.002925	(3.289025, 3.305197)	0.016172

According to Ref. (Wormsen, et al., 2015), the roughness factor k_r can be used to characterize the surface roughness of material (Liu et al., 2021), and it can be calculated as follows:

$$k_r = 1 - 0.22\lg(4R_a)\lg(\frac{S_{ut}}{200}) \tag{11}$$

where R_a is the roughness, S_{ut} is the tensile strength. In this paper, S_{ut} is taken as 771MPa for the low alloy steel AISI 8630M (Wormsen, et al., 2015).

According to the sample data of roughness R_a in Table 1 and Eq. (11), the sample data of roughness factor k_r can be obtained. Similar to the research method for R_a in above, k_r also follows the normal distribution, and the point estimation and confidence interval of μ_{k_r} by different methods are shown in Table 4.

As shown in Table 3 and 4, with the increase of significance level α, the errors between estimation and expectation gradually increase. Moreover, the confidence interval lengths of μ_{R_a} and μ_{k_r} increase under the same confidence level, that is, the interval estimation accuracy of μ_{R_a} and μ_{k_r} decreases.

Table 4 Point estimation and confidence interval of μ_{k_r} at different significance level α

α	Point Estimation			Confidence Interval	Interval Length
	Estimati-on	Expectati-on	Error		
MLE Method	0.8556	0.8556	0	(0.854200, 0.856900)	0.002700
Bootstrap Method	0.8556	0.8556	0	(0.854500, 0.856800)	0.002300
α=0.1%	0.855569	0.8556	0.000031	(0.855448, 0.855689)	0.000241
α=0.2%	0.855561	0.8556	0.000039	(0.855437, 0.855682)	0.000245
α=0.5%	0.855489	0.8556	0.000111	(0.855337, 0.855647)	0.000310
α=1.0%	0.855788	0.8556	0.000188	(0.855545, 0.856030)	0.000485
α=2.0%	0.855774	0.8556	0.000174	(0.855359, 0.856200)	0.000841
α=5.0%	0.856247	0.8556	0.000647	(0.855235, 0.857271)	0.002036

Comprehensively considering the interval estimation results of R_a and k_r, the significance level can be taken as α=0.1%. Therefore, the confidence intervals of R_a and k_r under the confidence level of 0.95 are (3.293157, 3.306800) and (0.855448, 0.855689), respectively. Comparing the confidence interval lengths by MLE method, Bootstrap method and the method of expanding sample data using the normal distribution, the interval estimation accuracy is effectively improved by 91.07% and 89.52%, respectively.

The point estimation of roughness coefficient k_r is 0.855569, and the confidence interval under the confidence level of 0.95 is (0.855448, 0.855689). Similar to the research method in above, 0.855569 is taken as the center to generate some data in the interval (0.855448, 0.855689) according to the normal distribution, and then the mean and standard deviation are estimated as 0.85557 and 0.000050 by MLE. That is, k_r=(0.85557, 0.000050)=0.85557(1, 0.000058). Similarly, the mean and standard deviation are expanded to the intervals under the interval level of $\gamma_2 = 0.5\%$, so μ_{k_r} =[0.85129, 0.85985], σ_{k_r} =[0.000049, 0.000051].

Surface Modified Factor

The determination of surface modified factor k_a depends on the processing method of sample surface and the tensile strength of material. For the deterministic variable, the relationship between k_a and S_{ut} can be shown as follows:

$$k_a = aS_{ut}^b \tag{12}$$

where a and b are constants, the determination method of which is shown in Table 5 (Budynas-Nisbett, 2006). In this paper, the sample surface is processed by machined, so a=2.70，

b=-0.265.

For the uncertain variable, k_a can be calculated as $k_a = 2.70(1, 0.060)S_{ut}^{-0.265}$ =0.776(1, 0.060)=(0.776, 0.047) (Budynas-Nisbett, 2006). Similarly, the mean and standard deviation are expanded to the intervals under the interval level of $\gamma_1 = 5\%$, so μ_{k_a} =[0.737, 0.815], σ_{k_a} =[0.045, 0.049].

