## Structural Damage Detection Based on Measurement Point Layout and Model Updating Method

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#### **Keywords:**

Iterative methods, sensitivity-based iterative model updating method, finite element model, damage detection

#### Abstract

In this study, a method is proposed for detecting the structural damage position and extent by combining the measurement point layout and model updating method. Initially, the finite element analysis (FEA) model of the structure is applied to evaluate the appropriate measurement point layout for the modal test. The method proposed in this study is based on the functional complementarity of the integrated modal kinetic energy (MKE) and Hemez methods. The method includes the MKE cumulative value for the number of measurement points and the Hemez method for reasonable positions. Model variables, such as the cross-sectional area and quadratic moment of inertia. are used to represent the geometrical or material properties of the structure. Further, the model updating method takes advantage of the eigensensitivity matrix, and the model variables of the FEA model are updated by iterative calculation. The results demonstrate that the analysis results of the updated model variables of the FEA model can approach the pre-simulated damage positions and extent of the reference model. The measurement and model updating methods developed in this study can be applied for the monitoring and early warning of structural damage.

#### Introduction

The detection of the structural damage positions and extent using measurement data under limited funding and instrument availability is a crucial consideration for structural health monitoring. Before

Engineer, Independent Researcher, No. 173, Henan Rd., Lingya Dist., Kaohsiung City 802027, Taiwan (R.O.C.). Email address: plinlin@yahoo.com.tw modal testing, the finite element analysis (FEA) model can be used to plan the setting of the measurement points. The measurement point layout is closely related to the degree of freedom (DOF) of the structure. The dynamic behavior of structures (e.g., motion and deformation) is often described with a few representative DOF. These selected DOF must completely exhibit the dynamic characteristics (modal parameters: natural frequencies and mode shapes) of the structure, else important dynamic response information will be lost. The number of points that can be measured by modal tests is limited; however, the FEA model can provide more points to present information on the characteristics of the structural dynamic behavior of unmeasured points. An example eigensensitivity algorithm for the finite element model (FEM) updating procedure has been presented (Lin RM et al., 1995; Taylan K et al., 2012).

The measurement point layout of the modal test should exhibit the dynamic characteristics of the modes of interest. Therefore, several researchers have proposed measurement point layout methods for obtaining the dynamic characteristics of the structure. Lim (1991) proposed the measurement point layout method for exploring the identification and control of the dynamic characteristics of the structure. Kammer (1991) proposed the effective independence (EFI) method for selecting the measurement positions in which the shapes of the modes of interest are rendered as linearly as possible. An iterative process was used for removing measurement DOF with a low linear independent contribution to the mode shape for obtaining the final measurement point position.

To obtain a good estimate of the dynamic characteristics, the determinant value or norm of the fisher information matrix (FIM) should be maximized. Yao et al. (1993) developed a gene algorithm program based on the concept of maximizing the FIM. However, they found that the results obtained by the algorithm for the EFI Method were limited. Hemez and Farhat (1994) configured the measurement position based on the contribution of the structural strain energy, which is also an application of the EFI Method. Udwadia (1994) discussed the placement of finite-point measuring points which maximize and apply the FIM to linear as well as nonlinear structural systems.

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Furthermore, Park and Kim (1996) proposed an iterative process for removing certain DOF to maximize the FIM; the DOF that were not removed was the measurement point location. Schedlinski and Link (1996) applied the eigenvector decomposition method for the measurement point layout scheme. Cobb and Liebst (1997) proposed the eigenvector sensitivity method, which uses the first-order derivative function of the model parameters to model variables for measurement point layout planning. Based on this method, Shi et al. (2000) proposed the selection of appropriate measurement point positions, based on the ability to locate damage. Xia (2002) improved this method, considering the influence of noise measurement when selecting the measurement point position.

