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The Application of the Design of Experiments Approach in Thermal and Structural Calculation of a Brake Disc

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Keywords: heat transfer, brake disc, ansys, Von Mises, Design of Experiments, Fisher test.

ABSTRACT

In this paper we have developed a thermal and mechanical study of an aeronautical brake disc, which we have made geometric changes. To inspect the effect of these disc modifications on thermal and mechanical behavior, a simulation series was done by the ansys software in order to calculate the heat transfer coefficient, temperature and Von Mises stress. In this study we used statistical tools such as the design of experiments approach to establish the experimental designs required for this study and to write the corresponding mathematical model for each response. Finally, Fisher test was applied to all the designs to determine the most influential factors on the thermal and mechanical behavior of the brake disc.

INTRODUCTION

The construction of the brake discs is the subject of numerous studies in the field of automotive, railway and aviation. Indeed, it involves the safety of passengers, which is a primary criterion. The research has focused on the contact of two rubbing parts where various phenomena may occur such as the rise in temperature, wear and noise emissions, many experimental studies have been donne to measure brake disc temperature distribution and thermal stress (Gao and Lin, 2002; Chung, Jung and Park, 2010; Okamura and Yumoto, 2006). Noting also that the FE approach is widely employed in solving such as problems related to braking phase (Shahzamanian et all, 2010; Li and Barber, 2002).

Based on previous research, it have been found that the following factors affect the brake disc behaviour: braking mode: single,emergency and repeated brake; the shape of the disc: full or ventilated, thickness variation and hole number; material properties : disc, pad (Adamowicz and Grzes, 2011; Lijun and Kun, 2011; Duzgun, 2002; Chiba, 2009; Belhocine and Bouchetara; 2013). A statistical design of experiments approach has indicated that the number of braking applications has the strongest effect on the interface temperatures in comparison with other factors, i.e. friction loads, sliding speeds and friction material composition Qi , Day (2007).

To examine the influence and the interactions of different parameters characterizing a brake disc on the thermal and mechanical behavior, we adopted the design of experiments method (DOE) Kowang,Long (2015). This statistical optimization techniques very useful in parametric analysis allows to obtain the analytical model which describes the relationship between the main parameters, their interactions and the response (the temperature or the mechanical constraints). According to the established mathematical model, the experimenter can thus deduce qualitative or quantitative information on the behavior of the object studied.

In the present study, we adopted the following approach:

1) Determination of the temperature distribution and equivalent stress for different geometrical brake disc configurations (Figure 1) during the braking phase using the FE comercial software ANSYS 14.5.
2) Application of the experimental design method to get the mathematical model describing the thermal or mechanical behavior of the brake disc , followed by the analysis of the variance (ANOVA test) in order to confirm the significant effects of the selected parameters on the response.

In general, the experimental design method allows to identify and to establish the relationships between two types of variables:

a) The response as output variable (temperature or mechanical stress),
b) The factors: physical variables modifiable by the experimenter, assumed to influence the varation of

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the response (e.g. the geometrical brake disc parameters) