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Study of Dynamic Behavior of a Bevel Gear Used in Vertical Axis Wind Turbine

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Keywords： Vertical axis wind turbine, Dynamic response, Straight bevel gear system, Aerodynamic torque.

ABSTRACT

The challenge of Vertical-Axis Wind Turbines (VAWT) is to extract the maximum power from wind. Therefore, the best modeling should take into account all elements constituting the turbine. In this work, both the dynamic model for one stage straight bevel gear and the aerodynamic model for VAWT are developed. For the first, we developed a lumped-mass dynamic model with 14 degrees of freedom. For the second, the actuator disk theory was used and leads to torque modeling. The aim of this study is to analyze the dynamic behavior of VAWT with variable aerodynamic excitation in the presence of a gearing system. Thus, the turbine is subject to external and internal excitations. The main factors of these excitations are the variability of the aerodynamic torque and the fluctuation of the gear mesh stiffness. Using the step-by-step time integration method (Newmark algorithm), dynamic equation was resolved. From frequency and time domains, the dynamic response in each bearing shows the persisting influence of gear mesh and wind fluctuation.

INTRODUCTION

Wind turbine is used to convert wind energy into electrical or mechanical energy. It can be divided into two groups of turbines, Horizontal Axis Wind Turbines (HAWT) and Vertical Axis Wind Turbines (VAWT). The VAWT attract many researchers for their benefits for domestic applications. Particularly, they have blades with uniform untwisted section, which makes them easy to manufacture. In addition, almost all the components of the mechanical power generation equipment are located at the ground levels, which facilitates their maintenance. The most apparent advantage of VAWT is their ability to operate in all wind direction without any yawing mechanism.

Wind speed variation is the main source of power fluctuating. Among the models found in the literature, the work of Abboudi et al. (2011) which introduces empirical approach to estimate the steady power coefficient. In fact, this is far from the reality where wind turbines are generally exposed on randomly varied wind speed. In addition, Malcolm (1988) and Paul (1984) in their works studied the dynamic response of vertical axis wind turbine subjected to turbulent flow whose stochastic wind model is based on the Kaimal spectra (Paul, 1984; Chad et al., 2014; Giuseppe, 2012; Pedro, 2003). Other researchers modeled the wind as a deterministic form by sums of several harmonics like the work of Ridha et al. (2013).

The literature is rich in theoretical and computational models achieved on aerodynamic modeling for VAWT. The most well-known models can be classified into two approaches. The first, called Computational Fluid Dynamics model (CFD), presents a numerical simulation used by many researchers like Marco (2011) to predict the performance of Darrieus wind turbine. The second category is based on the momentum theory, which presents an analytical investigation, and it includes three main mathematical models that are respectively: the double-multiple streamtube model, the Vortex model and the Cascade model. Each of these models has its strengths and weaknesses and they are well studied by Islam et al. (2008) and Asress et al. (2013). They analyzed the aerodynamic behavior of straight bladed VAWT using a comparative study between these models which are very expensive and become invalid in high solidity and high-speed rotation. Hansen et al. gives also a review of the works done by applying, on wind turbine rotors, the simple aerodynamic Blade Element Momentum, CFD, vortex and panel methods Hansen (2006, 2008).

In VAWT, the gearing mechanical system is used to transmit power from the input to the generator. However, several researchers have studied the dynamic behavior of mechanical systems with only internal excitation induced by the periodic variation of the mesh stiffness Khabou et al. (2011). Driss et al. (2014) presents the model of two-stage straight bevel

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gear system excited with only the periodic fluctuations of the gear meshes' stiffness. In addition to this research, there are other works in literature in which the bevel gear system transmission are modeled by a lumped masses spring parameters. Fujii et al. (1995) analyzed the dynamic behavior of straight bevel gear supported on angular bearings and tapered roller. Also, Li et al. (2003) studied the dynamic response of a spiral bevel-geared rotor-bearing system. Chang-jian (2011) studied the nonlinear dynamic behavior of bevel-gear system. Y. Wang et al. (2001) developed a new approach based on the finite element theory.

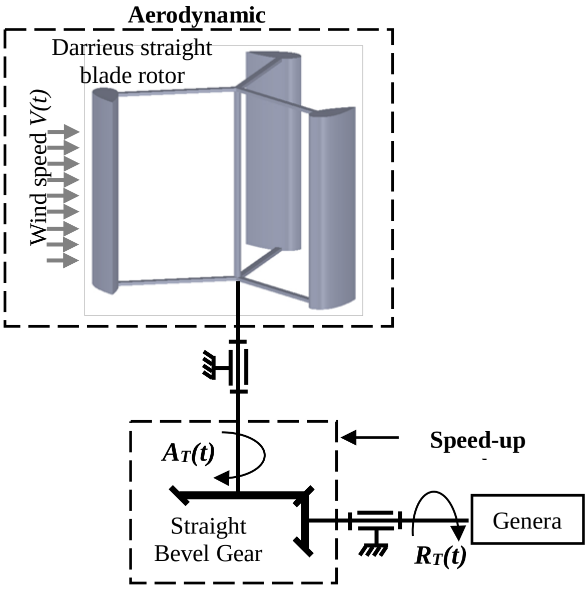


Fig. 1. Vertical axis wind turbine

In this paper we present the dynamic behavior of the one-stage straight bevel gear system used in vertical axis wind turbine and powered by two main sources of excitation which are the aerodynamic torque fluctuation and the periodic variation of the gear meshes' stiffness.

