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Temperature Stratification Failure Mechanism of LNG Tank Based on Complex Contourlet Finite Element Method

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Keywords：temperature stratification, failure, LNG tank, complex Contourlet finite element model.

ABSTRACT

Temperature stratification failure of LNG tank is a main security problem of LNG tank, to enhance computation accuracy and efficiency of temperature stratification failure of LNG tank, complex Contourlet finite element model is established by combing complex Contourlet transform and finite element method. Firstly, theory model of temperature stratification failure of LNG tank is constructed, Secondly, complex Contourlet finite element model is confirmed, and mathematical model is deduced. Finally, temperature stratification simulation analysis is carried out, temperature stratification of LNG tank is analyzed based on B-spline finite element model and proposed complex Contourlet finite element model, comparison between simulation results and existing testing results illustrate that proposed complex Contourlet finite element model has higher computation accuracy precision and efficiency. Stratification degree changing laws of LNG tank are obtained, and effect rules of heat leakage on temperature stratification failure are also obtained.

INTRODUCTION

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LNG is a multiple component cryogenic liquid, which exits temperature stratification during process of storage and transportation. Compared with pure single component working medium, temperature stratification procession of LNG has its own features. Temperature stratification of LNG will cause rollover accident inside LNG tank. When rollover of LNG tank happens, liquid in LNG tank will be violently mixed to release a lot of heat, and a lot of steam will be generated instantaneously, instantaneous impact will lead to great harm to performance and service life of LNG tank, and affect security during storage and transportation. LNG tank occupies core position in LNG terminal, which will lead to major potential hazards such as low temperature, fire, rollover. After LNG tank has stratification phenomenon, liquid in each layer maintains a relatively stable state. Mass and energy exchange occurs at interface. LNG density in upper layer is small, and boiling point of light component is also low. After absorbing heat leakage from environment and heat transferred from lower part, liquid will take away some of heat through evaporation and heat absorption, so that upper layer density will continue to increase. LNG in lower layer absorbs heat leakage from environment and transfers it to upper liquid, and density gradually decreases. During entire transfer process, when densities of upper layer and lower layer are close, the two layers of liquid will mix, and LNG in lower layer that has not yet evaporated will absorb heat and accelerate evaporation, and convection circulation will intensify. Two layers of LNG with different original density in tank will happen rollover pheromone. During rollover process, a large amount of BOG will be generated, and accumulated gas will make pressure in LNG tank continuously increase, which may cause serious accidents with casualties and economic losses.

In recent years, thermal stratification failure mechanism of tank has been studied by some scholars, Huimin et al. (2022) carried out numerical simulation on thermal stratification process for storage tank, and obtained effect rules of number of middle perforated obstacles and inlet velocity on thermal stratification process of tank. Wei et al. (2018) studied dynamic features of original interface in stratified multiple composition liquid tanks in process of rollover, effect rules of buoyant flow on temperature stratification and rollover were obtained. Yuxing et al. (2015) analyzed rollover mechanism of LNG tank, and confirmed rollover criteria for ensuring security of LNG tank. Jin et al. (2022) studied temperature stratification mechanism of liquid air energy storage tank based on experiment analysis, and put forward measures for avoiding rollover. Maksym and David (2018) studied rollover mechanism of LNG tank based on a novel rollover-magnitude-evaluation method considering saturated vapor pressure. Antoine et al. (2019) analyzed rollover rules of LNG tank based on developed and validated rolloverFoam, and designed analysis code based on Navier-Stokes equations. As seen from existing achievements, thermal stratification and rollover phenomenon of tank have been a major research direction that mainly focuses on thermal stratification rollover phenomenon, however stratification and rollover process of LNG tank is very complex, experiment is relative difficult to be carried out, therefore it is necessary to select an effective numerical means to analyze stratification process of LNG tank.