Table 5. The method of determining *a* and *b* (Budynas-Nisbett, 2006)

Surface Processing Method	*a*		*b*
	S_{ut} (kpsi)	S_{ut} (MPa)	
Ground	1.34	1.58	-0.085
Machined/Cold-Drawn	2.70	4.51	-0.265
Hot-Rolled	14.4	57.7	-0.718
As-Forged	39.9	372	-0.995

Dimension Modified Factor

The determination of dimension modified factor k_b mainly depends on the type of loading that the part bears (Mischke, 1987), and which is shown as Eq. (13).

$$k_b = \begin{cases} (d_0 / 0.3)^{-0.1133} \\ 1 \end{cases} \tag{13}$$

where d_0 is the sample diameter.

The wheel hub is mainly subjected to bending and torsion loading, so k_b=(0.315/0.3)-0.1133=0.994. Then k_b is expanded to the interval under the interval level of γ_2 =0.5%, that is k_b =[0.989, 0.999].

Loading Modified Factor

The determination of loading modified factor k_c also mainly depends on the type of loading that the part bears (Mischke, 1987). For the deterministic variable, k_c can be determined as follows:

$$k_c = \begin{cases} (1, 0) \\ \varphi_{ax} / \varphi_{d_0} \\ \varphi_t / \varphi_{d_0} \end{cases} \tag{14}$$

where φ_{ax} and φ_t are the proper factors in equation $S'_e = \varphi S_{ut}$ for axial and torsion loading, respectively, and φ_{ax} =0.390(1, 0.310), φ_t =0.295(1, 0.269) (Mischke, 1987), φ_{d_0} =0.502(1, 0.146).

And the wheel hub is mainly subjected to bending and torsion loading, so $k_c = \varphi_t / \varphi_{d_0}$ =0.587(1, 0.123)=(0.587, 0.072). Similarly, the mean and standard deviation are expanded to the intervals under the interval level of γ_1=5%, so μ_{k_c} =[0.558, 0.616], σ_{k_c} =[0.068, 0.076].

Temperature Modified Factor

The effect of temperature on fatigue strength is shown that the fatigue strength decreases with the increase of temperature and increases with the decrease of temperature. When the service temperature $70^\circ F \le T_F \le 1000^\circ F$, the temperature modified factor k_d can be determined as follows:

$$k_d = 0.975 + 0.432(10^{-3})T_F - 0.115(10^{-5})T_F^2 + 0.104(10^{-8})T_F^3 - 0.595(10^{-12})T_F^4 \tag{15}$$

Generally the temperature of wheel hub is $65^\circ C$, that is, $T_F = 149^\circ F$. According to Eq. (15), the temperature modified factor is obtained as k_d =1.017. k_d is expanded to the interval under the interval level of γ_2 =0.5%, so k_d =[1.012, 1.022].

Miscellaneous Effect Modified Factor

The miscellaneous effect on fatigue strength mainly including the residual stress, corrosion, electroplating, shot peening, case hardening and cold rolling. The fatigue strength is increased by the residual compressive stress and reduced by the residual tensile stress. Moreover, the fatigue strength can be improved by shot peening. In this paper, the effect of miscellaneous effect on fatigue strength is not considered. Therefore, the miscellaneous effect modified factor k_e =1.

In summary, the each modified factors that affect the fatigue strength are determined in Table 6.

