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Strength And Reliability Analyses For A Small Teeth Difference Mechanism Based On FEA

Yuo-Tern Tsai*，Kuo-Shong Wang** and Kuan-Hong Lin***

Keywords: speed-reduced mechanism, design, stress analysis, reliability.

ABSTRACT

Speed-reduced mechanisms designed with small teeth difference (STD) possess many excellent features such as small volume, compact structure, and high reduced ratios, etc. This paper presents the designed methods of STD mechanisms including interference analysis and stress evaluation of the paired gears. The interference conditions are investigated for giving proper shifting amounts for the gears to avoid meshing interferences. The geometric models of a STD mechanism are created using CAD software for performing interference analysis and stress evaluation. The stresses of the gear sets are simulated using Finite Element Analysis (**FEA**). An evaluated procedure of stress variations is proposed. The reliabilities of the mechanism are further rated according to the analyzed stresses so that the allowed loads can be decided accordingly. The study is useful in structural design, stress analysis and reliability evaluation for a reducer designed with involute gears.

INTRODUCTION

Speed-reduced devices are extensively applied in many modern industrial types of machinery such as automatic mechanisms, machine tools, and robots, etc. It stands for the development of the mechanical industry in transmission technology. Speed-reduced devices with great transmission ratios are frequently discussed due to their particular functions. Different speed-reduced devices are designed with various transmitted techniques. Several types of speed reducers such as wave gearing devices, trochoid gear reducers, hypo-cyclic gear reducers, and james ferguson-type planetary drives are commonly applied in industrial products. The comprehensive researches about the designing and strength calculation for these reducers had been reported by Li (2014). He studied the theories of a contact problem and numerical analysis of a planetary drive mechanism. Tsai et al. (2017) presented a new design of speed reducers and analyzed its structural stresses. The fatigue life of the reducer is further studied by Tsai et al. (2018).

The small teeth difference (STD) mechanisms are designed primarily based on two sets of involute gears for obtaining high speed reduced ratios. A pair of crank mechanisms connecting to the gears is designed to meet the needs of transmitting while the input shaft rotates. The advantages of one pair of cranks inputs are that the loads acting on the bearing can be dropped when the inner gear is regarded as the planetary wheel. The designing of STD mechanisms about motion, transmission ratios and efficiency had been reported by many scholars in the past. For example, Macovei et al. (2015) presented a short overview of the types of STD mechanisms designed with internal gears. Meshing interferences are frequently encountered because the tooth-number differences are very small. Maiti and Roy (1996) examined the possibility of lowering the difference as much as possible in the internal-external gear pair with the help of simple gear corrections and suggested a mathematical form to investigate the conditions of avoiding tooth tip interference. Sensinger (2013) proposed a method for analyzing stress and predicting efficiency based on varied torque ratio, which is useful for evaluating the benefits and faults of different types of reducers.

The cycloidal speed-reduced device is another application of STD mechanisms. Lin et al. (2014) presented the design of a new two-stage cycloidal speed reducer with tooth modifications. The topological structure of cycloidal drives is discussed and analyzed with the aid of graphs. Hsieh (2015) proposed a design with multi-tooth differences and derived a model to avoid undercutting problems of gears as well as improving the design. Xu et al. (2016) proposed a method for analyzing the contact dynamics of multi-tooth meshing by considering the influences

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* *Professor, Department of Mechanical Engineering, Hungkuo Delin University of Technology, 236, Taiwan, R.O.C.*

** *Professor, Department of Mechanical Engineering, National Central University, Chungli, Taiwan, ROC.*

*** *Dr., Department of Mechanical Engineering, Tungnan University, 222, Taiwan, R.O.C.*

of the turning-arm cylindrical roller bearing, established the dynamic model and modeled the multi-point contact using non-linear contact forces. Wang et al. (2016) proposed an optimization methodology based on genetic algorithm for simultaneously minimizing the volume and maximizing efficiency of a cycloid speed reducer.

The static and dynamic properties of the contact teeth are the primary analyzed jobs while designing the STD mechanisms. The mathematical models of multi-tooth contacts are a topic of studying. Huang and Tsai (2017) proposed a computerized approach of loaded tooth contact analysis based on the influence coefficient method, either for the contact tooth pairs of the involute stage or of the cycloid stage. Although many papers had reported the analyzed methods of gear transmission, the meshing problems of gears such as interference, profile design stress and reliability, etc., are worthy to be studied because they are important in designing a STD mechanism. Reliability is more and more emphasized in recent engineering design for ensuring the safety. For example, Tsai et al. (2013) proposed the methods of reliability design for practical applications based on modelling processes. Reliability analysis based on experimental data is another topic in predicting the mean time between failure of a design (Tsai et al, 2013, 2015).

In this paper, a STD mechanism designed with involute gears is proposed for performing interference checks, stress analysis, and reliability prediction. The interference conditions of the gears are investigated for giving proper shifting amounts for the tooth's profile so that the meshing interferences can be avoid. Finite element analysis (**FEA**) is used as a tool to evaluate the contact stresses. The studied results showed that the maximum stress occurs on the smaller gear set and the root bending stress would dominate the fracture of the gears. An evaluated approach of stress variation in **FEA** is proposed for performing reliability evaluation in cooperation with stress-strength interference theories. The reliabilities of the mechanisms with respect to the loads can then be decided for which provides an index of safety of the mechanism in use.