The connection between the EFI and modal kinetic energy (MKE) methods for sensor placement is of primary concern for damage detection in structural health monitoring. The MKE method is an iterated version of the EFI method, and the reduced mode shapes are orthonormalized repeatedly during iterations of the MKE method (Yi TH et al., 2012). Wang et al. (2007) applied the MKE method which provides a measure of the dynamic contribution of each FEA model DOF to each of the target mode shapes and an idea on where the maximum dynamic responses can be measured. The same MKE cumulative value under different modes will correspond to a different number of measurement points and their positions. The increasing number of measurement points should result in a high MKE cumulative value for each target mode. Moreover, measurement point layouts that provide the largest cumulative value of the corresponding MKE should be selected. However, research on the determination of the required MKE cumulative value for each target mode within the candidate measurement points has been limited. A value as less as 40-50% could be sufficient (Kammer DC, 1991).

Structural damage detection using modal test data is crucial in structural health monitoring. However, given that modal testing is limited by cost and instrument availability, existing research pays less attention to the relationship between the measurement point number and its positional configuration. The measurement points need to show as much linear independence of the mode shapes of interest as possible. The more important the measurement points, the more linearly independent the mode shapes of interest. The Hemez method ranks the importance of candidate measurement points. Although only a limited number of measuring instruments can be arranged according to the importance of the measurement points, the appropriate number of points requires the measurement maximum nondiagonal element change curve of the MAC matrix for evaluation. However, the Hemez method and the MKE method are functionally complementary. As the

number of measurement points gradually increases, the increase of the MKE cumulative value will tend to converge. This means that the MKE method can replace the MAC curve as an evaluation method for the number of measurement points. However, the order of the increase in the number of measurement points is not important for the degree of the linear independent contribution. This means that the limited number of measuring instruments cannot be deployed according to the importance of the position of the measuring points. In this study, considering the portal beam structure as an example, the FEA model is used for modal test planning to set the measurement points; this, combined with the Hemez method for the reasonable determination of positions and the MKE cumulative value for the determination of the measurement point number, enables the complete comprehension of the dynamic characteristics of the structure. The method first evaluates the importance of the measurement point positions and then evaluates the optimal number of measurement points.

Model variables, such as the cross-sectional area and quadratic moment of inertia, are used to represent the geometrical or material properties of the structure. Furthermore, it is verified that the FEA model updating procedure, which takes advantage of the eigensensitivity matrix, can detect the damage position and extent in the structure.

#### **Theoretical background**

#### **Measurement Point Layout**

In this study, the FEA model is used for evaluating the measurement point number and position for the modal test. The measurement data of the modal test finite-positions is used as the modified reference target of the FEA model. As the basis for modifying the FEA model, the measurement point number and position should be capable of exhibiting the natural frequencies and mode shapes of the modes of interest.

## Method for assessing the number of measurement points

In this study, the MKE of the FEA model is used for evaluating the appropriate number of measurement points for the modal test. The MKE is defined as follows (Cheng L et al., 2009):

$$MKE_{ii} = \phi_{ii} \sum_{t}^{u} M_{it} \phi_{ti}.$$
 (1)

where  $MKE_{ij}$  is the MKE of the *i*-th DOF of the FEA model in the *j*-th mode,  $\phi_{ij}$  is the *i*-th DOF of the FEA model in the *j*-th mode shape,  $\phi_{tj}$  is the *t*-th DOF value in the *j*-th mode shape, and  $M_{it}$  is the value of the modal mass matrix in the *i*-th column and *t*-th row. The measurement points are set within the *t*-*u* DOF range. If the *j*-th mode shape  $\phi_{ij}$  of (1) includes all the DOF of the FEA model, and the mass matrix is orthogonally normalized, the cumulative total value of  $MKE_{ii}$  is unity.

#### Method for assessing the measurement point position

The appropriate layout positions for the modal test are evaluated using the eigensensitivity matrix of the FEA Model. An FEA model with the deviation ratio of the cross-sectional area and quadratic moment of inertia of the correct structure is assumed. This model is regarded as the reference, and the analysis results are considered the modal test results. The FEA model of the correct structure is used as the initial analysis model, and it needs to be updated with the target modal test results. When the update procedure is complete, the updated model variables are evaluated for accurately detecting the deviation ratio of the cross-sectional area and the quadratic moment of inertia of the pre-simulated elements of the reference model. The eigenvalues and eigenvectors are simultaneously included in (2), as follows:

$$\begin{cases} \tilde{\theta}_l \\ \tilde{z}_l \end{cases} = \begin{cases} \phi_l \\ z \end{cases} + [S_l](\{\tilde{\theta}\} - \{\theta\})$$
 (2)

where  $\lambda_i$ ,  $\{\phi_i\}$ , and  $\tilde{\lambda}_i$ ,  $\{\tilde{\phi}_i\}$  are the *i*-th eigenvalues and eigenvectors of the analysis and modal test results, respectively.