Table 6. Each modified factors that affect the fatigue strength

k_a	k_b	k_c	k_d	k_e	k_r
0.776(1, 0.060)	0.994	0.587(1, 0.123)	1.017	1	0.85557(1, 0.000058)

According to Eq. (1), (4) and (14), the fatigue limit of actual part S_e can be converted as follows:

$$S_e = k_a k_b k_d k_r \varphi_t S_{ut} \tag{16}$$

According to the modified factors in Table 6, the coefficient of variation C_{S_e} can be obtained as Eq. (17), and then the mean μ_{S_e} and standard deviation σ_{S_e} can be obtained as Eq. (18) and (19).

$$C_{S_e} = \sqrt{C_{k_a}^2 + C_{k_c}^2 + C_{\varphi_t}^2} = 0.276 \tag{17}$$

$$\mu_{S_e} = \mu_{k_a} \mu_{k_b} \mu_{k_d} \mu_{k_r} \mu_{\varphi_t} S_{ut} = 25.489\text{kpsi} \tag{18}$$

$$\sigma_{S_e} = \mu_{S_e} C_{S_e} = 7.035\text{kpsi} \tag{19}$$

Therefore, S_e =(25.489, 7.035)=25.489(1, 0.276)kpsi. The mean and standard deviation are expanded to the intervals under the interval level of γ_2 =0.5%, so μ_{S_e} =[25.362, 25.616], σ_{S_e} =[6.990, 7.060].

FATIGUE STRENGTH PREDICTION OF WHEEL-HUB

The fatigue modified factors are determined, then the fatigue limit of wheel hub has been analyzed. For an actual part, the fatigue strength can be written as follows (Budynas-Nisbett, 2006):

$$S_f = cN^d \tag{20}$$

$$\begin{cases} c = \dfrac{(fS_{ut})^2}{S_e} \\ d = -\dfrac{1}{3}\lg(\dfrac{fS_{ut}}{S_e}) \end{cases} \tag{21}$$

where c and d are constants, S_f is the fatigue strength of wheel hub, S_{ut} is the tensile strength, S_e is the fatigue limit of wheel hub, f is the ratio of the sample fatigue strength $(S'_f)_{10^3}$ and tensile strength S_{ut} under the cycle number N=10^3.

The determination of f can be carried out by two methods, which are shown as follows:

(1) For the tensile strength S_{ut} <490MPa (70kpsi), f=0.9; For 490MPa (70kpsi) $\le S_{ut} \le$ 1400MPa (200kpsi), f can be obtained according to Fig.2 (Budynas-Nisbett, 2006). In this paper, S_{ut} =771MPa=110.143kpsi, so f=0.829.

(2) When S_{ut} =771MPa=110.143kpsi, the plastic stress $\sigma'_F = S_{ut} + 50$ =160.143kpsi (Budynas-Nisbett, 2006). For the deterministic variable, the fatigue limit of sample $S'_e = 0.5S_{ut}$ =385.5MPa=55.072kpsi, so $d = -\dfrac{\lg(\sigma'_F / S'_e)}{\lg(2N_e)}$ =-0.074 and $N_e = 10^6$. Therefore, $f = \dfrac{\sigma'_F}{S_{ut}}(2 \cdot 10^3)^d$ =0.828.

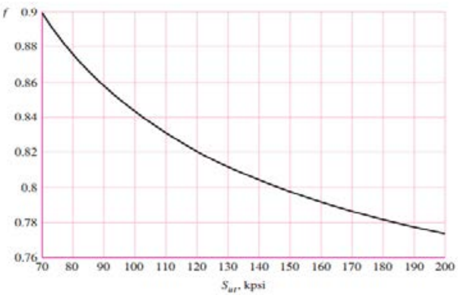

Fig. 2. The change curve of f

According to Eq. (21), the mean and standard deviation of c can be obtained as follows:

$$\mu_c = (fS_{ut})^2 / \mu_{S_e} = 327.092\text{kpsi} \tag{22}$$

$$\sigma_c = (fS_{ut})^2 \cdot \frac{\sigma_{S_e}}{\mu_{S_e}^2} = 90.278\text{kpsi} \tag{23}$$

So c=(327.092, 90.278)=327.092(1, 0.276)kpsi. The mean and standard deviation are expanded to the intervals under the interval level of γ_2 =0.5%, so μ_c =[325.457, 328.727], σ_c =[89.699, 90.601].

