中國機械工程學刊第四十卷第四期第 401~406 頁(民國一百零八年)
Journal of the Chinese Society of Mechanical Engineers, Vol.40, No.4, pp 401~406 (2019)

Study on the Design of Transformer Winding with Voltage Regulation and Interrupting Capacity of Switch Gear

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Keywords： leakage inductance, voltage regulation, short-circuit current, interrupt current, asymmetrical winding.

ABSTRACT

This paper proposes an approach for the design of transformer winding with voltage regulation and interrupting capacity of switch gear. Different from the symmetric winding structure of traditional transformer designs, the proposed method can provide a suitable leakage inductance for voltage regulation and short-circuit current in the transformer. To estimate a satisfactory structural for the winding configuration before manufacture, the leakage inductance is derived from the structural parameters of the transformer. Through the theoretical analyses, the leakage inductance is estimated in the multi-layered structure and a desirable winding configuration is presented that allows the appropriate voltage regulation and short-circuit current. Moreover, the inferential results are successfully obtained substantiation from the experiments.

INTRODUCTION

The magnitudes of the leakage inductance depend upon the structure of winding. The leakage inductance affects the voltage regulation and the short-circuit current. For getting appropriate leakage inductance to match the rating interrupting capacity of switch gears, the leakage inductance must be drawn up a limited range. This paper proposes a scheme based on changing the proportion of inner layer to outer layer in coil winding to obtain a suitable leakage inductance by modifying design as little as possible.

Several factors are considered in transformer design, including steady-state characteristics, cost, weight, volume, capacity and others. An actual transformer design represents a compromise among these factors [1]-[7]. The steady-state characteristics have been improved and corrected through many theories and field tests. Nevertheless, the magnitudes of the leakage inductance are changed from the asymmetrical winding configuration that can cause the inadvertent operation of the protective system has rarely been considered. Therefore, the leakage inductance must be understood from the winding configuration.

Paper Received August, 2018. Revised December, 2018. Accepted December, 2018.
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PROPOSED MATHEMATICAL MODEL

Many literatures have been dedicated to the formulae of leakage inductance [1]-[6]. According to concrete tests of these formulae, there is typically a 10% ~ 20% errors between calculated and actual values. the formula is obtained by using actual field test data [7]. In [8], It is discussed that the relations between the leakage magnetic field distribution under normal operating condition and the one under fault conditions. An analytical reluctance based model is presented in [9]. Based on the principles found in the literature, the following discussions will develop more accurate practical formulas for the derivation of leakage inductance. And it will stand on the structural parameters of the transformer.

This paper proposes a scheme based on modifying the structural alterations by changing the height of winding and the proportion of inner layer to outer layer in coil winding. Leakage inductance depend upon the asymmetric winding configuration of the transformer coil so the relationship between these inductances and transformer winding distribution must be considered and studied. And, for getting appropriate leakage inductance to match the rating interrupting capacity of breaker and low voltage regulation, the leakage inductance must be drawn up a limited range.

Ⅰ. Infinitely Long Solenoid

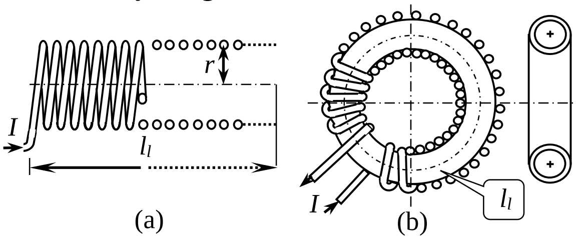

Fig. 1 (a) Infinitely long solenoid. (b) Toroidal winding. [10]

The infinitely long solenoid of Fig. 1 (a) may be regarded as a toroid of infinite radius as Fig. 2 (b); its magnetic field is thus also completely contained within the coil if the winding is idealized into an uninterrupted current sheet. Its inductance can be defined as:

$$L_{long} = \frac{\mu_0 N^2 \pi r^2}{l_l}, \quad (1)$$

where

l_i is the length of solenoid for the long solenoid,

r is the radius of the solenoid,

and N is the turns of the winding.

