

Stress Minimization of the Pressure Vessel by Optimal Shape Design

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Keywords: Pressure vessel, Stress concentration, Optimal method, SCGM.

ABSTRACT

This study proposes an optimal shape of the pressure vessel by the self-developed simplified conjugated gradient method (SCGM) for minimizing the Von Mises stress of this pressure vessel. The finite element analysis built from the ANSYS parametric design language is optimized by SCGM. The results are validated by the previous study. This study proves that the optimization of geometry can reduce the stress concentration of the pressure vessel effectively. It will benefit the design of the nuclear reactor for safe consideration. In addition, this proposed optimal method will build an effective way to simplify the engineering design procedure.

INTRODUCTION

In general, the pressure vessel is subjected to the complex environment, such as high pressure and high temperature. It does not only present a strong challenge about the physical and mechanical performance of structure, but also about the reliable and economical design. It will achieve a favorable safety performance under a perfect combination of design parameters. The demand for more lightweight, high-performance and low-cost in material drives the research of structural optimization. Many studies on shape design optimization attest that shape changes may lead to consider the mass saving, the improvement of structural performance and the minimization of the stress concentration which applies to the petrochemical industry and gas storage, et al. [Luo et al., 2008; Peng and Jones, 2008].

To the best of our knowledge, little previous paper has been published about the shape optimization for the axis-symmetric shells. Most

published studies examine the shape in the two-dimensional problems, a fairly detailed review is given by Ding [Ding, 1986], although no other general surveys have been published recently apart from that of Rozvany et al. [Rozvany et al., 1995] on the related problem of layout optimization.

In addition, the ASME Boiler and Pressure Vessel code does not only require detailed stress analysis but also sets the wall thickness to keep the basic hoop stress below the allowable stress. The safety factors and design rules [Faupel, 1956] are assumed to cater for the high localized stresses. This code provides a quick design method; a safer procedure will be to have the data analysis in detail [Kihui et al., 2007].

The extra requirements in various industries make it necessary to conduct the detailed stress analysis under primary loads for structure configurations. In particular, the well known codes such as ASME (1983), BS 5500 (1976), and Russian GOST (1989) do not contain enough information about nozzle connections on the pressure vessel heads [Skopinsky, 2000]. Therefore, there is a need for the overall optimal design in order to reinforce the regulation on the deficient part.

Most of the accidents (about 80%) of pressure vessel are resulted from the stress concentration. The associated stress concentration factor (SCF) depends on the size, and the shape of the vessel. The peak stress is occurring at the stress concentration and critical in determining the design life of a vessel [Makulsawatudom et al., 2004].

The stress concentration is a highly localized effect. The high stresses exist only in a very small region in the vicinity of the hole. In approaching the study of localized stresses, it is well to note that their significance does not depend solely on their absolute value. At the same time, it also depends upon [Harvey, 1974]: (i) the general physical properties of the material, (ii) the relative proportion of the member highly stressed to that under stressed which affects the reverse strength; it can develop in resisting excessive loads, (iii) the kind of loading to which the pressure vessel is subjected. Actually, the safety of pressure vessel is related to the design parameters such as radius, height [Oludare et al., 2014].

It is noted from the paper review cited above;

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despite its practical importance that studies of the optimization of a set of design parameters haven't received sufficient attention. This motivates this present investigation.

The optimization is used to search the extreme value of the objective function. The optimal methods currently used can be broadly divided into two categories: one is the gradient based techniques, such as the gradient search method (GSM) [Esparza et al., 2006] and the conjugate gradient method (CGM) [Cheng and Chang, 2003]. These methods can generate the local or global solution by the different initial values, and these methods have the advantage of the faster convergence. The other is the simulated evolutionary optimization, such as the genetic algorithms (GA) [Xu et al., 2009] and the simulated annealing (SA) [Sonmez, 2007], which can search the global solution, but needs a lot of iterations to convergent.

Darijani et al. find that the hoop of thick-walled vessel are become smooth after the equivalent stresses are optimized [Darijani et al., 2009]. Recently, the cost reduction of pressure vessels is proposed by reducing weight with adequate strength and stiffness [Hassan et al., 2014]. This research is to demonstrate how the application of numerical optimal simulation techniques can be used to search for an effective optimization of the pressure vessel design. Therefore, the optimal design of the pressure vessel for obtaining the minimum stress concentration is achieved in the present study. The numerical design approach is developed by combining a direct problem solver, ANSYS code, with an optimization method (the simplified conjugate gradient method, SCGM). A finite element analysis model ANSYS is used as the subroutine to solve the stress-strain profile associated with the variation of the geometry of the pressure vessel during the iterative optimal process. The SCGM method is capable of obtaining the minimized objective functions easily, and calculating fast than traditional conjugated gradient method. In the SCGM method, the sensitivity of the objective function resulted from the designed variables is evaluated first, and then by giving an appropriate fixed value for the step size, the optimal design can then be carried out without overwhelming mathematical derivation. This method is successfully employed in the composite pressure vessel for the laminate direction optimization [Lin et al., 2013] and the optimization of the heat concentration on the high power LED array [Hsieh et al., 2011]. This study is aimed at the optimization of the design parameters so that the minimized stress concentration of the high pressure vessel under some specified operating conditions can be acquired.