$$\left\{\tilde{\theta}\right\} - \left\{\theta\right\} = \left\{\Delta\theta\right\} = \left\{\Delta p_{1}, \dots, \Delta p_{n}\right\}^{T}$$

is the difference between the target and the analyzed mode variables, where  $p_r$  is the *r*-th model variable in a total of *n*. The eigensensitivity matrix  $S_i$  is defined as follows (Hemez FM et al., 1994):

$$[S_i] = \begin{bmatrix} \frac{\partial \{\phi_i\}}{\partial p_r} \\ \frac{\partial \lambda_i}{\partial p_r} \end{bmatrix}.$$
(3)

Udwadia and Garba proposed the FIM. FIM as a distribution of strain energy B is defined as the summation of the contribution of the selected modes (Udwadia FE et al., 1985), as follows:

$$[B_i] = [S_i]^T [S_i]. (4)$$

The measurement points should be deployed to obtain as much linear independent measurement data as possible. From the possible measurement position DOF of the structure, the measurement DOF that minimizes the variation of the FIM matrix determinant value are successively deleted. Thereby, the DOF that contributes to linear independence in the mode under consideration is retained.

Kammer (1991) has proposed that the diagonal terms of the following matrix be used to rank the importance of possible measurement points positions as follows:

$$[E_i] = \left( [S_i] [S_i]^T [S_i] \right)^{-1} [S_i]^T \right).$$
(5)

The larger the value of the diagonal term of matrix  $[E_i]$ , the more the modes of interest exist that can be presented; the corresponding DOF are used as

the measurement points. The DOF position of the structural analysis model that can be used as the measurement point is sorted according to its importance, in descending order. The DOF whose importance is less need not be measured. After determining the number of DOF to be measured and configuring the DOF of the measurement, a suitable measurement point layout can be obtained.

## Mode-shape relationship between the measurement and non-measurement DOF

Compared to the FEA model, the modal test has considerably less DOF. The FEA model can classify the DOF corresponding to the measurement and non-measurement points. The mass matrix [M] and stiffness matrix [K] of the FEA model are divided into two parts according to the DOF: measurement and non-measurement (Friswell MI et al., 1995):

$$\left[-\omega_{mj}^{2} \begin{bmatrix} M_{mm} & M_{ms} \\ M_{sm} & M_{ss} \end{bmatrix} + \begin{bmatrix} K_{mm} & K_{ms} \\ K_{sm} & K_{ss} \end{bmatrix} \right] \begin{pmatrix} \phi_{mj} \\ \phi_{sj} \end{pmatrix} = \begin{cases} 0 \\ 0 \end{cases},$$
(6)

where  $\omega_{mj}$  is the *j*-th natural frequency, and  $\{\phi_{mj}\}$  is the mode shape corresponding to each measurement DOF. If  $\{\phi_{sj}\}$  is the mode shape of each unmeasured DOF,

$$\{\phi_{sj}\} = -(-\omega_{mj}^2[M_{ss}] + [K_{ss}])^{-1}(-\omega_{mj}^2[M_{sm}] + [K_{sm}])\{\phi_{mj}\}.$$
(7)