Ⅱ. Leakage Inductance

For the convenience of descriptions, a winding three-layer structure(S-P-S structure) is defined by the presence of one primary winding between the inner and outer secondary windings.

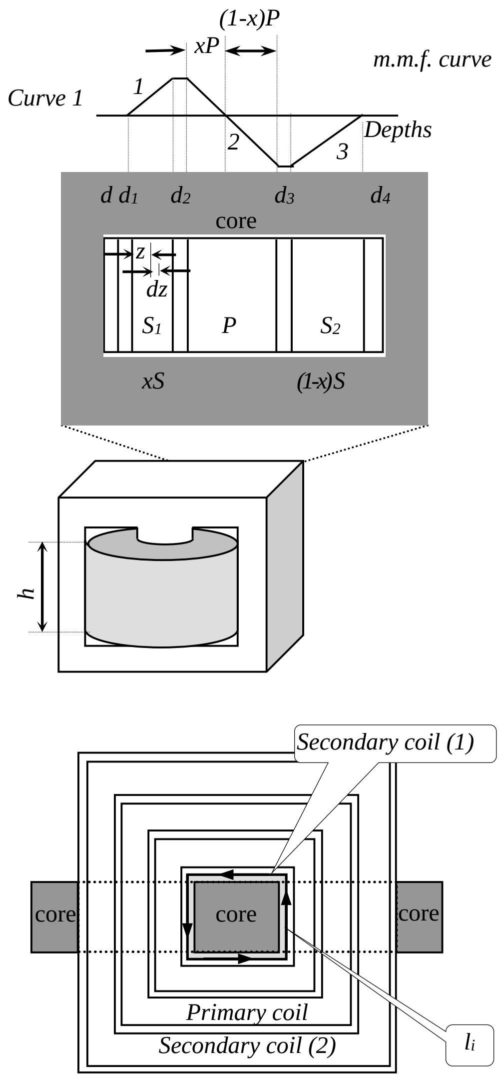

Fig. 2 Three-layer structure cutaway view.

In Fig. 2, which parameters as following.

d_1, d_2, d_3 and d_4 are the thicknesses of insulation paper,
$S_1=xS$ is the thickness of inner secondary winding,
P is the thickness of primary winding,
$S_2=(1\text{-}x)S$ is the thickness of outer secondary winding,
x is the distributive ratio of secondary winding,
d is the thickness of interval,
N_1 is the number of turns in the primary winding,
S is the total thickness of secondary winding,
h is the height of winding,
and l_i is the inner peripheral length of coil winding.

A shell-type transformer with the S-P-S structure describes in Fig. 2. The magnetomotive force between the top and bottom of the coils for different depths z is defined in the curve 1.

As the theory of the Eq. 1, the *m.m.f.* at depth z is

$$\frac{z}{S_1} I_2 x N_2 = \frac{z I_2 N_2}{S}, \quad (2)$$

where
N_2 is the number of turns in the secondary winding,
I_2 is the secondary current.

If the permeability of coil $\mu = \mu_0$, in the district of S_1, the relationship between the variation of flux φ_{S1}' and the thickness of winding z can be obtained as

$$d\varphi_{S1}' = \frac{z I_2 N_2}{S} \cdot \frac{\mu_0}{h} \left(l_i + 8d_1 + 8z \right) dz, \quad (3)$$

where h is the height of core window.

Multiplying (3) by $\frac{z}{xS} x N_2$, $d\lambda_{S1}'$ can be indicated as

$$d\lambda_{S1}' = \frac{z}{xS} x N_2 \cdot \frac{z I_2 N_2}{S} \cdot \frac{\mu_0}{h} \left(l_i + 8d_1 + 8z \right) dz. \quad (4)$$

$$\lambda_{S1}' = \int_0^{xS} \frac{z}{xS} x N_2 \cdot \frac{z I_2 N_2}{S} \cdot \frac{\mu_0}{h} \left(l_i + 8d_1 + 8z \right) dz$$

$$\lambda_{S1}' = \frac{x^2 N_2^2 I_2 \mu_0}{h} \left[\left(l_i + 8d_1 \right) \frac{xS}{3} + 2x^2 S^2 \right] \quad (5)$$