This paper includes five sections. In the first section, the current development of the investigation of the pressure vessel is introduced and the features

of the proposed method are also stated. In the second section, the characteristics and the process of the proposed method are illustrated. In addition, a computational algorithm is proposed to be implemented the method in the computer. In the third section, the results of the direct solver are verified by the previous publication and the optimal predictions at various conditions are employed to demonstrate the usage of the proposed method. At last, the overall contribution and possible applications are concluded in the fourth section. From this study, it can be concluded that the proposed method is an accurate, robust and efficient method to optimal the design problem.

NUMERICAL ANALYSIS AND OPTIMIZATION METHODS

The optimal shape design does not only depend on the objective function but also the geometric modeling methodology. In general, the design parameter can classify into three types: size variables, shape variables and topological variables describing the material distribution. In the general circumstances, the size variable approach is more applicable than others in the shapes design of the typical pressure vessel [Mackenzie et al., 2008; Yushan and Wang, 1996; Zhu and Boyle, 2000].

Validated model

A validated study is designed to explore the correctness, excellence of the proposed method in this study. First, we build a model and compare with the previous study [Sang et al., 2002] to validate our simulated model. The validated model consists of a cylinder, two elliptical heads and a nozzle. Figure 1 shows that the schematic diagram for pressure vessel, which the parameters of the model are shown in the Table 1. The average value of material properties for the young's modulus, the yield strength and the ultimate strength is 205E3MPa, 339.4MPa and 472 MPa, respectively. In addition, the thickness of wall is 6 mm in the vessel.

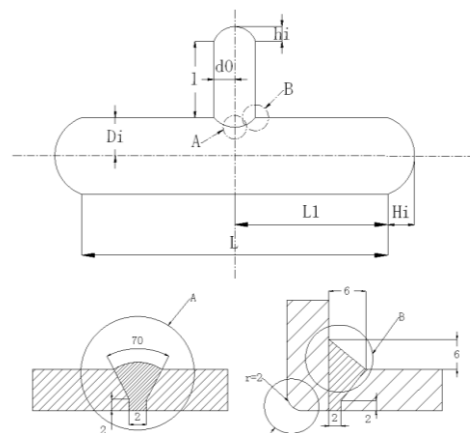


Figure 1 The schematic diagram of the verified model

Table 1 The parameter of pressure vessel in the verified study

Parameter	Hi(mm)	d0(mm)	Di(mm)	L1(mm)	Hi(mm)	l(mm)	L(mm)
	106	162.5	300	1200	175	600	2400

The comparison between the previous study [Sang et al., 2002] and our model provides the verification of our model. In the previous study, the finite element model is established using the finite element software ANSYS. The higher order 3D 20-node structural solid element SOLID186 is adopted to mesh the whole model. The FE mesh of the considered volume is presented, as shown in Figure 2. That comprises about 174,645 elements; a very dense meshed map is used. Figure 3 presents the stress distribution of the pressure vessel resulted from our simulated model by ANSYS and agrees with the result of the previous study [Sang et al., 2002]. According to this comparison, the above investigation makes a conclusion that the vessel deforms violently in the area A and B as shown in Fig. 1. This result is to be clear about the proof that credibility of simulation is sufficient for this research.

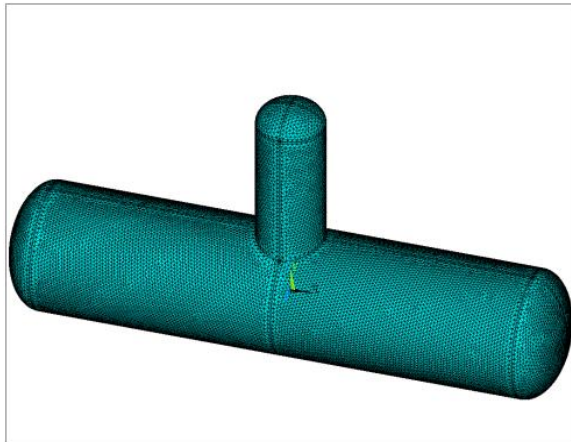
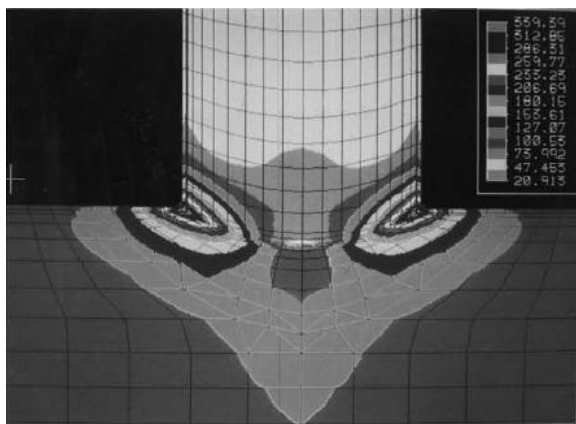
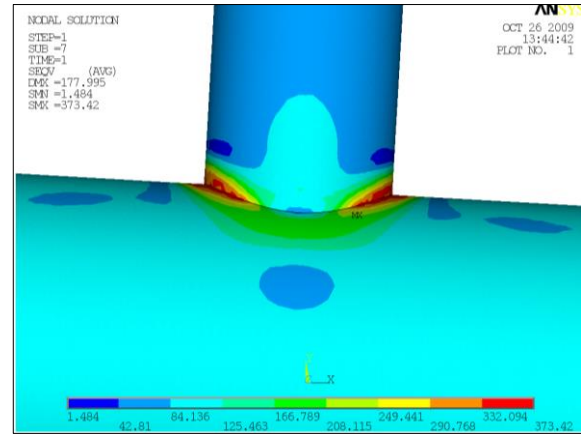