The bevel gear system is modeled by lumped parameters considering shafts and bearings flexibilities. Besides, brief explanation of the aerodynamic approach based on coupling the ‘Actuator disk model with Blade Element Theory is introduced. The dynamic response of the bearing is investigated due to numerical simulation with an implicit Newmark algorithm. This work presents a new methodology allowing the coupling between the aerodynamic torque fluctuation and the bevel gear lumped parameter models. Thus, complete simulation of dynamic behavior of VAWT was analyzed and presented.

DYNAMIC MODEL OF VERTICAL AXIS WIND TURBINE

Figure 1 shows the whole structure and ordinary mechanical transmission used in vertical axis wind turbine. The aerodynamic part includes the wind speed excitation *V(t)* acting on three straight-bladed Darrieus rotor. Resulting torque $A_T(t)$ is transmitted to the generator via a Speed-up gearbox constituted by the one-stage bevel gear.

Figure 2 shows the dynamic model based on lumped parameter to achieve the study of dynamic behavior of one-stage straight bevel gear system. This model includes two blocks. The first block is made up of the Darrieus rotor modeled by the mass (11) linked to the bevel gear (12) via a shaft with torsional rigidity $K_{\theta 1}$. The second block is composed of the bevel gear (21), the receiving wheel (22) and the second shaft with torsional rigidity $K_{\theta 2}$. The bevel gear (12) is linked to the bevel gear (21) via teeth mesh stiffness K(t). Each block is supported by a flexible bearing with different traction–compression stiffness Kxi, Kyi, Kzi and bending stiffness KΦi, Kψi.

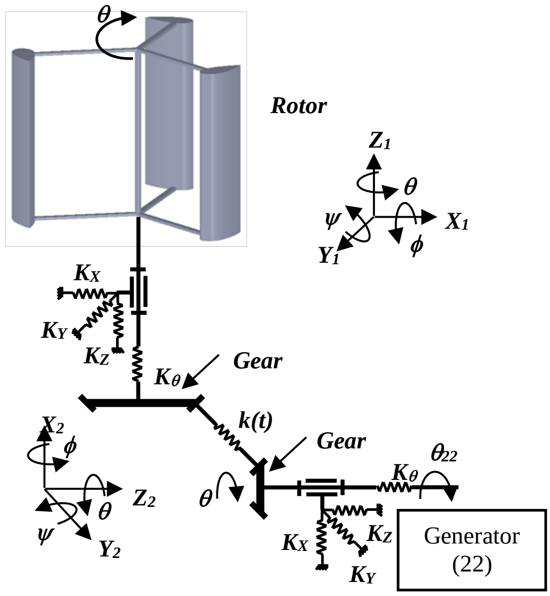


Fig. 2. Dynamic model of one-stage straight bevel gear system.

EQUATION OF MOTION

The equation of motion describing the dynamic behavior of the system was established using the formalism of Lagrange and it is presented in the following matrix form:

$$[M]\{\ddot{q}(t)\} + [C]\{\dot{q}(t)\} + ([K_S] + [K(t)])\{q(t)\} = \{F(t)\} \tag{1}$$

Where $\{q(t)\}$ is the generalized coordinate vector of the model, defined by:

$$\{q(t)\} = \begin{bmatrix} x_1, y_1, z_1, x_2, y_2, z_2, \phi_1, \psi_1, \phi_2, \psi_2, \\ \theta_{11}, \theta_{12}, \theta_{21}, \theta_{22} \end{bmatrix}^T \tag{2}$$

$\{x_i, y_i, z_i\}$ are linear displacements of bearing in each blocs (i=1:2), $\{\phi_i, \psi_i\}$ are angular displacements of the bearing following X and Y respectively and $\{\theta_{1i}, \theta_{2i}\}$ are angular displacements of the wheel and gear following Z direction (i=1:2).