In order to effectively analyze thermal stratification failure mechanism of LNG tank, an numerical analysis method should be established. Currently some intelligent algorithms have been proposed for numerical analysis. Machine learning algorithm is an good nonlinear analysis method, which has been applied in many engineering application fields (Cheng-Chi and Chih-Jer, 2023). Ping-Huan Kuo et al. (2020) proposed high precision random forest-based maximum lyapunov exponent algorithm, which can effectively solve nonlinear problem. Cheng-Chi and Chih-Jer (2020) applied machine learning algorithm to carry out bifurcation and nonlinear behavior analysis, and obtained better analysis effect. Cheng-Chi et al. (2020) applied machine learning algorithm to analyze nonlinear dynamic features of nonlinear system, and obtained better analysis results. According to thermal stratification failure real situation of LNG tank, finite element method is an effective numerical method, however thermal stratification failure process has high degree nonlinear characteristics, to improve analysis precision wavelet finite element method will be an effective method. Wavelet finite element method has been developed through combing wavelet transform and finite element method, which has good multiple resolution characteristics. Currently, wavelet finite element method has been successfully solve some engineering problems. Hao et al. (2021) proposed a unified wavelet finite element model for dynamical analysis of composite shell, and obtained higher analysis precision and efficiency. Joglekar (2020) proposed wavelet spectral finite element model for nonlinear frequency mixing analysis of beam, and efficacy of proposed model was validated. Hewei et al. (2021) proposed a novel finite element model centered B-spline wavelet for coping with first-order neutron transport equation, which has better analysis accuracy]. As seen from existing research achievements, wavelet finite element can get high analysis effect on nonlinear problems, therefore it is feasible to apply it to analyze temperature stratification failure process of LNG tank. In order to improve analysis effectiveness, complex Contourlet is selected to construct complex Contourlet finite element model due to its directional information expression ability.

This research proposed a complex Contourlet finite element model for thermal stratification failure analysis of LNG tank, which can improve analysis efficiency and precision of numerical analysis. Analysis results can offer suitable basis for avoiding thermal stratification failure of LNG tank.

MODEL OF COMPLEX CONTOURLET TRANSFORM

Complex transformation of wavelet is composed of two symmetrical tree-structured filter banks. Transformation principle is as follows: let $\phi(x,y)=\phi(x)\phi(y)$ and $\phi(t)$ be complex, and then

$$\phi(x,y)=[\phi_h(x)+i\phi_g(x)][\phi_h(y)+i\phi_g(y)]$$
$$=[\phi_h(x)\phi_h(y)-\phi_g(x)\phi_g(y)]+i[\phi_g(x)\phi_h(y)+\phi_h(x)\phi_g(y)] \quad (1)$$

Complex Contourlet transform concludes two components: Firstly, two-dimensional dual-tree complex wavelet transform (2DDTCWT) proposed by Kingsbury is applied to realize multiple scale decomposition, and fine node subspace W_j on scale 2^j contains a double-tree structure, and six directional sub-bands are obtained, and coefficient generated at position $x=(x_1,x_2)$ can be described by

$$\{\varphi_{n,q,e}^{e_1}\}_{n,x\in Z^2} \quad (2)$$

where $q\in\{1,2\}$ denotes real and imaginary parts of wavelet coefficients, $e_1\in\{1,2,3\}$, $e_1\in\{1,2\}$ can combine with q to describe 6 different direction information by using 12 sub-bands; $n=(n_1,n_2)$ denotes position offset. Output of complex wavelet transform can constitute a quasi-Hilbert transform pair, therefore, fine node subspace W_j has approximate shift invariance property.