Figure 2 The mesh of the verified model



(a)



(b)

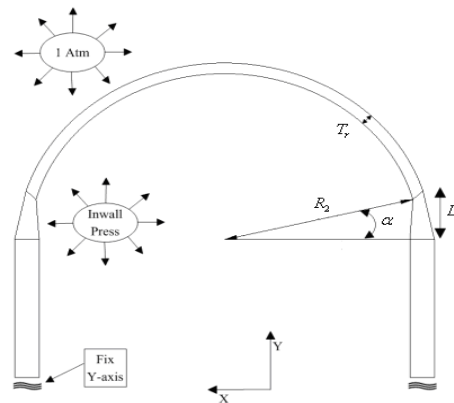
Figure 3 The stress concentration distribution of pressure vessel, (a) Sang et al. [Sang et al., 2002] (b) this study

Optimization model description

The typically design of pressure vessel usually includes the boundary conditions and operating conditions, to determine a reasonable of structure, to select the appropriate materials and the optimum structure size.

The model of the pressure vessel is analyzed by the commercial program ANSYS, that is used the three-dimensional solid brick element, SOLID186, and two-dimensional plane element, PLANE82, in this study. The whole structure is the two-dimensional, and the three-dimensional model of 90° for the whole structure to be analyzed in the axis-symmetric models.

For the present problem, the following formulation is used. The models and the design variables of the Case 1 and Case 2 are shown in Figure 4. A linear elastic material model is used in this study with poisson's ratio is 0.3, Young's modulus is 207E3MPa, ultimate strength is 620MPa and the density is 7,800 Kg/m^3 .



(a)

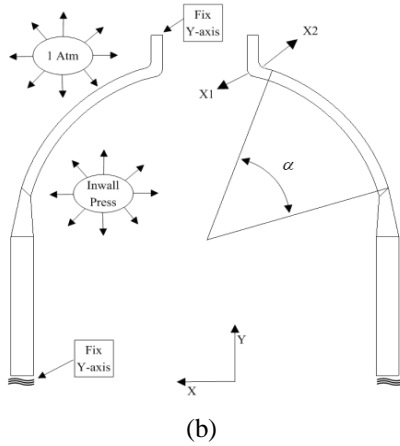


Figure 4 The schematic diagram of pressure vessel for (a) Case 1 and (b) Case 2

By the ASME chapter UG-32(f) [Ball and Carter, 2004], the definition of the minimum thickness T_r of the spherical dome is as follows:

$$T_r = \frac{PR_2}{2\sigma_{allow}\eta - 0.2P}$$

$$\sigma_{allow} = \frac{1}{n_s} \sigma_u$$

where P is the working pressure, R_2 is the inwall radius of spherical dome, η is the welding efficiency, n_s is the safety coefficient, σ_u is the ultimate strength, and σ_{allow} is the allowable stress of the material. In addition, the definition of the minimum thickness T_s in the cylinder part of the pressure vessel is as below:

$$T_s = \frac{2PR_1}{2\sigma_{allow}\eta - 1.2P}$$

where R_1 is the inwall radius of the cylinder part of vessel.

In this investigation, the thickness of cylinder part is fixed. The designed parameters of Case 1 are the radius R_2 of the spherical dome and the junction length of dome with cylinder. According the Equation (1), the thickness T_r of spherical dome is dependent on R_2 . In addition, the angle α will change with R_2 and L in the Equation (4) [Liang et al., 2002]. In other words, T_r and α are dependent with each other, and the others are independent variables.

$$\alpha = \arcsin\left(\frac{L}{R_2}\right)$$

In the Case 2, the designed variable $X1$ and $X2$ are the coordinate of y-axis in the opposite sides of the nozzle, and dependent with the angle α of the hemisphere.