$[M]$is the total mass matrix, expressed by:

$$[M]=\begin{bmatrix}[M_L] & 0\\ 0 & [M_A]\end{bmatrix} \tag{3}$$

$$[M_L]=diag[m_1, m_1, m_1, m_2, m_2, m_2] \tag{4}$$

$$[M_A]=diag\begin{bmatrix}I_{11x}+I_{12x}, I_{11y}+I_{12y}, I_{21x}+\\ I_{22x}, I_{21y}+I_{22y}, I_{11}, I_{12}, I_{21}, I_{22}\end{bmatrix} \tag{5}$$

m_i present the mass of each block. I_{11}, I_{12}, I_{21} and I_{22} are respectively the inertia moment of the drive wheel 11, the bevel gears 12 and 21 and the receiving wheel 22. I_{11x}, I_{12x}, I_{21x}, I_{22x} and I_{11y}, I_{12y}, I_{21y}, I_{22y} are the inertias of these elements following X and Y respectively.

$[C]$ is the proportional damping matrix (Khabou et al., 2011; Srikanth et al., 2015; Wilson, 1980; Closs, 2015). It is determined from the mass and stiffness matrix multiplied by the damping constants α and β. Where α and β are Rayleigh coefficients, α for mass proportional damping and β for stiffness proportional damping. These constants can be evaluated by assuming critical damping with natural frequencies of first and second modes of the system.

$$[C]=\alpha[M]+\beta[K_s] \tag{6}$$

where $\alpha=0.05 and \beta=10^{-5}$

$[K_s]$ is the average stiffness matrix of the structure defined by:

$$[K_s]=\begin{bmatrix}[K_p] & 0\\ 0 & [K_\theta]\end{bmatrix} \tag{7}$$

$$[K_p]=diag[K_{x1}, K_{y1}, K_{z1}, K_{x2}, K_{y2}, K_{z2}] \tag{8}$$

$$\left[K_\Theta\right]=\begin{bmatrix}K_{\phi1} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & K_{\psi1} & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & K_{\phi2} & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & K_{\psi2} & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & K_{\theta1} & -K_{\theta1} & 0 & 0\\ 0 & 0 & 0 & 0 & -K_{\theta1} & K_{\theta1} & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & K_{\theta2} & -K_{\theta2}\\ 0 & 0 & 0 & 0 & 0 & 0 & -K_{\theta2} & K_{\theta2}\end{bmatrix} \tag{9}$$

$\{F(t)\}$ is the external excitation force and it is expressed as

$$\{F(t)\}=\{0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,A_T(t)\,0\,0\,-R_T(t)\}^T \tag{10}$$

Where $R_T(t)$ is the electromagnetic torque that expresses the generator effect and $A_T(t)$ is the aerodynamic torque. $[K(t)]$ is the mesh stiffness matrix defined by Driss et al. (2014):

$$[K(t)]=k(t)\langle L\rangle^T.\langle L\rangle \tag{11}$$

$k(t)$ is mesh stiffness fluctuation and $\langle L\rangle$ presents the geometric parameters of the dynamic model, Driss et al. (2014).

Figure.3 describes the variation of the mesh stiffness, which has a periodic behavior with a constant period. Then, the mesh stiffness spectrum is presented, and we note the presence of meshing frequencies 'f_{e1}' corresponding to the internal excitation of the bevel gear system obtained from equation 12:

$$f_{e1}=\frac{Z_1 N}{60}\approx 100\,Hz \tag{12}$$

Where Z_1 is the number of gear teeth and N is the rotor speed.

In reality, the fluctuation of mesh stiffness is due to periodic variation of the teeth number in contact during meshing period Te. Indeed, mesh stiffness is maximal during the time ($(\varepsilon-1)\,T_e$) and minimal during the time ($(2-\varepsilon)\,T_e$).

$$T_e=\frac{1}{Z_1 f_e} \tag{13}$$

Where f_e is the frequency of rotation and ε is the meshing contact ratio.

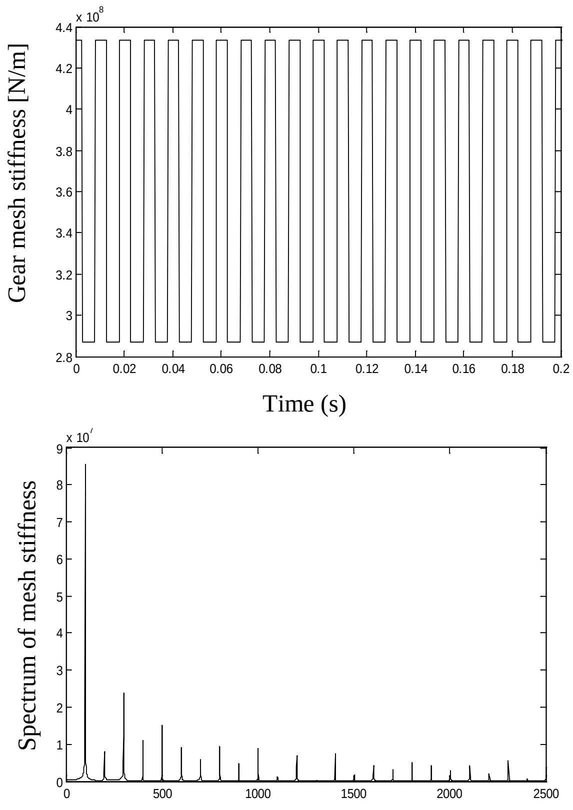


Fig. 3. Periodic meshes stiffness fluctuation and the corresponding spectrum.