#### **Model Updating Method**

#### Eigensensitivity matrix

This section presents the algorithm for the FEA model updating method. The eigensensitivity matrix is established from the modes of interest:

$$[S]\{\Delta\theta\} = \{\Delta z\}.\tag{8}$$

It can be expressed as follows:

$$\begin{array}{cccc} \frac{\partial (\lambda_{0})}{\partial p_{1}} & \frac{\partial (\lambda_{0})_{1}}{\partial p_{2}} & \frac{\partial (\lambda_{0})_{1}}{\partial p_{n}} \\ \frac{\partial (\lambda_{0})_{1}}{\partial p_{1}} & \frac{\partial (\lambda_{0})_{1}}{\partial p_{2}} & \cdots & \frac{\partial (\lambda_{0})_{1}}{\partial p_{n}} \\ \cdot & \cdot & \cdots & \cdot \\ \frac{\partial (\phi_{0})_{m}}{\partial p_{1}} & \frac{\partial (\phi_{0})_{m}}{\partial p_{2}} & \cdots & \frac{\partial (\phi_{0})_{m}}{\partial p_{n}} \\ \frac{\partial (\lambda_{0})_{m}}{\partial p_{1}} & \frac{\partial (\lambda_{0})_{m}}{\partial p_{2}} & \frac{\partial (\lambda_{0})_{m}}{\partial p_{n}} \\ \end{array} \right) \left\{ \begin{array}{l} \Delta p_{1} \\ \Delta p_{2} \\ \vdots \\ \Delta p_{n} \end{array} \right\} = \begin{cases} \{\phi_{2}\}_{1} - \{\phi_{0}\}_{1} \\ (\lambda_{1})_{1} - (\lambda_{0})_{1} \\ \vdots \\ (\lambda_{2})_{m} - (\lambda_{0})_{m} \\ (\lambda_{2})_{m} - (\lambda_{0})_{m} \\ \end{array} \right\}. \tag{9}$$

 $\lambda_a$  and  $\{\phi_a\}$  in the above equation are the eigenvalues and mode shapes obtained by FEM analysis, respectively; the first to *m* modes constitute the eigensensitivity matrix [*S*]. The difference in each model variable before and after each iteration in the modal updating procedure, for a total of *n* model variables  $(\Delta p_i)$ , constitutes  $\{\Delta \theta\}$ .

The eigensensitivity matrix of the model updating process is iteratively calculated until it meets the convergence stop criterion.

#### Convergence stop criterion

The FEA model updating method is discussed with respect to the natural frequency. The convergence stop criterion is established based on the correlation coefficient ( $CC_t$ ):

$$CC_t = \frac{1}{m} \sum_{i=1}^m C_{ri} \frac{|\Delta \omega_i|}{\omega_i},\tag{10}$$

where *m* is the number of natural frequencies,  $C_{ri}$  is the expected relative error value, and  $\omega^{al}$  and  $\omega^{x}$ are the natural frequencies obtained from the update analysis and modal test results.  $\Delta \omega_i = \omega_i^{al} - \omega_i^{x}$  is the difference in the natural frequency between the analysis and modal test results for each iterative procedure. Moreover,  $\omega_i = \omega_i^{x}$ . In addition,  $C_{ri} =$ 0.01 is used as the desired relative error value at all the natural frequencies.

The following two stop indicators are combined:  $CC_t < \epsilon_1$  and  $|CC_{t+1} - CC_t| < \epsilon_2$ , where  $\epsilon_1=0.005, \epsilon_2=0.001$  (FEM tools).

#### Case study



Figure 1 Portal beam structure: For members 1 and 3: I = 0.000084911 m<sup>4</sup> (204 in<sup>4</sup>), A = 0.0049355 m<sup>2</sup> (7.65 in<sup>2</sup>), and the Depth = 0.310388 m (12.22 in); for member 2: I = 0.000125285 m<sup>4</sup> (301 in<sup>4</sup>), A = 0.0049548 m<sup>2</sup> (7.68 in<sup>2</sup>), and the Depth = 0.398526 m (15.69 in).

#### Structural composition

#### **Correct structure**

The portal beam structure is shown in Figure 1. The left and right columns are divided into eight beam elements, respectively, whereas the upper structure is divided into 10. The positions of a total of 26 beam elements are indicated from 1 to 26. Assume that the bottom of the structure is fixed to the ground. The portal beam structure does not consider the axial deformation for each element of the structure, but only considers the lateral X-direction displacement and the angle of rotation around the Z-axis; hence, each node has 2 DOF.