DESIGN OF STD MECHANISMS

STD mechanisms are designed based on two sets of internal gears which exist small tooth number difference. The mechanisms possess the properties, large reduction ratios, structure compactness, and small volume compared with the traditional multiple wheel train. A design of STD mechanisms is shown in **Figure 1**. The speed reduced ratios of the mechanism are created by 2 sets of gear trains in the planetary rotation. The planetary gear trains include two stages, the front stage (input side) composed by gears (1, 2) and the rear stage (output side) formed by gears (3, 4).

The reduction ratios can be calculated according to the theories of the epicyclical gear train (Martin, 2002). The formula of the speed ratio of the mechanism can be derived as

$$s = 1 - \frac{z_1/z_2}{z_4/z_3} = 1 - \frac{i_a}{i_b} = \frac{z_2 z_4 - z_1 z_3}{z_2 z_4} \tag{1}$$

where (z_1, z_2) and (z_3, z_4) are the numbers of teeth in the front and rear stages, (i_a, i_b) standing for the gear ratios of the two stages, respectively. The rotating direction of the output shaft is influenced by the scales of i_a and i_b as shown in Eq.(1). If i_a is less than i_b, the output shaft rotates in the same direction of the input shaft, and if i_a is greater than i_b, it rotates reversely.

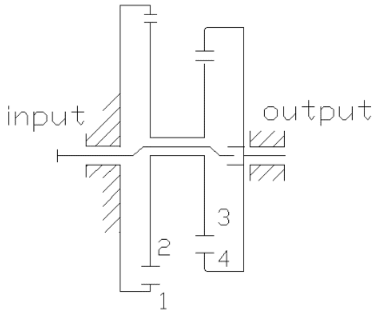


Figure 1. A speed-reduced mechanism

The eccentric amounts in the front stage and in the rear stage are usually designed to the same for obtaining smoothly meshing transmission. Based on the same eccentric amounts, various combinations including tooth's profile, modules and tooth number differences in each stage can be designed according to the speed-reduced ratios. Tooth number differences marked as Δz in the two stages can be set to the same or not the same. If the tooth number differences are set to the same, the modules of the gears in the two stages would be the same. If the tooth number differences between the two stages are not the same, the different modules need to be designed for the gears. It is expressed as

$$\Delta z_a m_a = \Delta z_b m_b \tag{2}$$

where Δz_a, m_a is the tooth number differences and the modules of stage a, respectively. This is necessary for satisfying the eccentric amounts of the two stages being identical.

Normally, the tooth number differences and the modules in the two stages are set to the same for convenience in manufacturing. The identical eccentric amounts of the two stages can be obtained by properly shifting the cutting positions using the same cutter. If the tooth number differences in the two stages are set to the same as (z_1 - z_2)= (z_4 - z_3)=Δz, Eq.(1) can be rewritten as

$$s = \frac{\Delta z(z_2 - z_3)}{z_2 z_4} \tag{3a}$$

$$s = \frac{\Delta z(z_1 - z_4)}{z_2 z_4} \tag{3b}$$

The above equations indicate that the maximum tooth number of the four gears is restricted, i.e. $z_2 > z_3$ or $z_1 > z_4$. The maximum reduction ratio can then be obtained as the condition either z_2-z_3=1 or z_1-z_4=1. As a result, the maximum reduced ratio would be

$$s_{\max} = \frac{\Delta z}{z_2 z_4} \tag{4}$$

Different tooth number differences and modules can be adopted for the two stages to obtain a high reduction ratio. If the tooth number differences in the two stages are set to not the same, an ultimately reduced ratio can be obtained by setting the tooth numbers of the gears satisfying the relation as $z_2 \cdot z_4 - z_1 \cdot z_3 = 1$. Then, the ultimately reduced ratios would be

$$s_u = \frac{1}{z_2 z_4} \tag{5}$$

On the other hand, if the tooth number differences between the two stages are all set to one (Δz=1), the ultimately reduced ratio can be obtained. However, the design with one tooth number differences (Δz=1) may be unpractical owing to interference problems of gear meshing. A feasible approach to eliminate interferences as well as obtaining good meshing transmission is through modifying tooth's profiles. A commonly adopted method for modifying tooth's profiles in involute gears is to use profiles shifting for processing the interference problems of the paired gears.

A high reduction scales from 1/30 to 1/10000 by allocating proper tooth numbers for the four gears. The reduction ratios of the mechanisms in some values may be unable to be obtained due to the constraints of the tooth number differences. However, an approximate ratio can be obtained by allocating the proper tooth numbers for the gear sets. For example, the reduced ratio 1/50 can't be obtained if the tooth number difference is Δz=5 and the teeth of the smallest one is z=45. An approximate ratio can be obtained by allocating proper tooth numbers for the gears. Several designs of gear sets for reduction ratio approximating 1/50 are listed in **Table 1**.

Table 1. Designs of the paired gears for speed ratio approximating 1/50

Tooth number differences (Δz)	Speed ratios (S)	Gears (z_2, z_1), (z_3, z_4)
5	1/50.9	(56, 61), (45, 50)
	1/49	(49, 54), (40, 45)
4	1/49	(60, 64), (45, 49)
	1/51	(51, 55), (40, 44)

A mechanism designed based on STD theories is proposed for obtaining high speed-reduced ratios. The mechanism primarily consists of two sets of paired internal gears and one off-center cam which are constructed using SolidWorks software as shown in **Figure 2**.