AERODYNAMIC TORQUE MODELING

In this section, the aerodynamic model used to calculate the performance of the Darrieus rotor is introduced; it is based on the Actuator Disk Theory (Hansen, 2006; Hansen, 2008; Khabou et al., 2011). Wind turbine power production is strongly related to the interaction between the blade rotor and the wind, which presents the external excitation of the wind turbine.

In this work, the time fluctuation of wind speed is modeled by deterministic form, as a sum of several harmonics. The wind speed is written as follows (Mirecki, 2005):

$$V(t) = 14 + 2\,sin(\omega t) - 1.75\,sin(3\omega t) + 1.5\,sin(5\omega t) - 1.25\,sin(10\omega t) + sin(30\omega t) + 0.5\,sin(50\omega t) + 0.25\,sin(100\omega t) \quad (14)$$

where ω is the angular speed

The relative flow velocity $W(t)$ can be written as a function of the wind velocity $V(t)$ [9]

$$W(t) = \sqrt{V_c{}^2 + V_n{}^2} = V(t)\sqrt{\left((1-a)\sin\theta\right)^2 + \left((1-a)\cos\theta + \lambda(t)\right)^2} \quad (15)$$

$$\begin{cases} V_c = R\omega + V_a \cos\theta \\ V_n = V_a \sin\theta \end{cases} \quad (16)$$

where V_c and V_n are the chordal velocity component and the normal velocity component respectively. R is the radius of the turbine and θ the azimuth angle.

V_a is the axial velocity (induced velocity) and can be expressed as:

$$V_a = V(t)(1-a) \quad (17)$$

where 'a' is the axial induction factor (Asress, 2013). $\lambda(t)$ is the Tip Speed Ratio, defined as the ratio between the tangential speed at blade tip and the wind speed $V(t)$. It can be expressed as (Asress et al., 2013; Prathamesh et al., 2013):

$$\lambda(t) = \frac{\omega R}{V(t)} \quad (18)$$

The angle of attack α is defined as the angle between the blade chord and relative velocity as shown in (Figure.4). In this Fig all possible positions of the blade are presented.

The local angle of attack can be expressed in terms of the azimuth angle by Hansen (2008):

$$\alpha(\theta) = tan^{-1}\left(\frac{V_n}{V_c}\right) = \left(\frac{(1-a)\sin\theta}{(1-a)\cos\theta + \lambda}\right) \quad (19)$$

According to equation 18, at θ=0° azimuth position and (θ=180°) position, the angle of attack α is set to zero, since the symmetrical airfoil are used. Thus, at this point, only a drag force exists (Habtamu et al., 2011). α increases with the increase of the azimuth angle which causes the rise in the lift force. This lift force is perpendicular to the resultant wind direction.