To obtain the mean and standard deviation of d, let $e = \dfrac{fS_{ut}}{S_e}$, $d = -\dfrac{1}{3}\lg e$, the mean and standard deviation of e are obtained as follows:

$$\mu_e = fS_{ut} / \mu_{S_e} = 3.582 \tag{24}$$

$$\sigma_e = fS_{ut} \cdot \frac{\sigma_{S_e}}{\mu_{S_e}^2} = 0.989 \tag{25}$$

According to the definition of expectation and variance, the mean and standard deviation of d can be calculated as follows:

$$E(d) = \int_{-\infty}^{+\infty} (-\frac{1}{3}\lg x) \cdot \frac{1}{0.989\sqrt{2\pi}} e^{-\frac{(x-3.582)^2}{2 \cdot 0.989^2}} dx = -0.181 \tag{26}$$

$$E(d^2) = \int_{-\infty}^{+\infty} (-\frac{1}{3}\lg x)^2 \cdot \frac{1}{0.989\sqrt{2\pi}} e^{-\frac{(x-3.582)^2}{2 \cdot 0.989^2}} dx = 0.035 \tag{27}$$

$$D(d) = E(d^2) - [E(d)]^2 = 0.002 \tag{28}$$

$$\sigma_d = \sqrt{D(d)} = 0.045 \tag{29}$$

So d=(-0.181, 0.045)=0.181(-1, 0.249). The mean and standard deviation are expanded to the intervals under the interval level of γ_1 =5%, so μ_d =[-0.190, -0.172], σ_d =[0.043, 0.047].

After determining the mean and standard deviation, it is also necessary to analyze the distribution type of c and d. As shown previously, S_e ~N (25.489, 7.0352), the interval for probability of 0.95 can be obtained as $[\mu_{S_e} - u_{0.975}\sigma_{S_e}, \mu_{S_e} + u_{0.975}\sigma_{S_e}]$ =[11.700, 39.278], and then 1000 sample data are generated randomly in this interval. According to Eq. (21), 1000 sample data of c are also obtained and the histogram is shown in Fig.3.

As shown in Fig.3, the sample data of c shows the unimodality, which can be described by the normal, skewed, student and hyperbolic secant distributions, respectively, and then the Cramer-Von Mises statistic is adopted to test the fitting effect of each distribution. The distribution parameter estimations and probability density functions of sample data of c under different distribution models are shown in Table 7 and Fig. 3.

Table 7. Distribution parameter estimations of sample data of c

Distribution Model	θ_1	θ_2	θ_3	W_n^2	p
Normal	360.098	100.652	-	3.511	5.637×10^{-9}
Skewed	231.529	163.282	12.232	0.247	0.192
Student t	345.919	81.260	5.318	1.314	0.0005
Hyperbolic secant	343.342	102.513	-	1.288	0.0005

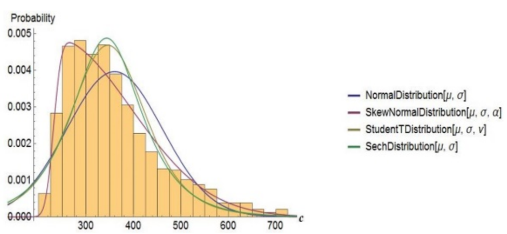

Fig. 3 Probability density functions of sample data of c under different distribution models

As shown in Table 7, the statistic W_n^2 for skewed distribution is minimum and p=0.192>0.05, so the fitting effect of skewed distribution is better. Besides, the fitting effect of different distribution model can be tested by *p-p* diagrams in Fig. 4-Fig. 7.