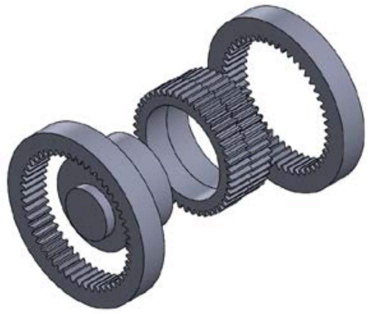


Figure 2. Design of an STD mechanism

The gear teeth are designed with 20° involute profile where the module is m=1 and the thickness is 10 mm. The tooth number differences between the two stages are set to the same as Δz=5 and the tooth numbers for the gear sets are (z_1=51, z_2=46), (z_3=45, z_4=50). The speed-reduced ratio of the design would be 1/460. The design is checked by interference analysis and motion simulation in CAD software. The motion simulations show the speed-reduced ratio being the same as the formula calculation representing the design being correct. Interference analysis indicates that the gear sets exist in interference conditions. The less the tooth number difference, the more the teeth occur interferences. To eliminate the meshing interferences, modifying the tooth's shape is necessary.

INTERFERENCES REMOVING

This section reported the interference problems of the inner gears and investigated the needed shifting amounts for the gear sets so that the meshing interferences can be eliminated.

Interference types

Meshing transmission of inner gears is taken place by the concave profile of the inner teeth and the convex profile of the external teeth. This kind of meshing is beneficial in motion conveying and stress of the contact teeth. The real segments of action of the inner gear are larger than those of the external gear. Engaging in the inner and external gears is shown in **Figure 3**. The meshing of inner gears occurs only on the inside of the engaging line if the paired gears have a minimum tooth number. To avoid the tooth tips of the bigger wheel intersect the base pinion teeth during meshing, the connection of the base circles of the external teeth to the inner circle must be designed with a special form.

There are three types of interference may occur for internal gears transmission: (a) involute interference, (b) trochoid interference and (c) trimming interference (KHK, 2015).

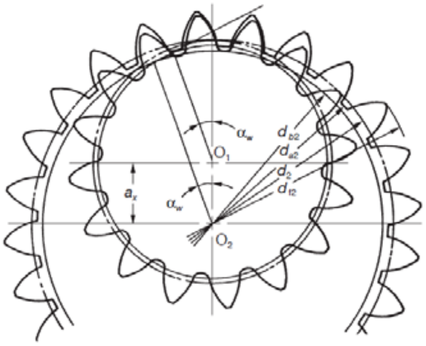


Figure 3. Engaging of the inner and the external gears

(a) Involute interference

This problem occurs when the number of teeth of the external gear is too small. The dedendum of the external gear and the addendum of the internal gear will generate interferences. The engaging of the paired gears must contact on the tangent line of the two base circles. This interference can be eliminated by designing the gears satisfying the condition,

$$\frac{z_1}{z_2} \geq 1 - \frac{\tan\alpha_{a2}}{\tan\alpha_w} \tag{6}$$

where z_1 and z_2 are the tooth numbers of the pinion and the wheel, respectively, α_{a2} being the pressure angle of a tooth tip of the internal gear and α_w being the working pressure angle.

(b) Trochoid interference

This problem happens when the differences between the teeth of the paired gears are too small. The addendum of the external gear can't smoothly engage with the dedendum of the internal gear. The tooth tips of the external gear will insert into the roots of the teeth of the internal gear. This interference can be avoided by satisfying the following equation

$$\theta_1 \frac{z_1}{z_2} + inv\alpha_w - inv\alpha_{a2} \geq \theta_2 \tag{7}$$

where θ_1, are θ_2, are half of the top land angles of the outside circle (pinion) and the inside circle (ring gear), respectively. The $inv\alpha_w$ indicates the involute function of pressure angles which are defined as $inv\alpha = \tan\alpha - \alpha$. In the meshing of external gear to a standard internal gear with α =20°, the trochoid interference can be avoided if the tooth number difference ($z_2 - z_1$) is larger than 9 (KHK, 2015).

(c) Trimming interference

This problem occurs in the radial direction to prevent pulling the gears apart. The gears must be engaged by sliding the gears in connection with the axial motion. This type of interferences takes place during engaging and disengaging of the paired gears. If the tooth numbers of the two gears are very closed, this interference tends to happen. The following equation needs to be satisfied for preventing this type of interferences.

$$\theta_1 + inv\alpha_{a1} - inv\alpha_w \geq \frac{z_2}{z_1}\left(\theta_2 + inv\alpha_{a2} - inv\alpha_w\right) \tag{8}$$

This type of interference can occur in the process of cutting an internal gear with a pinion cutter. If it happened, there is a danger of breaking the tools.

Interferences of gears involve many geometric factors such as addendum height, fillet, and backlash, etc. The profiles of the paired gears are constructed using macro codes built-in SolidWorks. The teeth of the big inner gear are fixed to 50 (z_2) and the teeth of the small external gear (z_1) are adjustable for generating the gear sets with tooth number differences. Interferences of the gear sets are checked one by one. For standard involute, the gear set would not occur interferences if the tooth number differences are larger than 25. Adding backlashes and fillets of the gears can improve the interference conditions. If the backlashes are set to 0.1 mm, the tooth number difference of no interference can reduce to 9 teeth. This denotes that the preceding three interferences can be eliminated if the gear set is designed with the backlashes and the tooth number difference. The engagement of the gear set designed with standard involute, backlashes 0.1 mm and 9-tooth difference is shown in **Figure 4**.