Optimization method

For the purpose of the optimum design, the objective function J of this study is the minimum Von Mises stress of the vessel. The Von Mises stress (as known as equivalent stress σ_{eqv}) is given by:

$$J = \sigma_{eqv} = \sqrt{\frac{(\sigma_i - \sigma_t)^2 + (\sigma_t - \sigma_r)^2 + (\sigma_r - \sigma_i)^2}{2}} \quad (5)$$

where σ_i , σ_t and σ_r is the longitudinal, tangential and radial stresses, respectively. Beside, IA is the iteration number in the optimal design process.

In addition, we assume $\{a_i, i = 1, 2, \dots, l\}$ be the set of the undetermined coefficients. The variables a_i are treated as the optimal variables designed in this study to minimize the objective function. Different combinations of these coefficients represent the variation of the pressure vessel's geometry. In other words, in the optimization process, the undetermined coefficients are updated iteratively toward the minimization of the objective function.

In this manner, as the objective function is approaching its minimum value in the optimization process, with the definition of J , the Von Mises stress gradually reaches a minimum value. This implies that the phenomena of stress concentration will be decreased.

The minimization of the objective function is accomplished by using the SCGM method. The method evaluates the gradient functions of the objective function and sets up a new conjugate direction for the updated undetermined coefficients with the help of a direct numerical sensitivity analysis.

We perform the direct numerical sensitivity analysis to determine the gradient functions $\left\{\left(\frac{\partial J}{\partial a_i}\right)^n, i = 1, 2, \dots, l\right\}$ in the n_{th} step. First, give a perturbation (Δa_i) to each of the undetermined coefficients, and then find the change of the objective function (ΔJ) caused by Δa_i . The gradient function with respect to each of the undetermined coefficients can be calculated by the direct numerical differentiation as:

$$\frac{\partial J}{\partial a_i} = \frac{\Delta J}{\Delta a_i} \quad (4)$$

Then, we can calculate the conjugate gradient coefficients, γ_i^n , and the search directions, π_i^{n+1} , for each of the undetermined coefficients with:

$$\gamma_i^n = \left[\frac{\left(\frac{\partial J}{\partial a_i} \right)^n}{\left(\frac{\partial J}{\partial a_i} \right)^{n-1}} \right]^2, i = 1, 2, \dots, l \quad (7)$$

$$\pi_i^{n+1} = \left(\frac{\partial J}{\partial a_i} \right)^n + \gamma_i^n \pi_i^n, i = 1, 2, \dots, l \quad (8)$$

The step sizes $\{\tau_i, i = 1, 2, \dots, l\}$ will be assigned for all the undetermined coefficients and leave it unchanged during the iteration. In this study, the fixed value is determined by a trial and error process, and the value is set to be 1.0×10^{-6} typically. The difficulty lies with the fact that how to decide the suitable value of the step size. The undetermined coefficients will be updated.

$$a_i^{n+1} = a_i^n - \tau_i \pi_i^{n+1}, i = 1, 2, \dots, l \quad (9)$$

The procedure for applying the SCGM method is described briefly in the following:

- (1) Make an initial guess for the shape profile by giving initial values to the set of undetermined coefficients. With initialization accomplished, the run itself can begin.
- (2) Use the direct problem solver to predict the stress concentration and stress distribution of the pressure vessel, and calculate the objective function J by Equation (5).
- (3) When the objective function reaches a minimum, that is to say, the relative criteria is satisfied, the solution process is terminated. Otherwise, proceed to step (4).
- (4) Through the Equation (6), to determine the gradient functions.
- (5) Through the Equations (7) and (8), to calculate the conjugate gradient coefficients, γ_i^n , and the search directions, π_i^{n+1} , for each of the undetermined coefficients.
- (6) Assign a fixed value to the step sizes for all the undetermined coefficients and leave it unchanged during the iteration.
- (7) According the Equation (9), to update the undetermined coefficients and re-new the geometry of the pressure vessel, and go back to step (2).

It is important to mention that the emphasis of present study is put on the optimization of the pressure vessel. To the authors' knowledge, it's a new view to deal the stress-strain problem by using the

optimal method of SCGM.

Figure 5 presents a flow chart of the optimization process. Note that the SCGM optimizer is integrated with the ANSYS code by means of a self-developed interface program written in APDL script. As shown in Fig. 5, the values of the undetermined coefficients suggested by the optimizer are sent to the direct problem solver in order to update the geometrical model and grid system. The direct problem solver then utilizes this updated information to determine the stress of the pressure vessel and to compute the corresponding value of the objective function. The outputs of the direct problem solver are then transferred back to the optimizer in order to calculate the new search direction.