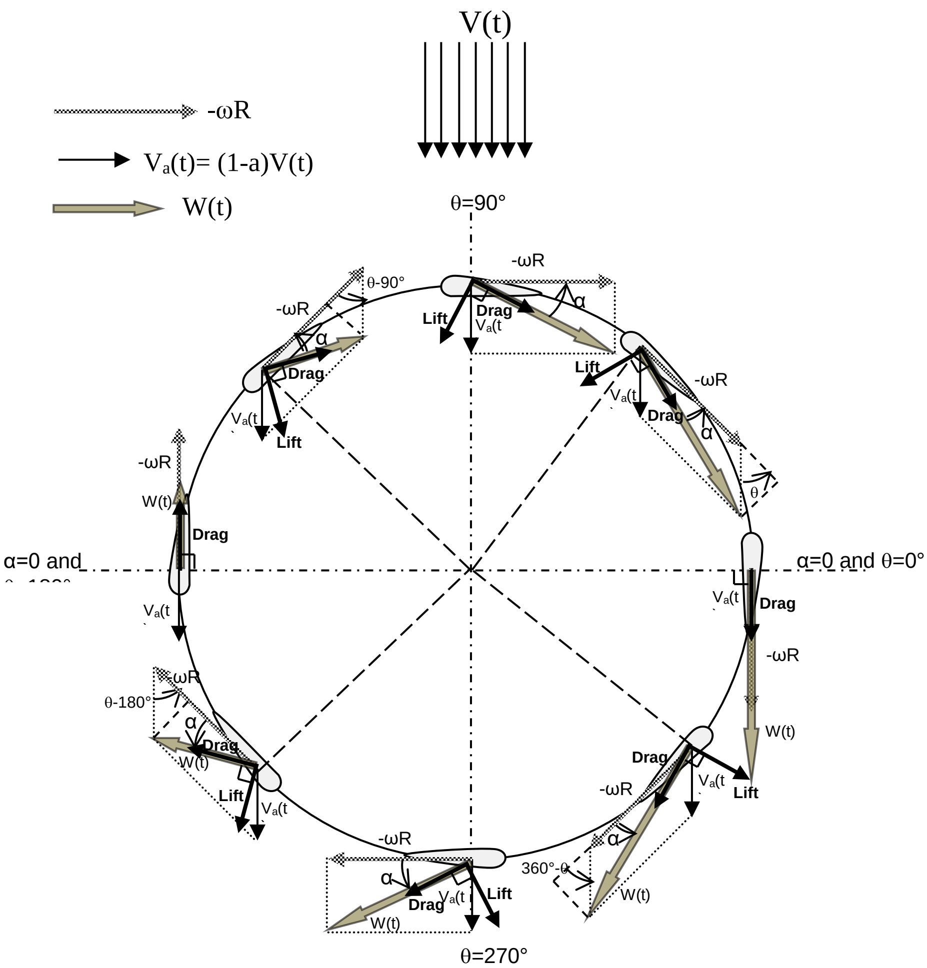


Fig. 4. Flow velocities and forces in the Darrieus rotor

Using the Blade-Element Momentum (BEM) theory (Asress et al., 2013), the normal and tangential forces acting in a single blade at azimuth location θ are established. These forces can be expressed as

$$F_{N_i}(\theta) = \frac{1}{2}\rho C_N W^2 hc$$
$$F_{T_i}(\theta) = \frac{1}{2}\rho C_T W^2 hc \quad (20)$$

Where h is the blade height, *c* is the blade chord length and *ρ* is the air density.

The tangential force coefficient C_T and normal force coefficient C_N are defined according to (Islam et al., 2008; Asress et al., 2013) as:

$$C_T = C_L \sin\alpha - C_D \cos\alpha$$
$$C_N = C_L \cos\alpha + C_D \sin\alpha \quad (21)$$

C_L and C_D are respectively the lift coefficient and the drag coefficient for angle of attack; they are expressed in terms of the local Reynolds number and the angle of attack (David, 1994).

The total tangential forces applied to the rotor are the sum of the forces acting on the blades.

As shown in Figure 5, the resulting force can be composed with the lift force (F_L) and drags force (F_D) or with the normal and tangential components which are F_N and F_T respectively.

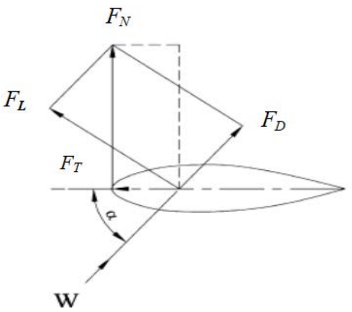


Fig. 5. Force diagram for a single blade of a Darrieus turbine

The torque $A_{Ti}(\theta)$ produced by a single blade (i) is calculated using the tangential force (Asress et al., 2013):

$$A_{Ti}(\theta) = F_{Ti}(\theta)R \quad (22)$$

And the total produced torque (for VAWT with n blades) is obtained from equation 23

$$A_T = \sum_{i=1}^{n} A_{Ti}(\theta) = \sum_{i=1}^{n} F_{Ti}(\theta)R \quad (23)$$

The power performance parameter of the wind turbine is calculated from the resulting torque $A_T(t)$, it can be expressed by (Islam et al., 2008):

$$P(t) = A_T(t).\omega = Cp\frac{1}{2}\rho A V(t)^3 \quad (24)$$

A is the swept area of the turbine and C_p is the power coefficient.

The shape of the swept area depends on the rotor configuration. For HAWT, the swept area has circular shape, while for a straight-bladed VAWT, the swept area has a rectangular shape which is calculated as (Habtamu et al., 2011; Federico, 2012):

$$A=2.R.h \quad (25)$$

DYNAMIC RESPONSE FOR ONE-STAGE STRAIGHT BEVEL GEAR SYSTEM

In order to study the dynamic behavior of bevel gear system used in vertical axis wind turbine and powered by the aerodynamic torque in addition to the periodic variations of mesh stiffness, we use numerical simulation based on the implicit Newmark method.

In the equation of motion (1) the variation of mesh stiffness was introduced in the first member of the equation but the fluctuation of the wind power appeared in the second member. Therefore, the coupling between the two phenomena exists when developing the equality of the two members.

The technological and dimensional parameters of the bevel gear transmission are presented in Table 1. Details of the vertical axis wind turbine are shown in Table 2.