In the district of d_2, the reluctance due to S_1 is

$$\Re_{S_1 d_2} = \frac{h}{\mu_0 \frac{d_2}{2} \left[l_i + 8d_1 + 8xS + 4d_2 \right]}. \quad (6)$$

The inductance includes the parameters of the height and the distributive ratio as

$$L_{\Re_{s1d2}}(x) = \frac{x^2 N_2^2}{\dfrac{h}{\mu_0 \frac{d_2}{2} \left[l_i + 8d_1 + 8xS + 4d_2 \right]}},$$

$$L_{\Re_{s1d2}}(x) = \frac{x^2 N_2^2}{h} \times \mu_0 \frac{d_2}{2} \left[l_i + 8d_1 + 8xS + 4d_2 \right]. \quad (7)$$

The total magnitude of leakage inductance due to S_1 is described in (8).

$$L_{S1}(x)=\frac{x^2N_2^2\mu_0}{h}\left[(l_i+8d_1)\frac{xS}{3}+2x^2S^2+\frac{d_2}{2}F_0\right], \quad (8)$$

where $F_0=l_i+8d_1+8xS+4d_2$.

The area under the product of the thickness "S" and the height "h" is a constant in the same type pecification.

Substituting $K_1=h\times S$ into Eq. 8, we get

$$L_{S1}(x)=\frac{x^2N_2^2\mu_0}{h^3}\left[(l_i+8d_1)\frac{xhK_1}{3}+2x^2K_1^2+\frac{d_2}{2}h^2F_1\right]. \quad (9)$$

where $F_1=l_i+8d_1+8x\frac{K_1}{h}+4d_2$.

Proceeding as (2)-(9), each leakage inductance due to S_2 and primary winding P are defined in (10) and (11).

$$L_{S2}(x)=\frac{(1-x)^2N_2^2\mu_0}{h^3}\left[F_2\frac{(1-x)hK_1}{3}-2(1-x)^2K_1^2+\frac{d_3}{2}h^2F_3\right], \quad (10)$$

$$L_P(x)=\frac{N_1^2\mu_0}{h^3}\left\{x^2\left(F_4\frac{xhK_2}{3}-2x^2K_2^2+\frac{d_2}{2}h^2F_1\right)+(1-x)^2\left[F_4\frac{(1-x)hK_2}{3}+2(1-x)^2K_2^2+\frac{d_3}{2}h^2F_3\right]\right\}, \quad (11)$$

where

$K_2=h\times P$

$F_2=l_i+8d_1+8\frac{K_1}{h}+8d_2+8\frac{K_2}{h}+8d_3$,

$F_3=l_i+8d_1+8x\frac{K_1}{h}+8d_2+8\frac{K_2}{h}+4d_3$,

and $F_4=l_i+8d_1+8x\frac{K_1}{h}+8d_2+8x\frac{K_2}{h}$.

In S-P-S structure, the relationship between the total leakage inductance and the structural parameters of the transformer are given by (12).

$$L_{l_{S-P-S}}(x)=L_P(x)+\left(\frac{N_1}{N_2}\right)^2\cdot\left[L_{S1}(x)+L_{S2}(x)\right] \quad (12)$$

Eq. (9)-(12) show that the leakage inductances of the transformer are directly proportional to the distributive ratio "x" and inversely proportional to the height of the transformer.

LIMITING FACTORS IN IMPEDANCE OF TRANSFORMER

The short-circuit current is limited by the line impedance and the transformer impedance in the power system. A lower value of impedance decreases the voltage regulation but increases the interrupting capacity of switch gears. Accordingly, the impedance of transformer must be restrained within an applicable range.

Eq. (13) is defined the normalized of the equivalent impedance.

$$Z\%=IZ\%_{75^0C}=\left[\left(IR\%_{75^0C}\right)^2+\left(IX\%\right)^2\right]^{\frac{1}{2}}, \quad (13)$$

And the voltage regulation $\varepsilon\%$ is described as Eq. (14).

$$\varepsilon\%=IR\%_{75^0C}\times\cos\theta+IX\%\times\sin\theta+\frac{\left(IX\%\times\cos\theta-IR\%_{75^0C}\times\sin\theta\right)^2}{200}, \quad (14)$$

where

$$IR\%_{75^0C}=\left(I_1^2R_1\frac{\frac{1}{\alpha_0}+75}{\frac{1}{\alpha_0}+T}\right)\cdot\frac{1}{P_{base}}\cdot 100\%, \quad (15)$$

P_{base} is the base value of power,
I_1 is the rate current in primary winding,
α_0 is the temperature coefficient of resistance,
θ is the power factor angle,
T is the room temperature,
$IX\%$ is the per-unit value of the leakage inductance at room temperature,
and R_1 is the total equivalent resistance referred to the primary side.