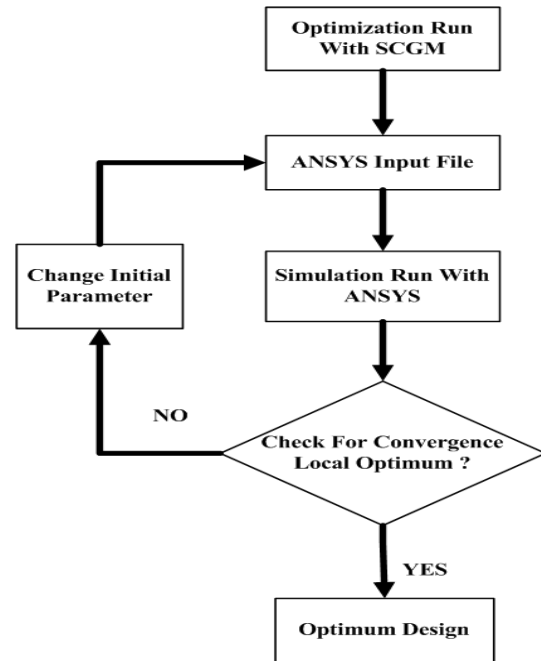


Figure 5 The process of optimization

RESULTS AND DISCUSSIONS

In the present study, the vessel is subjected to the internal pressure of $P = 2.5\text{MPa}$, the height and the wall thickness of cylinder part are fixed. In Figure 6, the convergence of the optimal iteration is shown as the relative criteria is 10^{-6} . In addition, we show the objective function of Case 1 and Case 2 in the optimal process. At the same time, we compare the results of 2D and 3D models in this figure. It requires approximately 105 iterations to reach the optimal design in the 3D model of Case 1. In Case 1, the Von Mises stress of this 3D model is reduced from 218MPa to 152.7MPa. In addition, we use the same initial conditions to simulate the 2D model in Case 1. Throughout the Fig. 6, we find that the 2D model requires 207 iterations to reach the optimal result and the objective function is down from 290MPa to

152.4MPa. The CPU time of 3D and 2D model is 15,452 s and 272 s, respectively. From the above illustrations, we demonstrate that the proposed method in this study is available to approach optimal result and the 2D model can reach the similar optimal result in the simplified geometry. The optimal process of the Case 2 can demonstrate these results apparently. The 2D and 3D model of Case 2 requires 35 and 33 iterations to reach the optimal result, respectively. In addition, the Von Mises stress of the 2D and 3D Case 2 model decreases from 162MPa, 160MPa to 156.7MPa, 156.3MPa, separately. The CPU time of the 2D and 3D Case 2 model is 132 s and 3364 s to reach the optimal result, respectively. Therefore, the simplified way by 2D model and the availability of this proposed method are shown in this figure clearly.

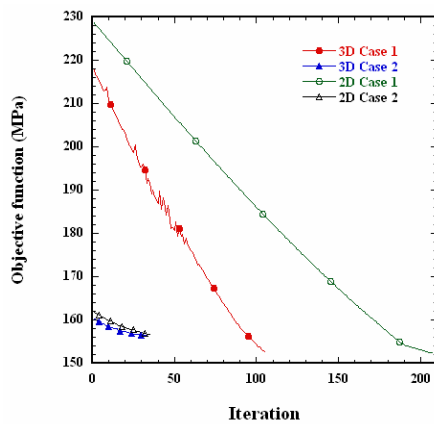


Figure 6 The convergence with iterations in the optimal process

Figure 7 presents the stress contour with the variation of R_2 and L in the 3D model of Case 1. The stress decreases as R_2 increases from 1,400 mm to 1,500 mm and increases as R_2 increases from 1,500 mm to 1,650 mm. In addition, the stress decreases apparently as the variable L increases from 100 mm to 500 mm. From the results of Fig. 7, the minimum result is 152.7MPa at R_2 is 1,509.68 mm and L is 158.58 mm. We observe that the model is a non-monotonic function.

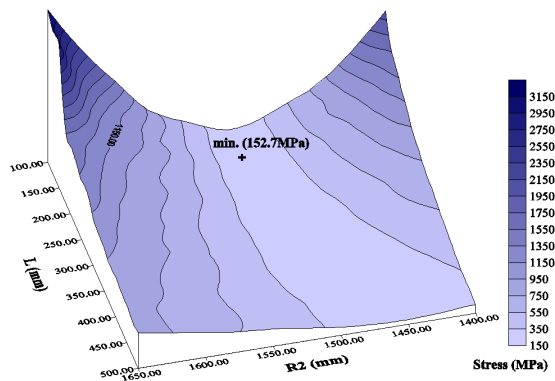
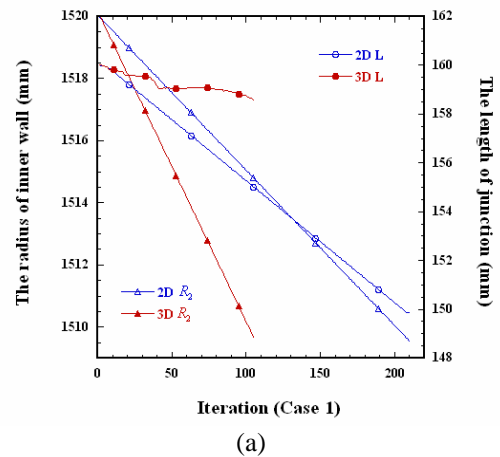