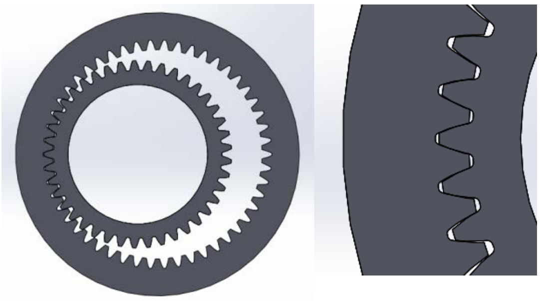
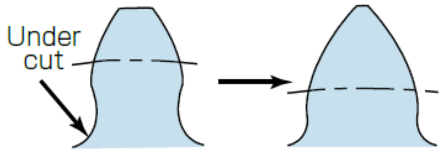


Figure 4. Engagement of the gear set with 9 teeth difference (m=1, z_1=41, z_2=50)

Profile shifting

Meshing Interferences of the gear teeth can be solved by profile shifting. Profile shifting can be used not merely to prevent undercut, but also to adjust the center distance between two gears. Diez-Ibarbia et al. (2016) reported the influence of profile shifting to the spur gear efficiency and found an increase in the profile shift would influence the load-sharing properties, thus lowering the transmission efficiency. Abderazek et al. (2015) suggested the profile shift coefficients in their optimization process and used a differential evolution algorithm to determine the optimal profile shift values for an arbitrary pair.

For a spur gear, the tooth profiles are changed as well as the tooth thickness increased while a positive shifting is added for the tooth profiles, meanwhile, the outside diameter (Tip diameter) also becomes larger. Positive correction is effective to prevent undercut of gear with small tooth number. A comparison of the tooth profiles with no-shifting and shifting is shown in **Figure 5**.



(a) no-shifting (b) shifting

Figure 5. Comparison of the tooth's profiles

According to the reports in KHK (2015), profile shifting to prevent undercut for a spur gear must satisfy

$$m - xm \le \frac{zm}{2} - \sin^2 \alpha \tag{9}$$

where xm is the extra feed of gear cutter (mm), x the profile shifted coefficient, m the module of gears and α the pressure angle. The shifted amount of correction is called the extra feed of gear cutter, xm when progressing gear cutting. A positive shift cutting for spur gear by rack form tool to prevent undercut is illustrated in **Figure 6**.

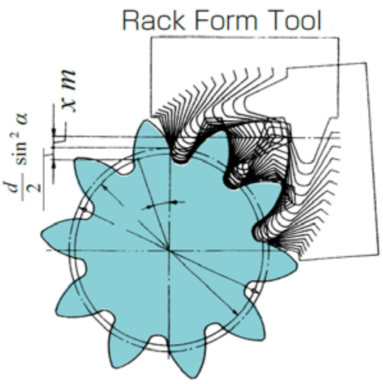


Figure 6. Generation of positive shifted gear (α=20°, z=10, x=+0.5)

This paper investigates the needed shifting amounts of the paired gears with backlashes 0.1 mm and various tooth number differences. The wheel is fixed to z_2= 50 where the tooth's profiles are changed depending upon the shifting amounts. The pinion is designed with standard involute, i.e. x_1=0, and the tooth numbers are adjustable (z_1). The geometric models of the paired gears are created using the macro codes built-in CAD software. The advantages of using macros are that the geometric models can be rapidly created just giving the depended designed variables. The shifting amounts (x_2) are set from 0 to 1.6 mm for generating the gears. The gear sets with one tooth number difference are first checked for giving proper shifting amount. The needed shifting amounts for the gear set with one tooth difference is x_2=0.8 mm. The engagement of the gear set is illustrated in **Figure 7**.

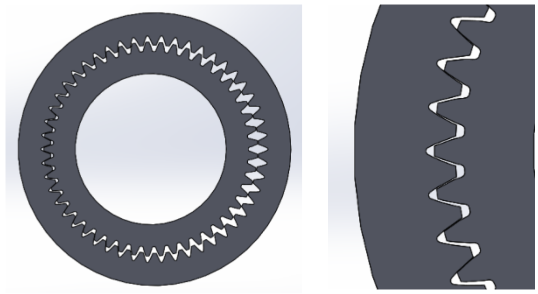


Figure 7. Engagements of the profile shifted gears (m =1, z_1 =49, z_2 =50, x_1 =0, x_2 =0.8)

The needed shifting amounts for the gear sets which tooth number differences are less than 9 are investigated one by one using the same procedures. The investigated results show that the shifting amounts of the gear sets are reversely scaled to the tooth number differences and proportional to the modules (m). The proper shifting amounts for the gear sets which the tooth number differences are less than 9, are shown in **Figure 8**.