The structure is composed of steel with a

Young's modulus  $E = 2.11 \times 10^{10} \left(\frac{\text{kgf}}{\text{m}^2}\right)$  (  $30 \times 10^6$  (psi)). The cross-sectional area *A* and quadratic moment of inertia *I* of the structure are also depicted in Figure 1. The left and right columns are denoted as member 1 and member 3, respectively, and are composed of W12×26 steel beams; the upper member 2 is composed of W16×26 steel beams (Moaveni S, 2008).

#### Reference damage model

The analysis results of the reference damagemodel are considered as the modal test data for this example. Stiff connectors are present at the ends of the members, and these, together with the nodes themselves, are considered perfectly rigid. During bending, the element stiffness and mass matrix for a member in the same plane with a flexible length (l)separated by a rigid portion of length (a) from the node can be expressed as follows (Mottershead JE, 2000):

$$[k] = \frac{El}{l^3} \begin{bmatrix} 12a & (12a+6l) & -12 & 6l \\ (12a+6l) & (12a^2+12al+4l^2) & -(12a+6l) & (6al+2l^2) \\ -12 & -(12a+6l) & 12 & -6l \\ 6l & (6al+2l^2) & -6l & 4l^2 \end{bmatrix}.$$
 (11)

$$[m] = \begin{bmatrix} 156 & (156a+22l) & 54 & -13l \\ (156a+22l) & (156a^2+44al+4l^2) & (54a+13l) & -(13al+3l^2) \\ 54 & (54a+13l) & 156 & -22l \\ -13l & -(13al+3l^2) & -22l & 4l^2 \end{bmatrix}$$
(12)

Suppose the following are the elements with the rigid portions (*a*):

(I) Member 1: Element numbers 1 and 8.

(II) Member 2: Element numbers 9 and 18.

(III) Member 3: Element numbers 19 and 26.

The other elements of members 1, 2, and 3 are considered as the flexible lengths. The rigid part (a) of the reference model is 0.45 times the length of each element.

The reference damage model is the structure in Figure 1 in which the geometric or material property variables can be changed. The Young's coefficient of the reference model is  $E = 2.11 \times 10^{10} (\frac{\text{kgf}}{\text{m}^2})$  (30 × 106 (psi)). During the welding of steel beams, the local uneven heating and cooling of the metal have different degrees of thermal expansion and contraction, resulting in changes in the cross-sectional area and secondary moment of inertia of the different elements. The deviation ratio from the correct value of the cross-sectional area (*A*) and quadratic moment of inertia (*I*) of the reference model are listed in Table 1. The cross-sectional area (*A*) and quadratic moment of inertia (*I*) of the initial analysis model need to be updated with those of the reference model.

Table 1 Deviation ratios of *A* and *I* in the reference model

Elemen order	ntal	2	4	5	11	13	16
А		10%	10%			15%	15%
Ι				- 20%	- 20%		

Elemental order	19	20	21	22	23	24
А		15%	15%		10%	10%
Ι	- 20%			- 20%		

#### Measurement-point number and position layout

#### **Problem description**

For structures, because of the limitations in the cost and measuring instruments for modal testing, the measurement points are limited. The number of measurement points in the modal test is often less than the number of DOF in the FEA model. Therefore, it is necessary to select specific DOF in the FEA model for measurement.

# Correct-structure FEA model to evaluate the appropriate measurement point layout for the modal test

It is assumed that only the modal test data of a specific DOF can be considered as the FEA model target for updating. Therefore, the number of measurement points and the configuration of the positions are critical. The FEA model pre-evaluates the appropriate measurement point layout method. The number of indices in Figure 2 and Table 2 are the same as the element position number in Figure 1. The lateral displacement is represented by x, and the rotation angle is represented by z.

The importance of the DOF of the structure takes advantage of the eigensensitivity of the first six modes according to Eq. (5). The diagonal values of matrix [E] are depicted in Figure 2. Table 2 presents the importance of the corresponding DOF of the diagonal term values in descending order.