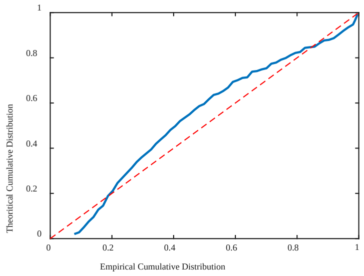

Fig. 4. *p-p* diagrams of sample data of *c* under normal distribution

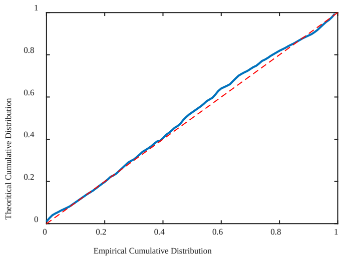

Fig. 5. *p-p* diagrams of sample data of *c* under skewed distribution

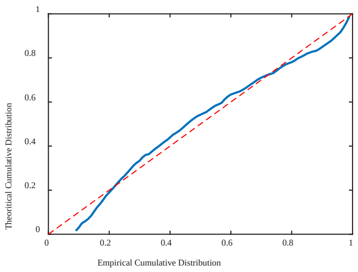

Fig. 6. *p-p* diagrams of sample data of *c* under student *t* distribution

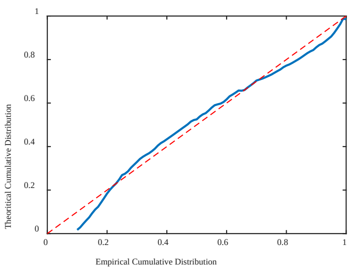

Fig. 6. *p-p* diagrams of sample data of *c* under hyperbolic secant distribution

It can be seen from Fig. 5 that the fitting effect of skewed distribution is better. Therefore, the constant *c* approximately follows the skewed distribution (231.529, 163.282, 12.232).

According to Eq. (21), 1000 sample data of d can also be obtained. Similar to the above research method, it can be known that the constant *d* approximately follows the skewed distribution (-0.147, 0.060, -3.756).

In practical engineering, the *S-N* curve can be described the cycle number of loading when fatigue failure happens, which provides a basis for the fatigue life prediction of part. To analyze the fatigue curve more conveniently, the *x*-coordinate and *y*-coordinate are processed with logarithm as Eq. (30), and then the *S-N* curve is approximated as a straight line in the double logarithmic coordinate.

$$\lg S_f = \lg c + d \lg N \tag{30}$$

As shown previously, the fatigue limit of wheel hub S_e ~*N* (25.489, 7.0352) and μ_{S_e} =[25.362, 25.616], σ_{S_e} =[6.990, 7.060]. Then 25.362, 25.489 and 25.616 are selected from the mean interval, and 6.990, 7.025 and 7.060 are selected from the standard deviation interval, so three normal distributions *N* (25.362, 6.9902), *N* (25.489, 7.0252) and N (25.616, 7.0602) are selected from a group of normal distributions, and that is following of S_e. And five data are selected as 23.362, 24.362, 25.362, 26.362, 27.362; 23.489, 24.489, 25.489, 26.489, 27.489; 23.616, 24.616, 25.616, 26.616, 27.616 from three normal distributions, respectively. According to Eq. (21), three sets of data of c and d can also be obtained, and then the *S-N* curve cluster can be obtained in the rectangular coordinate and double logarithmic coordinate, respectively. That can be shown in Fig. 8 and Fig. 9.

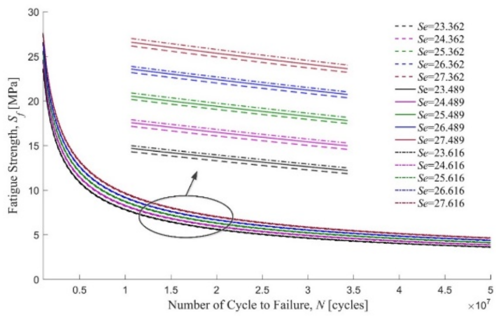

Fig. 8. *S-N* curve cluster in rectangular coordinate

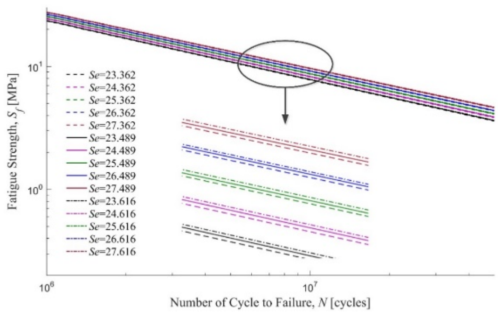

Fig. 9. *S-N* curve cluster in double logarithmic coordinate

CONCLUSIONS

In this paper, the low alloy steel AISI 8630M is taken as the research object, and the influencing factors of stochastic fatigue strength are analyzed, then fatigue strength is predicted. The main conclusions are summarized as follows:

(1) A method of expanding sample data using the normal distribution is introduced to analyze the parameter estimation of small sample data. Compared with the traditional and Bootstrap methods, the interval estimation accuracy are improved by 91.07% and 89.52%, respectively.