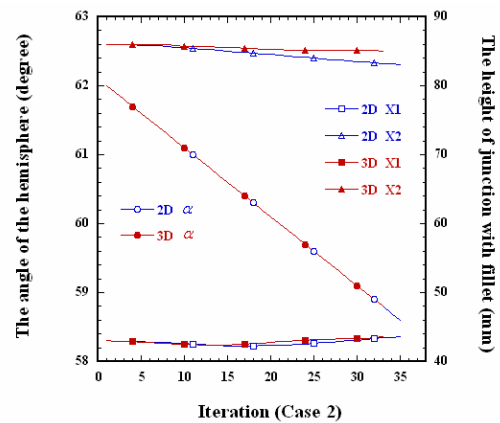
Figure 7 The stress distribution for varying the value

of R_2 and L in the 3D Case 1

The variations of the designed variables in the optimal process are shown in Figure 8. From these results, we observed that the optimal value of the designed parameter is different between the 2D and 3D cases, even the optimal results of the objective function are similar. That is due to the 2D finite element models, whose shape functions are also different with the ones of 3D model. Therefore, a 2D axis-symmetric finite element model seems not sufficient to interpret the global behavior even we can predict the optimal result by this model. The geometry of the 2D model still cannot use for the realistic optimal design. It is necessary to use the 3D optimal process to predict the optimal geometry and results in the pressure vessel design.



(a)



(b)

Figure 8 The variable with iterations in the optimal process (a) Case 1 and (b) Case 2

In Figures 9 and 10, the average Von Mises stress profiles along the y-axis in the inwall and the outer wall of pressure vessel are shown under the initial design and optimization in the 3D model of Case 1 and Case 2, respectively. Under the initial 3D model of Case1 in the Fig. 9, the stress concentration

of the inwall and outer wall at the interval between 100 mm and 500 mm of y-axis are larger than the safe working stress 155MPa obviously. This is not only high risk of accidents in the working environment but also reduces the lifetime of pressure vessel. The difference between the average stress and the peak stress is 79.53MPa. After the optimal procedure, the difference is down to 14.12MPa, and the stress concentration is reduced about 33.35%, the total volume of this pressure vessel also declines about 0.32%.

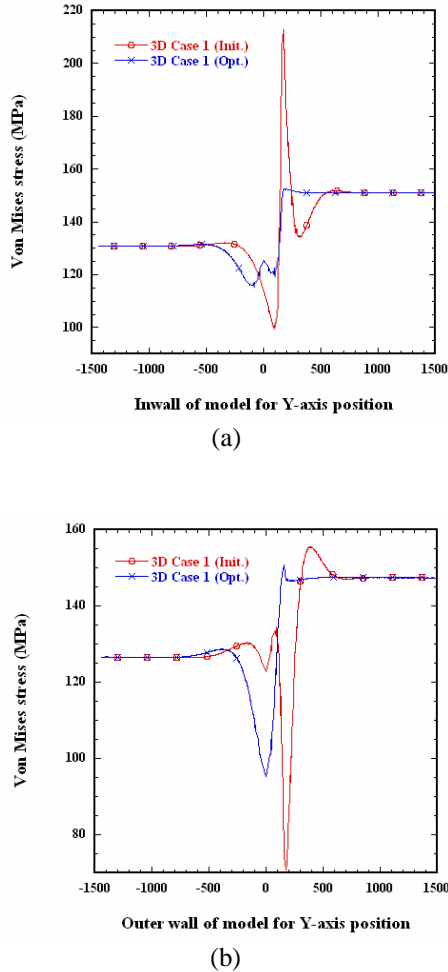


Figure 9 The Von Mises stress profiles of pressure vessel in the optimal process for Case 1, (a) inwall; (b) outer wall

In Fig. 10, the stress concentration of the inwall and outer wall at the interval between 1,000 mm and 1,300 mm of y-axis under the initial 3D model of Case 2 are larger than the safe working stress 155MPa (ASME code) obviously. The difference between the average stress and the peak stress is 27.98MPa, and change to 26.73MPa after the optimal process. The stress concentration is reduced about 6.9%, and the total volume is raised about 3.35% in this Case 2. There are the profiles to be constrained

after optimization and it is also controlled within the safe working stress under those optimal processes.