Figure 2 Matrix [*E*] diagonal term value for the corresponding DOF

Table 2 Order of importance of the measurement point DOF

Index	1	2	3	4	5	6	7	8	9	10	11	12	13
DOF	10x	<u>11x</u>	12x	<u>13x</u>	14x	15x	14z	15z	16z	23x	24x	<u>25x</u>	<u>1x</u>
Index	14	15	16	17	18	19	20	21	22	23	24	25	26
DOF	lz	<u>2x</u>	20z	21z	<u>3x</u>	<u>4x</u>	<u>5x</u>	<u>6x</u>	17z	<u>7x</u>	<u>8x</u>	<u>9x</u>	18z
Index	27	28	29	30	31	32	33	34	35	36	37	38	39
DOF	19z	2z	16x	3z	4z	5z	19x	20x	6z	7z	21x	8z	9z
Index	40	41	42	43	44	45	46	47	48	49	50		
DOF	10z	<u>17x</u>	<u>18x</u>	11z	12z	13z	22z	23z	24z	25z	22x		

Figure 3 indicates that when the measurement points increase in the order of importance listed in Table 2 until the increase to 38 measurement points, the curve of the largest nondiagonal elements of the MAC matrix gradually decreases until it stabilizes. This demonstrates that the measurement points with the first 38 orders of importance contribute to reasonably orthogonal modes. The maximum offdiagonal element MAC value, when 38 measurement points are included, is 0.022.



Figure 3 MAC maximum nondiagonal element change curve

The cumulative value of the MKE in Figure 4 is based on the order of the DOF listed in Table 2 and is established according to Eq. (1). The cumulative MKE for the DOF is based on the number of measurement points added. As observed in Figure 4, when the most important 38 DOF are included in the MKE cumulative value, the value approaches 89% of all the DOF. The 38 measurement points can be arranged according to the order listed in Table 2.



Figure 4 Relationship between the MKE cumulative value and the number of measurement points

The number of measurement points emerging from the maximum value of the nondiagonal elements of the MAC matrix in Figure 3 is the optimal number of measurement points with reasonably orthogonal modes. Then, the MKE value of the optimal number of measurement points is obtained from Figure 4.

#### Method for simulating non-measurement data

The reference model of this example includes measurement points with limited DOF (Not all the DOF of the FEA model are measured). Considering the portal beam structure as an example, the measurement points are arranged according to the DOF importance listed in Table 2. The structure has 50 DOF, and some of them have measurement data. If the DOF lack measurement data, the mode shape is replaced by the initial analysis results of the structure in the corresponding FEM DOF for the model update calculation.

#### Noise addition

From the measurement data, the modal parameters of the six modes are obtained. Noise is added to the natural frequency without noise under a normal distribution with a standard deviation of 0.2%. In addition, 1% is added to the mode shape without noise, which is  $\phi_{ij}^* = \phi_{ij}(1 + r_i |\phi_{max,j}|)$ , where  $\phi_{ij}^*$  and  $\phi_{ij}$  are the *j*-th mode in the *i*-th DOF of the structure and the mode shape in the presence or absence of noise, respectively.  $r_i$  is 1%, and  $\phi_{max,j}$  is the maximum value among the DOF of the *j*-th mode shape.

#### **Results and discussion**

According to the MKE cumulative value, the number of measurement points is selected, and the positions of the measurement points are determined according to the importance of the DOF. The measurement data from the modal test is used as the FEA model target for updating.

The core consideration regarding the number of measurement points is to sufficiently contribute to the linear independence of the modal shapes. As the number of measurement points increases, the degree of linear independence contribution also increases. Although the maximum value of the nondiagonal elements of the MAC matrix in Figure 3 and the MKE value in Figure 4 tend to converge, the former requires fewer measurement points than the latter. This indicates that the MKE method is more conservative in evaluating the number of measurement points. In contrast with the maximum nondiagonal element change curve of the MAC matrix for evaluation, when the MKE cumulative value was greater than 0.89, this example adopts the corresponding optimal measurement point number.

The analysis results of model updating, with and without noise, detect the pre-simulated damage position and extent for the reference model. This case can accurately detect the deviation ratio of the crosssectional area (A) and quadratic moment of inertia (I) of the reference model and the correct structure through the model updating procedure.