Estimating leakage inductance and a limited range of standards for impedance are obtained, an optimum winding configuration will be presented.

DESIGN DEMONSTRATION

The method of considering voltage regulation and short circuit current in transformer design is discussed below.

Step 1: Find out the leakage inductance of various coil structures in actual transformer.

Five transformers with the S-P-S structure are used for demonstration. Their structural parameters of the 220V/110-220V 5kVA transformer are as follows:

d= 0.00250m	d_1= 0.00287m
d_4= 0.00146m	N_1= 120 Turns
S= 0.01696m	μ_0 = 4πE-7
d_2= 0.00146m	d_3= 0.00146m
N_2= 120 Turns	P= 0.01696m
l_i= 0.32000m	h= 0.16000m

Table 1 presents the leakage inductances $L_{le_{S-P-S}}(x)$ that is obtained by experiment results.

Table 1 Experimental leakage inductance vary with the value of *x* in the S-P-S structure

x	$L_{le_{S-P-S}}(x)$
1/6	0.45751 mH
1/3	0.26762 mH
1/2	0.19397 mH
2/3	0.24484 mH
5/6	0.42486 mH

Step 2: Calculate the leakage inductances by (8), (9), (10), (11) and (12) using the structural parameters of transformer in graph of fulfillment plan.

For proving the accuracy of the values in leakage inductances, five patterns were developed as demonstrations in this paper. And, experiments and simulations carry out to examine the parameters of the proposed model.

The value of leakage inductances developed from the (12) and the structural parameters of the transformer as following.

Table 2 Calculated leakage inductance vary with the value of *x* in the S-P-S structure

x	$L_{le_{S-P-S}}(x)$	error between calculated and actual values
1/6	0.45751 mH	2.23%
1/3	0.26762 mH	-1.17%
1/2	0.19397 mH	-1.68%
2/3	0.24484 mH	3.12%
5/6	0.42486 mH	5.58%

There is less than 6% error between calculated and actual values.

Step 3: Calibrate the Regions in Which the Leakage Inductances Satisfy (13), (14) and (15).

The impedance of transformer determines the interrupting capacity of switch gears and voltage regulation in loads. The values of parameters in foregoing example are as follows.

$1/\alpha_0$ =234.5 for copper

I_1 =22.7A $R_1 = 164.14 m\Omega$

P_{base} =5kVA T =25^{0}C

Because the 220/220-110V 5kVA transformer is not normalizer in standard, the similar specification is cited to support this argument. Complying with the Chinese National Standard CNS 598, a 3.3kV/220-110V 5kVA the voltage regulation $\varepsilon\%$ should be below 2.55% at a power factor of 1.0. And with the norms of the Grand Power River Co. the per-unit valve of the equivalent impedance Z% must be the region 1.5%~2.5%. Using (13), (14) and (15), the leakage inductance is prescribed in the following.

$$L_l \le 0.3787 \quad mH \tag{16}$$

DISCUSS

Using (12), the diagram in Fig. 3 illustrates that the leakage inductance varies with the value of x. The relationship between the $L_{l_{S-P-S}}(x)$ and the value of x is shown in Table 3.

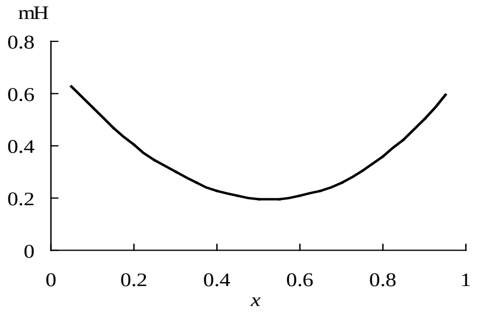

Fig. 3 The calculated values of leakage inductance vary with the value of x.