Secondly, to further improve directional resolution of complex wavelet transform, directional filter (DFB) can be connected with its fine sub-band, and detail sub-space can be further decomposed. Each sub-band can be expanded to $2l_j$ sub-band spaces, which is expressed by

$$\eta_{\kappa,n,q,e}^{e_1 I_j}(x) = \sum_{n \in Z^2} h_k^{I_j}[n - C_k^{I_j} w]\varphi_{n,q,e}^{e_1}(x) \quad (3)$$

where $\eta_{\kappa,n,q,e}^{e_1 I_j}(x)$ denotes direction sub space family on scale 2^j, $\kappa \in [1, 2, \cdots, 2^{I_j}]$, $h_k^{I_j}(\cdot)$ denotes impulse response of reconstruction filter. In this case, each direction subspace $V_{j,\kappa}^{I_j}$ contains a complex dual-tree frame, which can more flexibly represent the geometric information in the image than the detail subspace V_j of DTCWT. Combining dual-tree complex wavelet with directional filter architecture can be named as complex contourlet transform.

THERMAL STRATIFICATION FAILURE MODEL OF LNG TANK

Because the thermal stratification process belongs to turbulence, in order to improve calculating precision of numerical analysis, the large eddy simulation method is applied to describe the thermal stratification of LNG tank, the pulsation with small scale in turbulence can be eliminated by using filtering method, and the filtering process can be achieved by integral operation, the corresponding expression is listed as follows (Mi et al., 2023):

$$\bar{u}(x,t) = \int_V u_i(\xi,t)G(x-\xi)dV \quad (6)$$

where $G(x-\xi)$ is the filtering function, and the cartridge filtering function is used in this research, which is expressed as follows:

$$G(x-\xi) = \begin{cases} 1/\Delta, |\eta| \le \Delta/2 \\ 0, |\eta| > \Delta/2 \end{cases} \quad (7)$$

where Δ is the grid average scale, η is the dissipation scale, ξ is the initial horizontal coordinate.

The controlling equation of large eddy simulation can be obtained through filtering process, which is expressed as follows:

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j} + \mu \frac{\partial \bar{u}_i}{\partial x_i \partial x_j} \quad (8)$$

$$\frac{\partial \bar{u}_i}{\partial x_j} = 0 \quad (9)$$

where ρ represents density of LNG, μ represents kinematic viscosity of LNG, τ_{ij} represents sub grid stress. Corresponding equations can be deduced by using strain rate of large scale flow field as (Chenglong et al., 2022):

$$\tau_{ij} - \frac{1}{3}\tau_{kk}\delta_{ij} = -2\mu_t \bar{S}_{ij} \quad (10)$$

$$\bar{S}_{ij} = \frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x} + \frac{\partial \bar{u}_j}{\partial y}\right) \quad (11)$$

where δ_{ij} represents Kronecker signal, $\bar{S}_{ij}$ represents large scale deformation ratio tensor, μ_t represents eddy viscosity factor of sub grid, which is computed by Smagorinsky model, and its formulation is as follows (Jianyun et al.,2013)

$$\mu_t = (C_S \bar{\Delta})^2 |\bar{S}| \quad (12)$$

where $|\bar{S}| = \sqrt{2\bar{S}_{ij}\bar{S}_{ij}}$, C_S represents empirical Smagorinsky factor, C_S =0.2.

Status expression of mass density is formulated by:

$$\rho = \rho_0[1 - \alpha(T - T_0)] \quad (13)$$

where α represents heat expansion factor, ρ_0 .and T_0 represent original density and temperature respectively.

Volume ratio equation of q th phase of LNG is formulated by

$$\frac{\partial \alpha_q}{\partial t} + \bar{V}\alpha_q = \frac{S_{\alpha_q}}{\rho_q} \quad (14)$$

CONSTRUCTION OF THERMAL STRATIFICATION FAILURE COMPLEX CONTOURLET FINITE ELEMENT MODEL

Complex Contourlet functions $\hat{c}_{l,m}^1(\alpha)$ and $\hat{c}_{l,m}^2(\beta)$ generate multiple resolution subspaces $\{\Theta_j^1\}$ and $\{\Theta_j^2\}$, tensor product of subspaces form higher order space that is formulated by

$$\Theta_j = \Theta_j^1 \otimes \Theta_j^2 \quad (15)$$

where Θ_j represents tensor space, j=0,1,...,N-1; $\otimes$ represents Kronecker signal, α and β represent local coordinates.