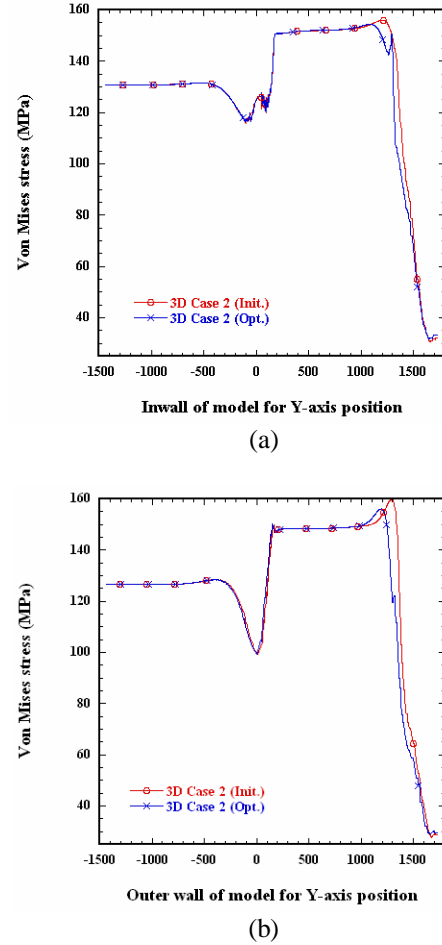


Figure 10 The Von Mises stress profiles of pressure vessel in the optimal process for Case 2, (a) inwall; (b) outer wall

The optimization algorithms are divided into the line search methods and the random search methods. The SCGM is one kind of the line search method. The disadvantage of the line search method is that the optimal result maybe is the local extreme value. Therefore, it needs to try many different initial values to modify this disadvantage and make sure the result is the global solution. Different initial values for the Case 1 and Case 2 are used in this research and get the different optimal results shown in the Table 2, Table 3 and Figures 11 and 12. In Case 1, we can find the direction of the variation of the R_2 in the Tab. 2 and Fig. 11. The value of R_2 starts from 1,500 mm or 1,490 mm and stops nearby 1,502 mm, and it starts from 1,525 mm stops nearby 1,509 mm. It is clearly that the profile of the objective function exists more than two extreme values. In Case 2, the more complex variations related to three designed parameters are shown in the Tab. 3 and Fig. 12 and illustrate the effects of the initial guess again.

This method proposes positively a sequence of process steps can be followed to achieve the optimal results. It is evident that both of pressure vessel designs are successful through SCGM and make the maximum stress under the optimum design to satisfy the safe working stress.

Table 2 The different initial values for case 1
(The * star is optimization value)

Initial value		$R_2=1490\text{ mm}$ $L=155\text{ mm}$	$R_2=1500\text{ mm}$ $L=150\text{ mm}$	$R_2=1525\text{ mm}$ $L=165\text{ mm}$	$R_2=1520\text{ mm}$ $L=160\text{ mm}$
2D	$R_2\text{ (mm)}$	1502.522	1503.106	1509.155	1509.7
	$L\text{ (mm)}$	169.495	154.938	159.912	149.919
	Function (MPa)	207.292	211.335	152.831	152.424(*)
3D	$R_2\text{ (mm)}$	1502.741	1502.757	1509.286	1509.676
	$L\text{ (mm)}$	174.164	170.609	164.206	158.583
	Function (MPa)	199.094	200.315	153.433	152.762(*)

Table 3 The different initial values for case 2
(The * star is optimization value)

Initial value		$\alpha=65^\circ$ $X2=40\text{ mm}$ $X3=85\text{ mm}$	$\alpha=62^\circ$ $X2=43\text{ mm}$ $X3=86\text{ mm}$	$\alpha=55^\circ$ $X2=48\text{ mm}$ $X3=78\text{ mm}$	$\alpha=68^\circ$ $X2=31\text{ mm}$ $X3=80\text{ mm}$
2D	$\alpha\text{ (}^\circ\text{)}$	59	58.6	57.413	59.627
	$X1\text{ (mm)}$	38.664	43.622	50.589	33.761
	$X2\text{ (mm)}$	80.69	83.045	80.7	75.527
	Function (MPa)	157.363	156.684(*)	168.278	157.922
3D	$\alpha\text{ (}^\circ\text{)}$	59.1	58.8	54.964	59.906
	$X1\text{ (mm)}$	37.998	43.553	60.805	31.136
	$X2\text{ (mm)}$	84.459	85.074	120	80.388
	Function (MPa)	157.059	156.319(*)	159.326	157.98

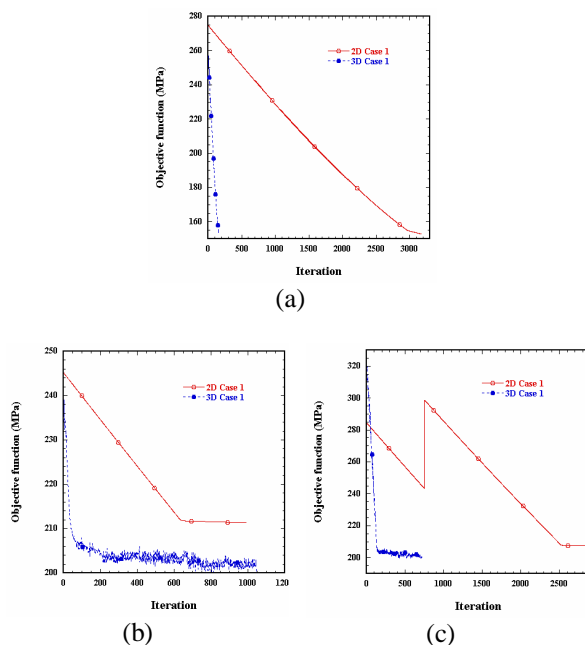


Figure 11 The different initial values with iterations in Case 1, (a) $R_2=1,525\text{ mm}$, $L=165\text{ mm}$; (b) $R_2=1,490\text{ mm}$, $L=155\text{ mm}$; (c) $R_2=1,500\text{ mm}$, $L=150\text{ mm}$