Table 3 Inferred leakage inductance varies with the value of x.

x	0.05	0.10	0.15	0.20
$L_{l_{S-P-S}}(x)\,mH$	0.62657	0.54441	0.47021	0.40424
x	0.25	0.30	0.35	0.40
$L_{l_{S-P-S}}(x)\,mH$	0.34680	0.29815	0.25859	0.22840
x	0.45	0.50	0.55	0.60
$L_{l_{S-P-S}}(x)\,mH$	0.20785	0.19723	0.19683	0.20693
x	0.65	0.70	0.75	0.80
$L_{l_{S-P-S}}(x)\,mH$	0.22781	0.25975	0.30303	0.35794
x	0.85	0.90	0.95	
$L_{l_{S-P-S}}(x)\,mH$	0.42477	0.50379	0.59528	

Using the least-squares method, a quadratic equation for x can replace the values for leakage inductance. This is given as

$$E_r = \sum_{i=1}^{19}\left[L_{l_{S-P-S}}(x) - f(x_i)\right]^2$$
$$= \sum_{i=1}^{19}\left[L_{l_{S-P-S}}(x) - \left(a_0 + a_1 x_i + a_2 x_i^2\right)\right]^2 \tag{17}$$

where

E_r is the sum of the error values,

x_i denotes the distributive ratio of the secondary winding,

a_0, a_1 and a_2 are the respective term coefficients, and $L_{l_{S-P-S}}(x)$ is obtained using (12).

An approximate quadratic equation of x can be described as follows:

$$L_{l_{S-P-S}}(x) \approx \overline{L}_l(x) = a_0 + a_1 x + a_2 x^2$$
$$= \left(0.7382 - 2.1033x + 2.0429x^2\right)\cdot 10^{-3}. \tag{18}$$

According to (16), (18) must be limited in the following.

$\left(0.7382-2.1033x+2.0429x^2\right)\cdot 10^{-3} \leq 0.3787\cdot 10^{-3}$;

$$0.2164 \leq x \leq 0.8132 . \quad (19)$$

The objective function f_{obj} of the distributive ratio " x " for this example is

$$f_{obj}=\left\{x \middle| \max\left(L_{in\ air_{S-P-S}}(x)\right), \text{ and } 0.2164 \leq x \leq 0.8132 \right\} \quad (20)$$

According to Table 1 and (19), in this sample, the feasible distributive ratio in the actual transformer is $1/3$, $1/2$ and $2/3$.

INFERENCES

Table 4 Estimated leakage inductance varies with the value of the height of winding "h" and x.

h (m) \ x	0.1	0.2	0.3
0.080	2.63666 mH	1.96368 mH	1.43898 mH
0.096	1.71573 mH	1.27593 mH	0.93590 mH
0.112	1.20288 mH	0.89373 mH	0.65633 mH
0.128	0.88975 mH	0.66075 mH	0.48589 mH
0.144	0.68518 mH	0.50874 mH	0.37465 mH
0.160	0.54441 mH	0.40424 mH	0.29815 mH
0.176	0.44350 mH	0.32939 mH	0.24333 mH
0.192	0.36873 mH	0.27397 mH	0.20270 mH
0.208	0.31180 mH	0.23177 mH	0.17176 mH
0.224	0.26743 mH	0.19891 mH	0.14764 mH
0.240	0.23219 mH	0.17281 mH	0.12847 mH
h(m) \ x	0.4	0.5	0.6
0.080	1.07991 mH	0.90387 mH	0.92821 mH
0.096	0.70576 mH	0.59566 mH	0.61572 mH
0.112	0.49712 mH	0.42252 mH	0.43898 mH
0.128	0.36952 mH	0.31597 mH	0.32959 mH
0.144	0.28599 mH	0.24583 mH	0.25725 mH
0.160	0.22839 mH	0.19723 mH	0.20693 mH
0.176	0.18701 mH	0.16216 mH	0.17049 mH
0.192	0.15627 mH	0.13601 mH	0.14324 mH
0.208	0.13281 mH	0.11598 mH	0.12232 mH
0.224	0.11448 mH	0.10028 mH	0.10588 mH
0.240	0.09989 mH	0.08774 mH	0.09273 mH
h(m) \ x	0.7	0.8	0.9
0.080	1.17031 mH	1.64755 mH	2.37730 mH
0.096	0.77610 mH	1.08692 mH	1.55832 mH
0.112	0.55294 mH	0.77082 mH	1.09906 mH
0.128	0.41474 mH	0.57576 mH	0.81699 mH
0.144	0.32334 mH	0.44715 mH	0.63178 mH
0.160	0.25975 mH	0.35794 mH	0.50379 mH
0.176	0.21371 mH	0.29354 mH	0.41168 mH
0.192	0.17930 mH	0.24551 mH	0.34320 mH
0.208	0.15288 mH	0.20872 mH	0.29090 mH
0.224	0.13214 mH	0.17991 mH	0.25003 mH
0.240	0.11555 mH	0.15691 mH	0.21749 mH