Complex Contourlet function in sub space is formulated by

$$\vec{C}_{l,m}^1 = \{\hat{c}_{l,m}^1(\alpha), \hat{c}_{l,m}^1(\alpha+1), \cdots, \hat{c}_{l,m}^1(\alpha+(N-2))\} \quad (16)$$

$$\vec{C}_{l,m}^2 = \{\hat{c}_{l,m}^2(\beta), \hat{c}_{l,m}^2(\beta+1), \cdots, \hat{c}_{l,m}^2(\beta+(N-2))\} \quad (17)$$

Complex Contourlet function on $\{\Theta_j\}$ is formulated by

$$\vec{C}_{l,k} = \vec{C}^1_{l,k} \otimes \vec{C}^2_{l,k} \quad (18)$$

Complex Contourlet function is considered as interpolation function to confirm Complex Contourlet finite element model, and temperature function $\theta(\alpha,\beta)$ is defined by

$$\theta(\alpha,\beta)=\vec{\psi}\vec{b} \quad (19)$$

where $\vec{C}$ is complex Contourlet function, $\vec{b}$ is complex Contourlet coefficient vector, $\vec{b}=(b_0,\dots,b_{-(N-2)})$, $b_0,\dots,b_{-(N-2)}$ represent elements of $\vec{b}$.

Thermal conductive model is formulated by (Max et al., 2019)

$$\rho c\frac{\partial\theta}{\partial t}-\frac{\partial\theta}{\partial\alpha}(k_\alpha\frac{\partial\theta}{\partial\alpha})-\frac{\partial\theta}{\partial\beta}(k_\beta\frac{\partial\theta}{\partial\beta})-\rho Q=0 \quad (20)$$

Equivalent integral form of 2D heat conduction is formulated by

$$\int_{S_e}[\delta\theta\cdot\frac{\partial\theta}{\partial t}+\frac{\partial\delta\theta}{\partial\alpha}(k_\alpha\frac{\partial\theta}{\partial\alpha})-\frac{\partial\delta\theta}{\partial\beta}(k_\beta\frac{\partial\theta}{\partial\beta})-\delta\theta\rho Q]d\alpha d\beta$$
$$-\int_{\Gamma_e}\delta\theta q dl-\int_{\Gamma_e}\delta\theta h(\theta_e-\theta)dl=0 \quad (21)$$

where e is element, S is surface of element, l is line boundary of element.

Putting $\theta(\alpha,\beta)=\vec{\psi}\vec{b}$ into formula (21) considering arbitrariness of $\delta\theta$, complex Contourlet finite element model is deduced as follows (Daniel et al., 2023):

$$T\frac{\partial\theta}{\partial t}+H\theta=L \quad (22)$$

where T is thermal capacity matrix, H is thermal conduction matrix, L is thermal load volume.

THERMAL STRATIFICATION FAILURE SIMULATION ANALYSIS OF A LNG TANK

To validate effectiveness of proposed complex Contourlet finite element model, comparison analysis is carried out, height of tank is 510mm, diameter of tank is 325mm, armored thermocouple is used to measure temperature, MD-G pressure sensor is used to measure pressure. Proposed complex Contourlet finite element model and B-spline wavelet finite element model are used to carry out thermal stratification failure simulation analysis. θ_1 is gas phase point, and θ_2 is liquid phase point. Filling rate is 80%. Liquid phase layer 1 is meshed by 400 complex Contourlet finite elements and 2260 B-spline wavelet finite elements, and liquid layer 2 is meshed by 450 complex Contourlet finite elements and 2046 B-spline wavelet finite elements. The temperature of LNG is measured by temperature sensor, and pressure of LNG is measured by pressure sensor. Temperature changing rules of θ_1 and θ_2 are illustrate in figure 1 and figure 2 respectively.