(2) The fatigue modified factors with uncertainty are determined from six aspects: surface, dimension, loading, temperature, miscellaneous effect and roughness, and then the stochastic fatigue strength is obtained as $S_f = cN^d$, where c and d approximately follows the skewed normal distribution (231.529, 163.282, 12.232) and (-0.147, 0.060, -3.756), respectively.

(3) This data analysis method proposed in this paper can also be applied to the statistics inference of small sample data in other engineering fields. Moreover, the distribution characteristic of fatigue strength and *S-N* curve cluster obtained in this paper can provide a basis for the fatigue life prediction of wheel hub.

CONFICTS OF INTEREST

The authors declare no conflicts of interest.

REFERENCES

Budynas-Nisbett, "Shigley's mechanical engineering design", 8th edition. McGraw-Hill Primis (2006).

Chen W, Kitamura T, Feng M, "Creep and fatigue behavior of 316L stainless steel at room temperature: experiments and a revisit of a unified viscoplasticity model," International Journal of Fatigue, Vol. 112 pp. 70-77 (2018).

Coretto P, Hennig C, "Maximum likelihood estimation of heterogeneous mixtures of Gaussian and uniform distributions," Journal of Statistical Planning & Inference, Vol. 141 No. 1 pp. 462-473 (2011).

Cumming JA, Goldstein M, "Small sample Bayesian designs for complex high-dimensional models based on information gained using fast approximations," Technometrics, Vol. 51, No. 4, pp. 377-388 (2009).

Dwivedi AK, Mallawaarachchi I and Alvarado LA, "Analysis of small sample size studies using nonparametric bootstrap test with pooled resampling method," Statistics in Medicine, Vol. 36, No. 14, pp. 2187-2205 (2017).

Efron B, "Bootstrap methods: another look at the Jackknife," The Annals of Statistics, Vol. 7, No. 1, pp. 1-26 (1979).

Fisher RA, "On the mathematical foundations of theoretical statistics," Philosophical Transactions of the Royal Society, Vol. 222, pp. 309-368 (1922).

Fu WJ, Carroll RJ and Wang S, "Estimating misclassification error with small samples via bootstrap cross-validation," Bioinformatics, Vol. 21, No. 9, pp. 1979-1986 (2005).

Ge HY, Liu XT, Wang X, Wang XL, Wang HJ, "Effect of fisheye failure on material performance inducing error circle under high cycles," International Journal of Damage Mechanics, doi: 10.1177/10567895211020068 (2021).

Guagliano M and Vergani L, "An approach for prediction of fatigue strength of shot peened components," Engineering Fracture Mechanics, Vol. 71, No. 4, pp. 501-512 (2004).

Hansen AM, Garde AH and Eller NH, "Estimation of individual reference intervals in small sample sizes," International Journal of Hygiene & Environmental Health, Vol. 210, No. 3-4, pp. 471-478 (2007).

Leuders S, Vollmer M, Brenne F, et al., "Fatigue strength prediction for Titanium alloy TiAl6V4 manufactured by selective laser melting," Metallurgical & Materials Transactions A, Vol. 46, No. 9, pp. 3816-3823 (2015).

Li SS, Liu XT, Wang XL, Wang YS, "Fatigue life prediction for automobile stabilizer bar," International Journal of Structural Integrity, Vol. 11, No. 2, pp. 303-323 (2019).