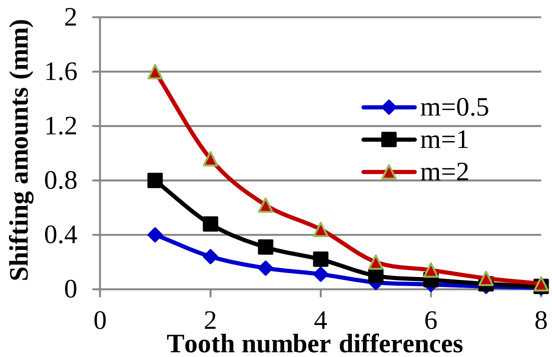


Figure 8. The proper shifting amounts for the gear sets with various tooth differences

The analyzed results can be applied in designing the STD mechanism to avoid meshing interferences of the gears. The interference problems can be dispelled by modifying the tooth's profiles. The larger the shifting amount is, the bigger the tooth belly would be and the tooth tip will become sharp. This phenomenon would easily cause the problems of tooth bursting apart while transmitting.

On the other hand, applying profile shifting on the teeth can also change the center distance of the paired gears. The center distance of the standard gears (without shifting) is half of the sum of the diameters of the two gears. The center distance of the gears will be enlarged when the tooth profiles are shifted. The positive shifting would enlarge the center distance as well as the negative shifting would reduce the center distance. The characteristics of profile shifting gears are as follows:

(A) Positive shifting

(1) The tooth thickness becomes thicker at the root so that the more bending strength will be formed for the teeth.

(2) The center distance of the gears will be increased, meanwhile, the contact ratio becomes smaller and the working pressure angle becomes larger.

(3) The more the shifting is applied, the more sharpen the tooth tip is. If the corrections exceed the limit of shifting, the tooth width at the tip becomes smaller, even turns into sharpening.

(B) Negative shifting

(1) The tooth thickness becomes thinner at the root representing the bending strength of the teeth is smaller compared with the standard teeth.

(2) The center distance of the gears is decreased, meanwhile, the contact ratio becomes larger and the working pressure angle becomes smaller.

(3) The more shifting is applied, the smaller the tooth width at root is. Undercut will occur when the shifting amount exceeds its limit.

STRESS ANALYSIS

The gear rings are the key components of failure since they are the main parts of stress-induced while the mechanism is running. The acting and reaction forces of the mechanism can refer to **Figure 9**.

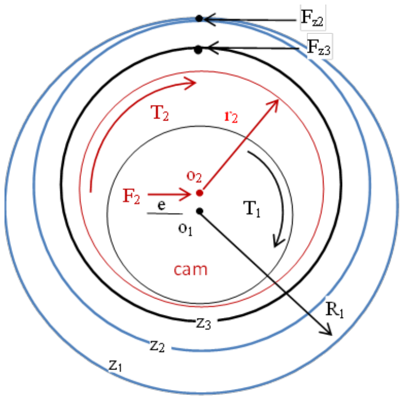


Figure 9. The loading analysis of the gear sets.

The input torque acting on center O_1 being T_1 can be decomposed into both one torque T_2 and one lateral force F_2 acting on center O_2 The lateral force can be expressed as

$$F_2 = \frac{T_1}{(r_2 + e)} \tag{10}$$

where e is the off-center distance. The acting torque T_2 would be

$$T_2 = \frac{r_2}{(r_2 + e)} T_1 \tag{11}$$

The input torque expressed with T_2 and F_2 of the off-center cam would drive the gear rings, (z_2, z_3) to rotate. The contact points on gear rings, (z_1, z_4) would generate one reaction force, F_{z2}, F_{z3}, respectively, to resist the gear rings z_2, z_3 moving.

The induced stresses of the gear sets primarily have both, tooth Root Bending Stress (RBS) and tooth Surface Contacting Stress (SCS). The detailed formulas about stress calculation of gears can refer to the technical booklets (ANSI, 2004). The disadvantages of formula calculation are the mathematical models always involve many unknown coefficients. For simplifying the complexity of analysis, the respective gear set is extracted from the mechanism for performing stress analysis in ANSYS. The geometric models were then imported into DesignModeler to generate the line and surface bodies for analyzing. Structural steels are set as the materials of the models. The augmented Lagrange formulation method is selected for the nonlinear analysis since it involves the nonlinear problems in the interface connection. The inner gear is set as the driving component and the outer gear as the driven component according to the energy flow of power transmission. The connections of the inner-outer teeth are set to frictionless contact. The driving component is set to frictionless support and the driven component is set to fix.

The geometric models of the front and rear gear sets (z_1, z_2)=(53, 48), (z_3, z_4)=(45, 50) are constructed according to the analyzed results in profile shifting. In this example, the shifted amounts of the gear sets are set to 0.1 mm for module 1 according to the results of **Figure 8**. The forces are given according to the loading analysis so that a moment and a bearing force are added to simulate the loads of the off-center cam acting on the gear ring. The half-plane models of the gear sets are adopted in analyzing for simplifying the analyzed procedures and ensuring the converged solutions can be obtained in **FEA.** The meshes on the contacted teeth are densed for obtaining a fine solution. The settings on supporting and loading are shown in **Figure 10**.