As seen from figure 1 and figure 2, complex Contourlet finite element solution is closer to testing value than B-spline wavelet finite element solution, therefore proposed complex Contourlet finite element model has higher computation accuracy. Temperature rise rate and change trend of gas and liquid phase in LNG tank are different, which leads to obvious temperature difference between gas-liquid phases and at different heights in each phase zone, which is reflected in figure 1 and figure 2 as obvious temperature gradient.

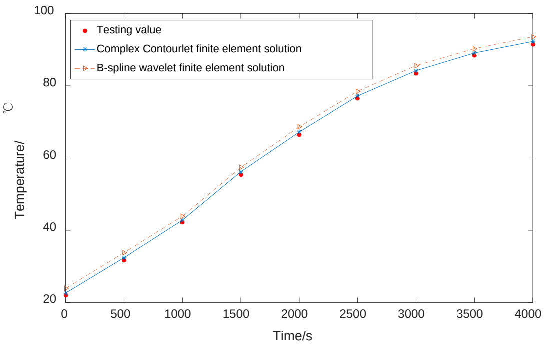

Fig.1. Temperature changing rules of θ_1

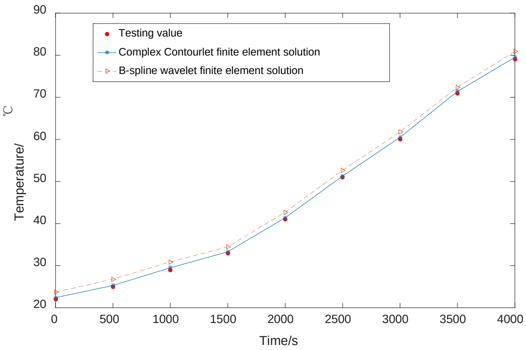

Fig.2. Temperature changing rules of θ_2

Internal pressure of LNG tank is also calculated by B-spline wavelet finite element model and complex Contourlet finite element model. Simulation and experimental results are illustrated in figure 3. As seen from figure 3, complex Contourlet finite element solution is closer to testing results than B-spline wavelet finite element solution, therefore it has higher computation accuracy than B-spline wavelet finite element model. In addition, complex Contourlet finite element model uses less elements to obtain higher computing precision, therefore complex Contourlet finite element has also higher calculation efficiency.

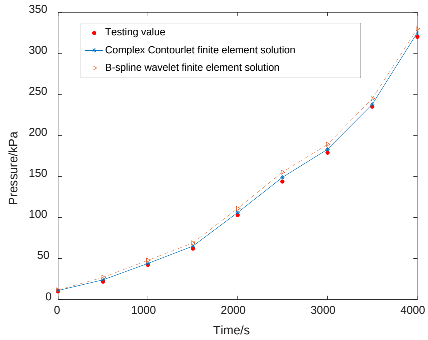

Fig.3. Changing curves of internal pressure

In order to effective evaluation temperature stratification characteristics, stratification degree is introduced in this research, which is ratio of internal pressure to saturated vapor pressure of medium, complex Contourlet finite element model is used to analyze stratification degree changing rules of LNG tank. Results are listed in table 1.

Table 1. Stratification degree of LNG tank

Time/s	Stratification degree
0	1.00
500	1.53
1000	1.76
1500	1.98
2000	2.23
2500	1.96
3000	1.68
3500	1.59
4000	1.42

As seen from table 1, stratification degree increases first and then decreases, stratification degree reaches to maximum value at 2000s, therefore temperature stratification of LNG tank tends to be eliminated with continuous heating.

Environmental heat flux intensity is a main factor of affecting temperature stratification failure process, which concludes heat leakage of side wall and bottom. Effect of heat leakage of side wall and bottom on temperature stratification of LNG tank is studied. Failure time is used to reflect rollover due to temperature stratification. Analysis results are shown in figure 4. As seen from figure 4, failure time of LNG tank due temperature stratification decreases with increase of heat leakage of side wall, and failure time of LNG tank is inversely proportional to heat leakage of bottom, therefore controlling heat leakage of LNG tank can effectively avoid failure due to rollover.