Liu XT, Kan FC, Wang HJ, Xin XF, Wang ZQ, Huang H, "Fatigue life prediction of clutch sleeve based on abrasion mathematical model in service period," Fatigue & Fracture of Engineering Materials & Structures, Vol. 43, No. 3, pp. 488-501 (2020).

Liu XT, Wang HJ, Wu Q, Wang YS, "Uncertainty-based analysis of random load signal and fatigue life for mechanical structures," Archives of Computational Methods in Engineering, doi: 10.1007/s11831-021-09579-6 (2021).

Liu XT, Wu Q, Su SC, Wang YS, "Evaluation and prediction of material fatigue characteristics under impact loads: review and prospects," International Journal of Structural Integrity, doi: 10.1108/IJSI-10-2021-0112 (2022).

Liu XT, Yu XG, Tong JC, Wang X, Wang XL, "Mixed uncertainty analysis for dynamic reliability of mechanical structures considering residual strength," Reliability Engineering & System Safety, Vol. 209, pp. 107472 (2021).

Liu YJ, Kang GZ, Gao Q, "A multiaxial stress-based fatigue failure model considering ratcheting-fatigue interaction," International Journal of Fatigue, Vol. 32, No. 4, pp. 678-684 (2010).

Luo SH, Nie XF, Zhou LC, et al., "High cycle fatigue performance in laser shock peened TC4 Titanium alloys subjected to foreign object damage," Journal of Materials Engineering & Performance, Vol. 27, No. 3, pp. 1466-1474 (2018).

Marin J, "Mechanical behavior of engineering materials," Journal of the Franklin Institute (1962).

Mossaab BM, Zemri M, Arab M, "Effect of medium carbon steel microstructure on tensile strength and fatigue crack growth," International Journal of Structural Integrity, Vol. 10, No. 1, pp. 67-75 (2019).

Mischke CR, "Prediction of stochastic endurance strength," Journal of Vibration Acoustics Stress & Reliability in Design, Vol. 109, No. 1, pp. 113 (1987).

Nykänen T, Björk T, Laitinen R, "Fatigue strength prediction of ultra high strength steel butt-welded joints," Fatigue & Fracture of Engineering Materials & Structures, Vol. 36, No. 6, pp. 469-482 (2013).

Park YS, Sohn IS, Bae DH, "Fatigue strength assessment including welding residual stress of spot welded joints subjected to cross-tension loads," International Journal of Automotive Technology, Vol. 15, No. 5, pp. 765-771 (2014).

Sakamoto J, Lee YS, Cheong SK, "Effect of surface flaw on fatigue strength of shot-peened medium-carbon steel," Engineering Fracture Mechanics, Vol. 133, pp. 99-111 (2015).

Sheng Z, Xie QS, Xie CY, "Probability theory and mathematical statistics," 4th ed. Biejing: High Education Press 149-171 (2008).

Shigley JE, Mitchell LD, "Mechanical engineering design," 4th edition. McGraw-Hill (1983).

Tsai YT, Wang KS, "A study of reliability analysis of fatigue life for dental implants," Journal of the Chinese Society of Mechanical Engineers 36(5): 439-448 (2015).

Temme K, Verstraete F, "Quantum chi-squared and goodness of fit testing," Journal of Mathematical Physics, Vol. 56, No. 1, pp. 1804-4490 (2015).

Vořechovský M, "Correlation control in small sample Monte Carlo type simulations II: Analysis of estimation formulas, random correlation and perfect uncorrelatedness," Probabilistic Engineering Mechanics, Vol. 29, No. 7, pp. 105-120 (2012).

Vořechovský M, Novák D, "Correlation control in small-sample Monte Carlo type simulations I: A simulated annealing approach," Probabilistic Engineering Mechanics, Vol. 24, No. 3, pp. 452-462 (2009).

Wang HJ, Liu XT, Wang XL, Wang YS, "Numerical method for estimating fatigue crack initiation size using elastic-plastic fracture mechanics method," Applied Mathematical Modelling, Vol. 73, pp. 365-377 (2019).