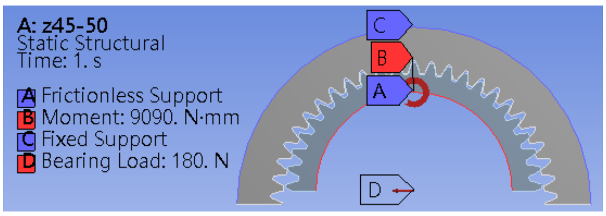
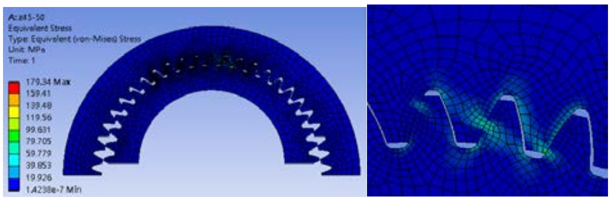
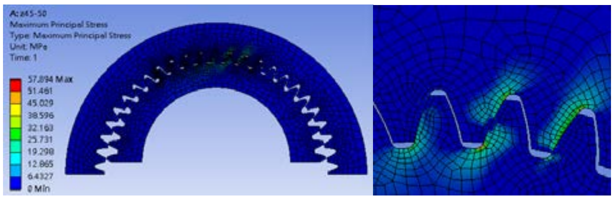


Figure 10. Settings of the supports and the loadings

A converged solution can be obtained based on the settings. The analyzed results for the equivalent stress indicating Surface Contacting Stresses (SCS) and the principle stresses representing Root Bending Stresses (RBS) (Tsai, 2018) are shown in **Figure 11**.



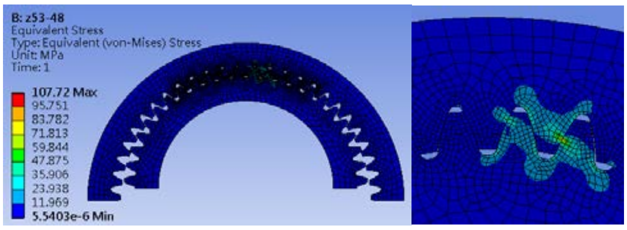
(a) Equivalent stress



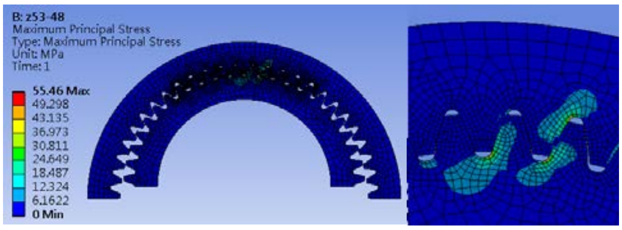
(b) Maximum principal stress

Figure 11. Stresses of the rear gear set z(45-50)

The induced stresses for the front gear set (z_1, z_2) under the same loads can also be obtained as shown in **Figure 12**.



(a) Equivalent stress



(b) Maximum principal stress

Figure 12. Stresses of the front gear set z(48-53)

The analyzed results show that the induced stresses at the rear gear set are larger than those at the front gear set. This implies that the possible failed components of the mechanism would occur at the rear stage, i.e. the smaller gear set. On the other hand, the RBS is obviously smaller than the SCS in the two gear sets.

To picture out the dominated stresses, materials S35C is reviewed where the yielding and tensile strengths are **304 MPa** and **510 MPa,** the allowable bending and contacting stresses are about **180 MPa** and **490 MPa**, respectively, according to the data reported in ANSI (2014). Comparing the strengths with the allowable stresses, we can find that the allowable RBS is about one-third of the tensile strength and the allowable SCS is about equal to the tensile strength (Tsai, 2017). The loading of the rear gear set is added step by step to observe the variety of the stress rising. Particularly, the maximum RBS and SCS would be **180.7 MPa** and **444.2 MPa**, respectively, when the loading is **36 Nm.** The information reveals that bending fracture would prior to contacting fracture for the gear set since the maximum SCS is still lower than its allowable value when the maximum RBS meet to its boundary-value. The results denote that the strength of the mechanism can be evaluated based on the RBS of the rear gear set.

The stresses of the gear sets with different tooth number differences are further studied to observe the influence of tooth number difference to stress. The gear sets are designed with 50 teeth for the outer gears using profile shifting and the teeth of the inner gears are varied from 42 to 49. The geometric models with different tooth number differences are imported into ANSYS one by one to fulfill stress analysis. The RBS and SCS with respect to the tooth number differences for the gear sets are illustrated in **Figure 13**.

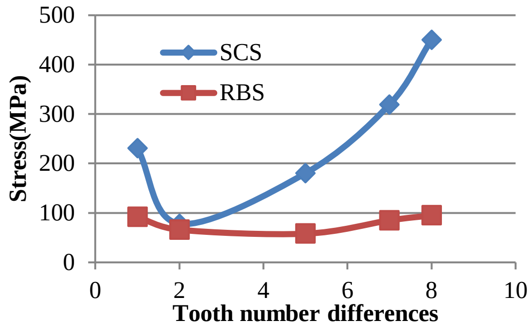
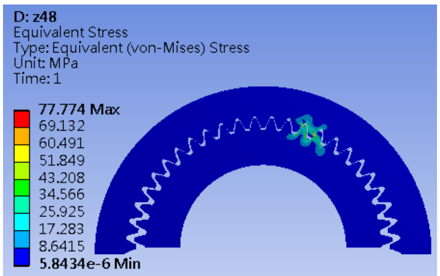


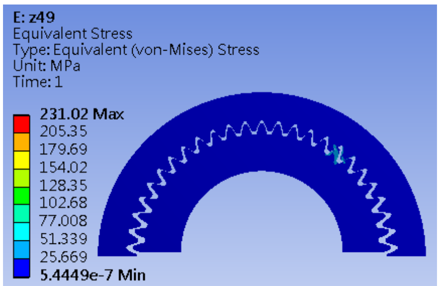
Figure 13. Varying of the stresses corresponding to the tooth number differences