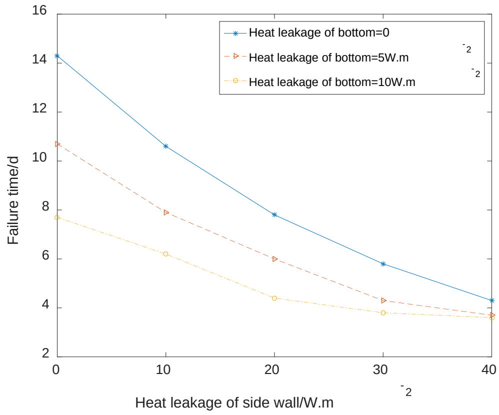

Fig.4. Failure time changing curve of LNG tank with heat leakage

CONCLUSIONS

Temperature stratification process will lead to rollover of LNG tank, which can cause failure of LNG tank. Complex Contourlet finite element model is constructed to enhance computation accuracy and efficiency of temperature stratification failure process. Compared with existing experimental analysis results, effectiveness of proposed complex Contourlet finite element model is verified. Simulation results illustrate that complex Contourlet finite element model has higher computation accuracy and efficiency than B-spline wavelet finite element model. Temperature stratification degree increases first and then decreases with heating. failure time of LNG tank due temperature stratification decreases with increase of heat leakage of side wall, and failure time of LNG tank is inversely proportional to heat leakage of bottom. Accuracy results can provide effective guidance for establishing prevent measurements of temperature stratification failure.

REFERENCES

Antoine, H., Siaka, D., Petr, D., Jennifer, W., "Predicting Liquefied Natural Gas (LNG) rollovers using Computational Fluid Dynamics," Journal of Loss Prevention in the Process Industries, Vol.62, No. 103922(2019).

Cheng-Chi, W., Chih-Jer, L., “Dynamic Analysis and Machine Learning Prediction of a Nonuniform Slot Air Bearing System, ” Journal of Computation and Nonlinear Dynamics, Vol.18, No.011007(2023).

Cheng-Chi, W., Chih-Jer, L., “Bifurcation and Nonlinear Behavior Analysis of Dual-directional Coupled Aerodynamic Bearing Systems,” Symmetry-Basel, ,Vol.12, pp.1521–1543(2020).

Cheng-Chi, W., Rong-Mao, L., Chih-Jer, L., Chih-Yung, H., Tsui-Er Lee, “Research on the Nonlinear Dynamic Characteristics of Opposed High-Speed Gas Bearing

Systems, ”Journal of Low Frequency Noise Vibration and Active Control, Vol.39, No.3, pp.502–522(2020).

Chenglong, S., Jiayu, L., Jin, L., “Finite Element Analysis of the Influence of Tide Velocity on the Borehole Entering of Drilling Riser,” Journal of Petrochemical Universities, Vol.35, No.3, pp.75-80(2022).

Daniel, A., W., Jun, K., Kenneth, S., Daniel A., T., Seth, W., “A reduced order model approach for finite element analysis of cellular structures,” Finite Elements in Analysis and Design, Vol.214, No. 103855(2023).

Hao, Z., Yixin, C., Feng, J., Zhibo, Y., “Unified wavelet finite element formulation for static and vibration analysis of laminated composite shells, ” Composite Structures, Vol.272, 2021, No.114207(2021).

Hewei, G., Wei, C., Xinbiao, J., Duoyu, J., Xinyi, Z., “A discontinuous finite element method based on B-spline wavelet on the interval for solving first-order neutron transport equation with discrete ordinate (SN) angular discretisation,” Annals of Nuclear Energy, Vol.162, No.108511(2021).