The leakage inductance is discussed by the height of winding besides the distributive ratio " x " in this section. Considering the height of winding in Eq. (8) ~ (12), the values of the leakage inductance are shown in Table 4. And the magnitudes of the leakage inductance those conform to the Eq. (16) are marked.

The run chart in Fig. 4 illustrates that the leakage inductance varies with the value of the height of winding "h" and x.

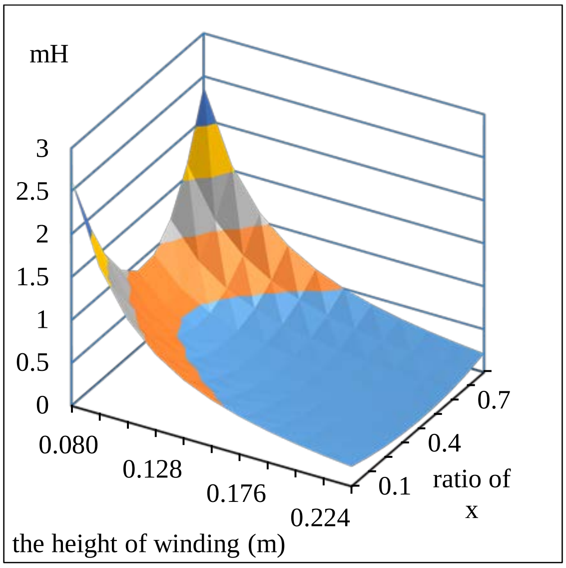

Fig. 4 Trend of the magnitudes of the leakage inductance with the height of winding and the distributive ratio of secondary winding

Using the theoretical analyses, various optimum distributive ratio and the height of winding for the coil winding has been presented. Proving by experiments, the appropriate voltage regulation and short-circuit current can be obtained in the transformer for the traditional transformer design.

CONCLUSIONS

This paper presents an approach to transformer design that involves a change the height of winding and an asymmetric winding configuration to obtain perfect designs for transformer.

For determining suitable levels of short-circuit current and voltage regulation, the formulae for the leakage inductance of the multiple asymmetric winding configurations are derived. According to these restrictions on the voltage regulation $\varepsilon\%$ and the per-unit value of the equivalent impedance $Z\%$, an optimum winding configuration is presented. Experiments and calculations both show that the proposed design is successful.

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電壓調整率與開關設備啟斷容量於變壓器繞組設計之研究

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摘要

本文提出了一種藉由變壓器繞組結構配置改變漏磁電抗的方法。與傳統變壓器設計的對稱繞組結構不同，本方法對變壓器的電壓調整率、開關設備啟斷容量及短路電流限制，提供合適的漏磁電抗值。為了滿足製造前即對繞組特性有準確的預估，論述中以設計時所獲得的變壓器各類材料的尺寸結構參數，預先推導出漏磁電抗值。於製造前即可提供使用者完整的訊息，免於製作完成後再修正，也降低後續所衍生出的材料與時間浪費。經由理論分析，對多層結構中的漏電電感進行了估計，提出了一種理想的繞組結構，使其具有適當的電壓調整率和短路電流。最後也於實驗中成功地印證推理結果。