The analyzed results show that the varying of the SCS is more sensitive than that of the RBS with respective to the tooth number differences. Theoretically, the lower the tooth number difference is, the higher the contact ratio, a higher contact ratio has lower stress. In this case, the stresses on one tooth difference are higher than those on two tooth difference which seems not meeting the theories. This condition may be the stresses on one tooth difference being more concentrated on the tooth tips and those on two tooth difference being shared by the more teeth. The varieties of the stresses can be observed from the stress contours. The stress contours of the gear sets with one and two tooth number differences are shown in **Figure 14.**

The stress contours show that the stress areas on one tooth difference are obviously smaller those on two tooth difference. This property explains why the stresses on one tooth difference are higher than those on the two teeth difference. According to the analyzed results, the SCS of the gear set with 2 teeth difference is minimum. It means that the optimal one of the design may be 2 teeth difference when the two stresses are considered simultaneously and the SCS and RBS are 78 and 66 MPa, respectively.



(a) Two teeth difference



(b) One tooth difference

Figure 14. The stress contours of the gear sets

RELIABILITY PREDICTION

Reliability prediction of the mechanism is done by integrating the analyzed stresses in FEA with stress-strength interference (SSI) theories. Reliability evaluation is formulated based on probabilistic distributions of the strengths and the stresses. The strength random variable of a design, X, is supposed as a normal distribution with a mean value, μ_X, and standard deviation, σ_X, as well as the stress random variable Y is also normal distribution with parameters μ_Y and σ_Y. The reliability can be defined as

$$R = \Phi(z) = \Phi\left[\frac{\mu_X - \mu_Y}{\sqrt{\sigma_X^2 + \sigma_Y^2}}\right] \qquad (12.a)$$

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) dz \qquad (12.b)$$

where $\Phi(\,)$ means the cumulative distribution function of normal distribution and z is the reliability index.

A mechanical system usually has many failure modes such as fatigue, wear out and corrosion, etc., because it always consists of many components or units. Different components have various contributions to the failure modes of the system as well as they have various weights to the system failures. The reliability of a system can be evaluated based on the probabilities of the failure modes occurring. If the failure modes are mutually independent, the failure probabilities of a system can be regarded as a combination of all failure modes in a series relationship. The reliability of the system in a series relationship is expressed as

$$R_s = R_1 R_2 \cdots R_n = \prod_{i=1}^{n} R_i \qquad (13)$$

where R_s is the reliability of the system, R_i is the reliability of the i-th failure mode or component. Aiming to the differential mechanism, the possible failure modes of the gear sets have two, tooth root breakage and surface fracture caused by the repeated RBS and SCS respectively. Considering the failure modes, the reliability of the system can be defined as

$$R_s = R_b R_c \qquad (14)$$

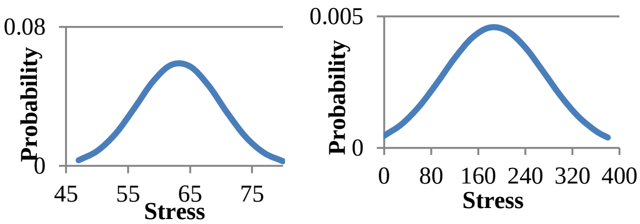
where R_b, R_c is the reliabilities of resisting bending and contacting fractures, respectively.

The geometric errors of the gear sets are frequently taken place in manufacturing such as backlashes, tooth tip fillets, and assembly errors. The geometric errors usually lead to the contact points changing which induces various stresses during transmission. The geometric errors are simulated in **FEA** by setting an offset value of the interface connection to the models. The offset values are set to ±0.1 mm for simulating the geometric errors. The geometric models with offset values are loaded into ANSYS one by one to evaluate the stresses. The evaluated values for RBS and SCS under loading 10 Nm at various offsets are listed in **Table 2.** The evaluated stresses exhibit a linear increasing depending upon the offset values.

Table 2. The evaluations of the RBS and SCS at different geometric errors

Offsets (mm)	RBS(MPa)	SCS(MPa)
-0.1	62.7	94.2
-0.075	62.8	92.8
-0.05	62.3	95.5
-0.025	60.5	150.5
0	**57.9**	**179.3**
0.025	55.1	211.5
0.05	60.9	247.0
0.075	69.1	285.9
0.1	78.1	328.2
μ_Y	**63.25**	**187.21**
σ_Y	**6.75**	**87.47**
COV	**0.11**	**0.47**

Considering the random properties of the geometric errors, the offsets occurring in real conditions can be regarded as normal distribution so that the induced stresses can be expressed as normal distribution (**see Figure 15**).



(a) RBS (b) SCS

Figure 15. Stress distributions at load 10 Nm

The means (μ_Y), standard deviations (σ_Y) including the variation coefficients (COV) of the stresses can then be obtained according to the evaluations. The COV of the SCS is about 4 times of that of the RBS. This implies that the SCS has a larger variation than the RBS. Making use of the COV, the stress distributions of the gear rings at the other loads can also be calculated when the stress means are given. No sooner is the stress distribution established, the reliabilities of the mechanism can be computed in cooperation with the strength distributions.