Figure 5 Updated results of the deviation ratios of *A* and *I* in the FEA model

For this example, the updated results after the iterative calculation of the model updating method converge accurately, and the natural frequency is obtained. The differences in the natural frequency before and after updating are listed in Table 3. After updating the model, the percentage of maximum difference between the modal test results and FEA model is 2.115%, whereas that of the minimum difference is 0.864%

Table 3 Differences between the analysis and measurement values of the natural frequency (Unit: Hz)

Mode	Modal test results	Before updating	Natural frequency difference (%)	After updating	Natural frequency difference (%)
1	13.846	14.988	8.248	13.643	-1.465
2	54.658	58.824	7.622	55.757	2.011
3	133.894	141.241	5.487	135.402	1.126
4	222.851	236.147	5.966	220.926	/-0.864
5	236.156	254.264	7.668	232.004	-1.758
6	313.726	336.528	7.268	320.361	2.115

## General procedure for checking the applicability in other structures

The flowchart in Figure 6 presents the important steps of the proposed method used in this case study. This case takes a portal beam structure as an example to illustrate the feasibility of the method. It is still necessary to further verify the feasibility of this method for other structures.



Figure 6 Flow chart

#### Conclusions

The measurement and model updating methods developed in this study can be applied for monitoring and early warning with respect to structural damage. The measurement point layout method includes the MKE cumulative value for the number of measurement points and the Hemez method for reasonable positions. The FEA model is established using the model variables (geometric or material property variables) of the structure. When the modal parameters (natural frequency and modal shape) of the structure are changed, the FEA model updating method, using the eigensensitivity matrix, can detect the damage position and extent in the structure.

Although modal testing can depict the actual dynamic behavior of the structure, there are limits regarding costs and instrumentation. Therefore, it is necessary to use FEA modeling to plan the measurement point layout for the modal test. Before modal testing, the FEA model can pre-evaluate the number and position configuration of the measurement points for completely understanding the dynamic characteristics of the structure and confirm the feasibility and economy of the modal test.

The modal test presents the true structural dynamic characteristics. Because of the uncertainty of the model variables of the FEA model, it is necessary to render the modal parameters of the FEA model and modal test consistent by updating the model variables. The updated FEA model can be used for structural design and analysis.

In this study, a combination of the measurement point layout and model updating method was used to detect the damage position and extent in the structure. A case study involving a portal beam structure confirmed that the proposed method can detect the damage position and extent. The optimal number of measurement points and the corresponding MKE were obtained using the largest nondiagonal element in the MAC matrix that gradually converges. The considerably high number of obtained measurement points (38), with respect to the total number of DOF (52), were aimed to optimize the linear independence of the modes of interest. However, if the number of measuring instruments is limited, they can be arranged according to the importance of their position.

Although the case study demonstrated that the proposed method can be practically implemented and provide adequate results, it includes certain shortcomings. In this study, the method for evaluating the measurement point layout is based on positions that can be measured. Certain DOF can present the important dynamic characteristics of the structure. However, because it is difficult to obtain the measured values of the DOF within the structure, this method cannot be used for evaluation. There is a continual need for an adequate theoretical basis for the practical application of measurement planning.

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## 基於量測點佈局與模型更新 方法的結構損傷檢測

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#### 摘要

本研究提出了一種結合量測點佈局與模型更新方 法作為檢測結構損傷位置與程度的方法。最初, 結構的有限元分析(FEA)模型用於評估模態測試 的適當量測點佈局。本研究中提出的方法是基於 結合模態動能(MKE)與 Hemez 方法的功能互補性。 該方法利用 MKE 累積值求取量測點數量,及 Hemez 方法求取合理量測位置。模型變量(例如:橫截面 積和二次慣性矩)用於表示結構的幾何或材料特 性。此外,模型更新方法利用特徵靈敏度矩陣, 並且透過送代計算以更新 FEA 模型的模型變量。 結果顯示更新 FEA 模型後,模型變量的分析結果 可以逼近參考模型預先模擬的結構損傷位置與程 度。這項研究發展的量測與模型更新方法可應用 於結構損傷的監測和預警。