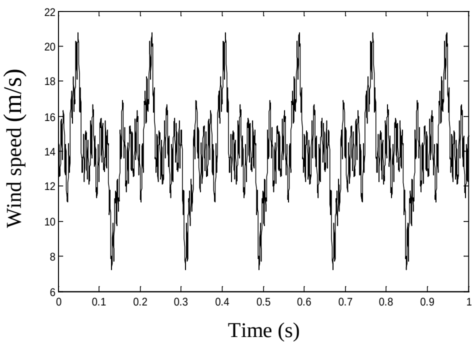
Table 1. Characteristics of the studied bevel gear system

Number of teeth	18 / 45
Module(m)	0.004
Bearing stiffness (N/m)	$K_{x1}=K_{y1}=K_{x2}=K_{y2}=2.10^8$, $K_{z1}=K_{z2}=4.10^8$
Torsional stiffness (N/rd/m)	$K_{\theta 1}=K_{\theta 2}=310^8$
Pressure angle	$\alpha = 20°$
Contact ratio	ε=1.56
Density (42CrMo4)	7860kg/m^3

Table 2. Wind turbine specification

Type	Straight blade Darrieus
Airfoil profile	NACA0018
Airfoil chord(mm)	480
Blade length (m)	3.66
Turbine diameter (m)	4
Number of blades	3
Rotor Speed (rev/min)	334

The wind speed vibration and its spectrum are shown in Figure.6.



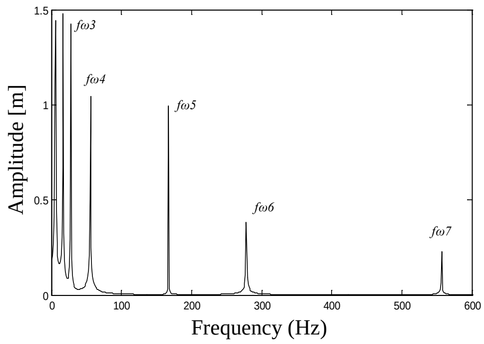


Fig. 6. Temporal evolution of the wind speed and the corresponding spectrum

We note the presence of seven frequencies in a band [0 600Hz]. These frequencies (*fwi*) are very important since they present an external excitation of the bevel gear system. Besides, their effect will be clearly apparent in the dynamic response of the VAWT.

Figure. 7.a. shows the variation of the tangential force (F_{T2}) acting on a single rotor blade (blade 2). Fig.7.b. shows the aerodynamic torque fluctuations, which present a periodic external source of excitation to the bevel gear system. It is generated from the total tangential force obtained via equation 23. The torque fluctuates between the maximum value of 3500N.m and the minimum value of 800N.m.

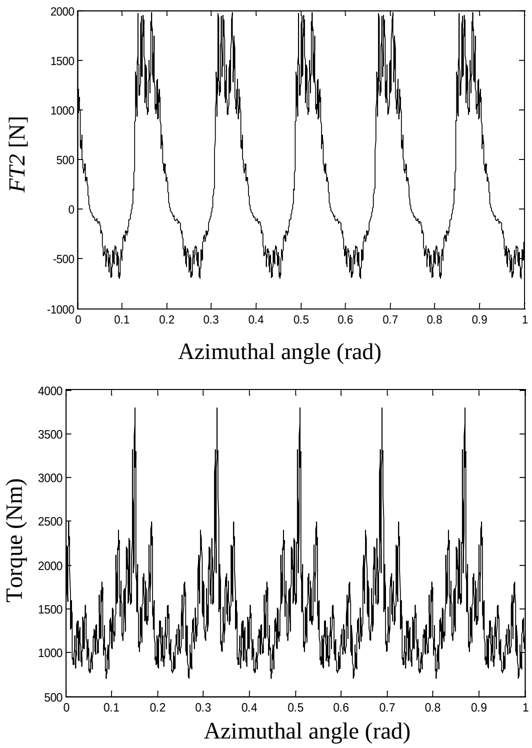


Fig. 7. a. Tangential force for one blade; b. Evolution of the total aerodynamic torque

Figure .8 shows the variation of the power output function of time, which is related to the aerodynamic torque according to equation (24). The case considered was run for optimum C_p that is for TSR about 5. For such wind conditions, the turbine generates its nominal power (Mohammad, 2014).

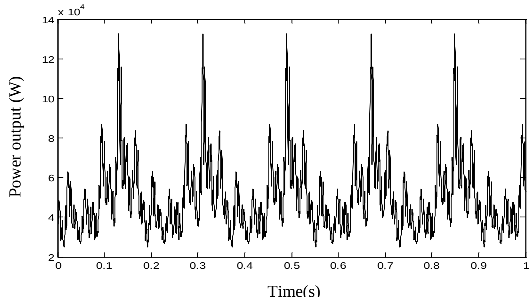


Fig. 8. Power fluctuation of Darrieus wind turbine for TSR =5

The dynamic responses to non-stationary excitation on the first bearing are presented in Figure.9 and Figure.10. In the spectrum, we note that the most important peaks are due to wind frequencies *fwi*.