In this case, the yielding strength (bending strength, μ_{XB}) for materials S35C are 304 MPa, and the allowable RBS and SCS are (180, 490) MPa, respectively (ANSI/AGMA, 2004). The strength can be regarded as resisting bending fracture. The contacting strength can be rated by setting the same scales as the yielding to the allowable RBS. As a result, the contacting strength will be 828 MPa. The strength variations are set to 20 percent of the means. The strength information of RBS and SCS for this example would be $(\mu_X, \sigma_X)_B = (304, 61)$ and $(\mu_X, \sigma_X)_C = (828, 166)$ MPa, respectively.

The induced RBS and SCS indicating the means (μ_Y) for the gear set z(45-50) under no connection offset conditions for various torques are further evaluated by **FEA** as listed in **Table 3**.

Table 3. The RBS and SCS of the gear ring at different loads.

Input T(Nm)	RBS (MPa)	SCS(MPa)
5	33.8	104.6
10	**57.9**	**179.3**
15	82.2	253.6
20	**106.5**	**326.3**
25	131.1	398.9
30	153.9	418.9
35	177.1	439.2
40	202.8	459.4
45	229.8	479.2
50	256.8	492.6
55	283.7	506.3
60	310.7	517.6

The stress variations of the RBS and SCS can be decided according to the COV obtained in the previous paragraphs. For example, input torque T=20 Nm, the stress variations would are $\sigma_{YB} = 0.11 \times 106.5 = 11.7$ MPa and $\sigma_{YC} = 0.47 \times 326.3 = 153.4$ MPa. Combining the strength and stress information, the reliabilities of the mechanism corresponding to the loads can be obtained by Eq.(12). For example, the load 20 Nm, the RBS and SCS would be $(\mu_Y, \sigma_Y)_B = (106.5, 11.7)$ MPa and $(\mu_Y, \sigma_Y)_C = (326.3, 153.4)$ MPa, respectively. The reliabilities can be calculated by Eq.(12) as

$$R_b = \Phi\left(\frac{304 - 106.5}{\sqrt{61^2 + 11.7^2}}\right) = \Phi(3.18) = 0.999$$

$$R_c = \Phi\left(\frac{828 - 326.3}{\sqrt{166^2 + 153.4^2}}\right) = \Phi(2.223) = 0.987$$

The system reliability can be obtained by Eq.(14) as R_s =0.986. The reliability changings corresponding to the loads are illustrated as shown in **Figure 16**.

The reliability degradation to the RBS is faster than that to the SCS. This denotes that the reliabilities of the reducer are primarily dominated by the RBS. Making use of the information, the safety of loading can be further decided for the reducer. Considering bending fracture, the allowable load for the design would be 36 Nm. The system reliability would be R_s=0.9 under the load. In contrast, the corresponding RBS and SCS including the loads can be decided according to the curves if the reliability need given.

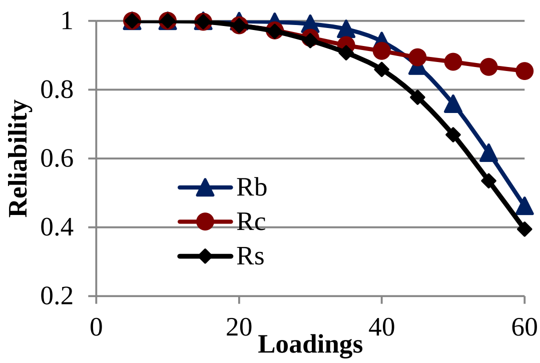


Figure 16. Reliability changings with respect to the loads

CONCLUSIONS

This paper reported the methods of designing and analyzing based on two-stage gear sets designed with STD. A high reduction ratio from 1/30 to 1/10000 can be obtained for the speed-reduced mechanism by allocating proper tooth numbers. A parametric designed approach is programed based on macros so that the geometric models of the paired gears can be generated rapidly in CAD software. The structural stresses including RBS and SCS are evaluated using **FEA** based on nonlinear contact analysis. An evaluated procedure of loading-related reliability is proposed for giving a risk index of the mechanism used at various loads. Several remarks are drawn out as follows.

1. The proper shifting amounts of the gear sets with STD are studied. The need shifted amounts of the gear sets are reversely scaled to the tooth number differences and are proportional to the modules.
2. The gear sets designed with two teeth differences may be the best one from the aspect of stress failure. The SCS is more sensitive than the RBS on the varying of stress to tooth number difference.
3. The maximum stresses of STD mechanism occur at the rear gear set (the smaller gear set). RBS would

dominate the failure of the gear set.

4. An evaluated method of stress variation based on **FEA** is proposed for simulating the effects of the geometric errors. The stress variation of the SCS is about 4 times of that of the RBS.

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小齒差機構基於有限元素法的強度和可靠度分析

蔡有藤

宏國德霖科技大學機械系

王國雄

國立中央大學機械系

林寬泓

東南科技大學機械系

摘要

減速機構設計用小齒差（STD）具有很多優良特性，如體積小、結構緊湊，減速比高等。本研究發表小齒差機構的設計方法，包括干涉分析，應力估計，使用移位齒形消除干涉，本文研究不同齒數差齒輪齧合無干涉移位量，以為設計 STD 機構齒輪基礎。用 CAD 軟體創建一 STD 機構以進行干涉分析和應力估計，使用有限元素法（FEM）分析齒輪齧合在不同齒數差下的應力，以為評估機構應力失效優先次序。本研究提出了一應力變異估計法，結合先前應力分析評估機構可靠度，以為決定容許負載基礎。綜合而言，本研究之應力分析和可靠度評估技術可用於漸開線齒輪減速器之設計開發。