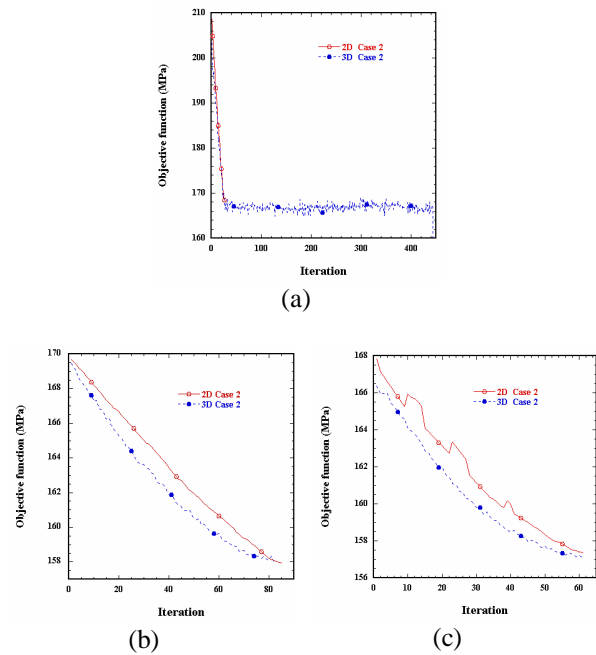


Figure 12 The different initial values with iterations in Case 2, (a) $\alpha=55^\circ$, $X1=48\text{ mm}$, $X2=78\text{ mm}$; (b) $\alpha=65^\circ$, $X1=40\text{ mm}$, $X2=85\text{ mm}$; (c) $\alpha=68^\circ$, $X1=31\text{ mm}$, $X2=80\text{ mm}$

CONCLUSIONS

The purposes of this study are to optimize the shape of pressure vessel and simultaneously to satisfy the requirements of the ASME code. In this research, it proposes a optimal algorithm for shape design of pressure vessel, the proposed method is based on the finite element method combines with the simplified conjugated gradient method. In this research of the verification part, it compares with the results of the previous study and verifies the accuracy of our numerical model. The significance of this study shows that the approach agrees with the experimental analysis for a good determination of the shape in the complete pressure vessel. In addition, this study illustrates many different results solved by the difference initial variables. It proves that the results reach is the optimal value in this optimal algorithm. Through the optimal process, the stress concentration is reduced 33.35% and 6.9%, and the total volume is also decreased 0.32% and 3.35%, respectively.

This proposed method can raise the efficiency of the design. This approach can process not only simple shape but also complex shape design. In addition, this approach is evidently superior to typical trial-and-error approaches commonly followed in the industrial environment. With the sophistication of the analysis software, coupled with simplified techniques such as this proposed method for 3D pressure vessel under the complex load and the composite layers

should be feasible in the industrial design.

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NOMENCLATURE

- T_r = minimum thickness of spherical dome (m)
 T_s = minimum thickness of cylinder part of pressure vessel (m)
 P = working pressure (MPa)

R_1 = inwall radius of cylinder part of vessel (m)
 R_2 = inwall radius of spherical dome (m)
 n_s = safety coefficient
 J = objective function

問題。此方法將提供一有效且快速之工程設計流程。

Greek Symbols

τ = search step size
 γ = conjugated gradient coefficient
 π = search direction
 η = welding efficiency
 σ_{eqv} = equivalent stress (MPa)
 σ_u = ultimate strength (MPa)
 σ_l = longitudinal stress (MPa)
 σ_t = tangential stress (MPa)
 σ_r = radial stress (MPa)
 α = angle of the hemisphere

Subscripts

i, j = indices
 n = number of iterations

壓力容器應力最小化之結構

設計最佳化研究

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摘要

本研究以自行開發之簡易共軛梯度法(SCGM)設計壓力容器之參數，使整體壓力容器之最大應力值最佳化之最小。研究中提出一最佳化流程，以SCGM 控制 ANSYS 之有限元素模型，進而達成最佳化之設計。本研究證實在壓力容器設計過程中經由型狀設計可以有效改善應力分布、降低應力集中之現象。此將有益於廣泛應用於核反應器、石化工業以及瓦斯等壓力容器結構之設計而考慮其安全