Huimin, F., Haotian, L., Suoying, H., Jianhui, Q., Kuihua. H., Ming, G., "Numerical simulation on thermal stratification performance in thermocline water storage tank with multi-stage middle perforated obstacles", Thermal Science and Engineering Progress, Vol.35, No.101473(2022).

Jianyun, S., Jingjie, R., Peng, L., Mingshu, B., “Experimental research on the effects of fluid and heater on thermal stratification of liquefied gas,” Experimental Thermal and Fluid Science, Vol.50, pp.29-36(2013).

Jin Y.H., Jung Hwan, P., Jeong Ik, L., "Experimental investigation of tank stratification in liquid air energy storage (LAES) system", Applied Thermal Engineering, Vol.202, No. 117841(2022).

Joglekar, D.M., “Analysis of nonlinear frequency mixing in Timoshenko beams with a breathing crack using wavelet spectral finite element method,” Journal of Sound and Vibration, Vol.488, No.115532(2020).

Max, B., Houman, O., Mathieu, D., “Operator-adapted wavelets for finite-element differential forms,” Journal of Computational Physics, Vol.388, pp.144-177(2019).

Maksym, K., David A., W., "LNG rollover challenges and their mitigation on Floating Storage and Regasification Units: New perspectives in assessing rollover consequences", Journal of Loss Prevention in the Process Industries, Vol.54, pp.352-372(2018).

Mi, Z., Oriol Costa, G., Shuhao, M., Naiqiang, Z., “Numerical investigation of turbulent thermal stratification at a horizontally oriented 90° pipe-elbow with varied elbow radiuses,” International Journal of Thermal Sciences, Vol.185, No.108092(2023).

Ping-Huan, K., Rong-Mao, L., Cheng-Chi, W., “A High-Precision Random Forest–Based Maximum Lyapunov Exponent Prediction Model for Spherical Porous Gas Bearing Systems,” IEEE Access, Vol.8, pp.168079-168086(2020).

Wei, S., Jingjie, R., Cheng, W., Mingshu, B., Yaofen, B., "Dynamic characteristics of the initial interface in stratified multi-composition liquid tanks during rollover", Applied Thermal Engineering, 145, 2018, 396-406.

Yuxing, L., Zhenglong, L., Wuchang, W., "Simulating on rollover phenomenon in LNG storage tanks and determination of the rollover threshold", Journal of Loss Prevention in the Process Industries, Vol.37, pp.132-142(2015).

NOMENCLATURE

W_j fine node subspace

$\{\varphi_{n,q,e}^{e_1}\}_{n,x\in Z^2}$ wavelet coefficient

$q \in \{1,2\}$ real and imaginary parts of wavelet coefficient

$n=(n_1,n_2)$ position offset

$2l_j$ sub-band space

$\eta_{\kappa,n,q,e}^{e_1 I_j}(x)$ direction sub space family

$h_k^{I_j}(\cdot)$ impulse response of reconstruction filter

$V_{j,\kappa}^{I_j}$ direction subspace

$G(x-\xi)$ filtering function

Δ grid average scale

η dissipation scale

ξ initial horizontal coordinate

ρ density of LNG

μ viscosity of LNG

τ_{ij} sub grid stress

δ_{ij} Kronecker signal

$\overline{S}_{ij}$ large scale deformation ratio tensor

μ_t eddy viscosity factor of sub grid

C_S empirical Smagorinsky factor

α heat expansion factor

ρ_0 original density

T_0 original temperature

$\widehat{c}_{l,m}^1(\alpha)$, $\widehat{c}_{l,m}^2(\beta)$ complex Contourlet functions

Θ_j tensor space

$\otimes$ Kronecker signal

α, β local coordinate

$\theta(\alpha, \beta)$ temperature function

$\vec{b}$ complex Contourlet coefficient vector

e element

S surface of element

l line boundary of element

T thermal capacity matrix

H thermal conduction matrix

L thermal load volume.