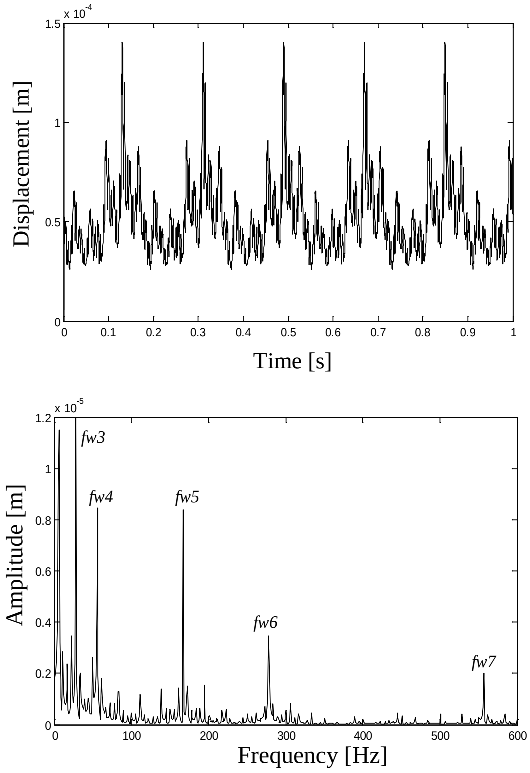
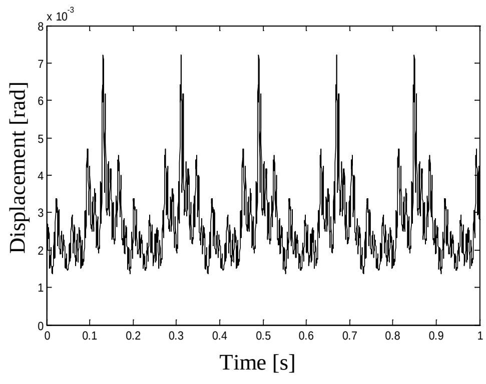


Fig. 9. Temporal and frequency dynamic responses of Y- direction



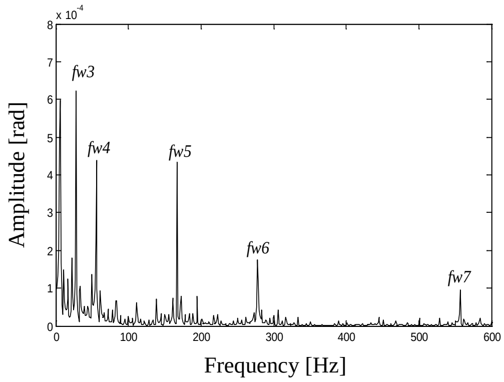
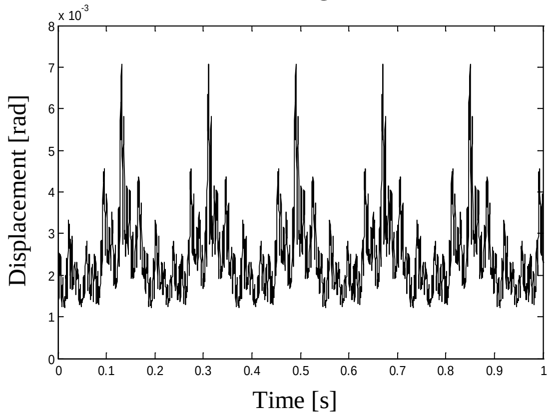


Fig. 10. Temporal and frequency dynamic responses of Angular displacement Φ

Figure 11 shows the linear displacement and it frequency spectrum in the second bearing. Like the first bearing, the dynamic response is dominated by wind effect. In fact, we don't find any apparent effect of the bevel gear.



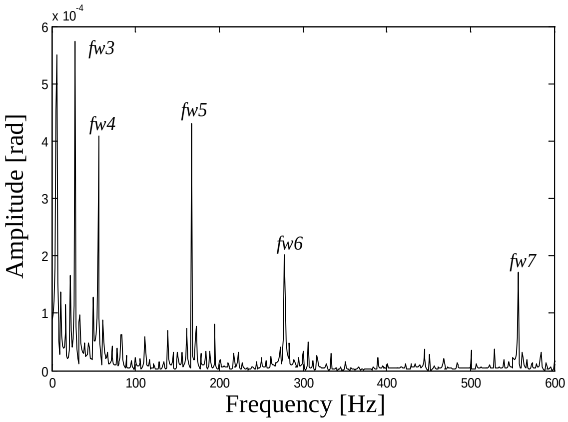
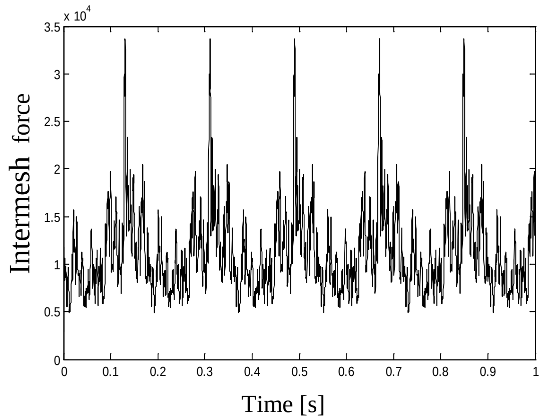