Wang HJ, Liu XT, Zhang MH, Wang YS, Wang XL, "Prediction of material fatigue parameters for low alloy forged steels considering error circle," International Journal of Fatigue, Vol. 121, pp. 135-145 (2019).

Wang HJ, Xuan FZ, Liu XT, "Prediction and evaluation of fatigue life under random load based on low load strengthening characteristic," International Journal of Fatigue, Vol. 151, pp. 106346 (2021).

Wang ML, Liu XT, Wang XL, Wang YS, "Probabilistic modeling of unified S-N curves for mechanical parts," International Journal of Damage Mechanics, Vol. 27, No. 7, pp. 979-999 (2018).

Wang ZQ, Chen W, "Confidence-based adaptive extreme response surface for time-variant reliability analysis under random excitation," Structural Safety, Vol. 64, pp. 76-86 (2017).

Wormsen A, Avice M, Fjeldstad A, et al., "Base material fatigue data for low alloy forged steels used in the subsea industry. Part 1: In air S–N data," International Journal of Fatigue, Vol. 80, pp. 477-495 (2015).

Wu H, Hamada S, Noguchi H, "Fatigue strength prediction for inhomogeneous face-centered cubic metal based on Vickers hardness," International Journal of Fatigue, Vol. 48, No. 3, pp. 48-54 (2013).

Wu JN, Yan SZ, Zuo M, "Evaluating the reliability of multi-body mechanisms: A method considering the uncertainties of dynamic performance," Reliability Engineering and System Safety, Vol. 149, pp. 96-106 (2016).

Yuan XL, Li CY, "An engineering high cycle fatigue strength prediction model for low plasticity burnished samples," International Journal of Fatigue, Vol. 103, pp. 318-326 (2017).

Zhang MH, Liu XT, Wang YS, Wang XL, "Parameters distribution characteristics of material fatigue life using improved bootstrap method," International Journal of Damage Mechanics, Vol. 28, No. 5, pp. 772-793 (2019).

Zhao S, Engelhardt BE, Mukherjee S, et al., "Fast moment estimation for generalized latent Dirichlet models," Journal of the American Statistical Association, Vol. 113, No. 524, pp 1528-1540 (2018).

Zhu SP, Ding L, Liu Q, Correia JAFO, Jesus AMPDe, "Nonlinear fatigue damage accumulation: Isodamage curve-based model and life prediction aspects," International Journal of Fatigue, Vol. 128, pp. 105185 (2019).

Zeng D, Tian G, Liu F, et al., "Fatigue strength prediction of drilling materials based on the maximum non-metallic inclusion size," Journal of Materials Engineering & Performance, Vol. 24, No. 12, pp. 4664-4672 (2015).

NOMENCLATURE

d_0 sample diameter
f the ratio of sample fatigue strength and tensile strength
k_a surface modified factor
k_b dimension modified factor
k_c loading modified factor
k_d temperature modified factor
k_e miscellaneous effect modified factor
k_r roughness factor
N number of cycles
P_e exact probability
R stress ratio
R_a roughness
$S_e^{'}$ fatigue limit of sample
S_e fatigue limit of actual part
S_f fatigue strength
$S_{\max}$ maximum stress
$S_{\min}$ minimum stress
S_{ut} tensile strength
T_F service temperature
W_n^2 carmer-von mises statistic
ΔS stress range
α significance level

基於材料力學性能的疲勞強度預測與評估

劉新田 王海傑 楊小兵
上海工程技術大學

賴家鳳
北部灣大學

摘要

零件的疲勞失效除受應力影響外，還受資料疲勞強度的影響。因此，在進行疲勞壽命分析之前，有必要對資料的疲勞強度進行預測。考慮影響疲勞強度的不確定性因素，確定了疲勞修正係數，並對隨機疲勞強度進行了預測。其中，通過改進Bootstrap方法分析了粗糙度因數的分佈類型和參數，為小樣本分析提供了一種方法。得到了表徵疲勞效能的S-N曲線簇，為疲勞壽命預測提供了依據。