Fig. 11. Temporel and frequency dynamic responses of Angular displacement ψ

In order to verify the safety condition on teeth, we examine the intermesh forces. Figure 12 presents the time evolution of intermesh forces, which presents the multiplication, at each time *ti*, the meshing stiffness $K(t)$ by the teeth deflection. The corresponding spectrum presents both the mesh frequency and its harmonics (*fei*) and the wind frequencies (*fwi*).



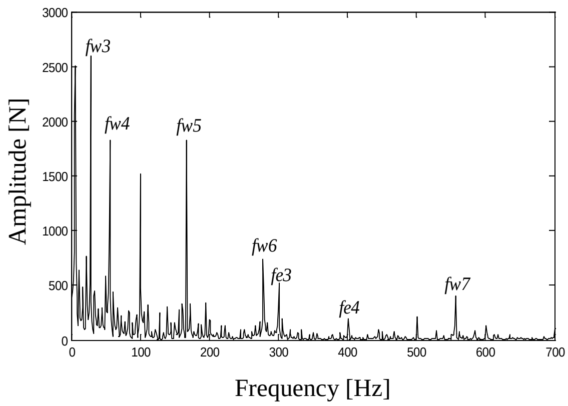


Fig. 12. Dynamic force fluctuation and the corresponding spectrum

From these numerical results, we conclude that the displacement in each bearing is variable and fluctuates around an acceptable amplitude. Besides, we remark that bearing displacement has the same shape of the aerodynamic torque excitation.

In addition, on the spectrums results, we clearly observe the presence of several peaks corresponding on the wind frequencies (*fwi*). In fact, these frequencies are more interesting than meshing ones. These results are very useful to control the dynamic behavior of wind turbines and to initiate maintenance tasks when necessary.

CONCLUSIONS

In this paper, a three dimensional model of one-stage straight bevel gear system is used in a vertical axis wind turbine. This model was developed in order to study the dynamic response of this kind of gearbox subjected to internal and external sources of excitation, which are presented by the periodic variation of the mesh stiffness and the aerodynamic torque fluctuation respectively. The aerodynamic model for the Darrieus-type straight-bladed vertical axis wind turbines was studied using actuator disk theory and Blade-Element Momentum (BEM) theory. The simulation of the dynamic response was done by a step-by-step time integration method (Newmark algorithm). The frequency dynamic response clearly shows the domination of peaks corresponding to the wind speed frequencies to the detriment of the mesh frequencies.

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NOMENCLATURE

a: Axial induction factor
A: Swept area of the turbine
$A_T(t)$: Aerodynamic torque
A_{Ti}: Single blade torque
c: Blade chord
$[C]$: Proportional damping matrix
C_T: Tangential force coefficient
C_N: Normal force coefficient
C_L and C_D : Lift and drag coefficients
C_p : Power coefficient
$F(t)$: External force
fe: Frequency of rotation
fei: Mesh frequency and its harmonics
F_L and F_D: Lift and drag forces
F_N and F_T: Normal and tangential forces
fwi: Wind frequencies
h: Blade height
$k(t)$: Teeth mesh stiffness
I_{11}, I_{12}, I_{21}, I_{22}: Inertia moments
$K_{\theta 1}$, $K_{\theta 2}$: Torsional rigidities
K_{xi}, K_{yi}, K_{zi}: Traction–compression stiffness
$K_{\Phi i}$, $K_{\psi i}$: Bending stiffness
$[K_s]$: Average stiffness matrix
$[K(t)]$: Mesh stiffness matrix
$<L>$: Geometric parameters of dynamic model
$[M]$: Total mass matrix
m_i: Mass of block i
N: Rotor speed
$P(t)$: Power
$q(t)$: Generalized coordinate vector
R: Radius of the turbine
R_T(t): Electromagnetic torque
$V(t)$: Wind speed
Va: Axial velocity
Vc: Chordal velocity component
Vn: Normal velocity component
$W(t)$: Relative flow velocity
xi, yi, zi: Linear displacements of bearing
Z: Gear teeth number
α: Angle of attack
θ: Azimuth angle
ω: Angular speed
ε : Meshing contact ratio
ρ: Air density
$\lambda(t)$: Tip Speed Ratio
$\phi i, \psi i, \theta_{1i}, \theta_{2i}$: Angular displacements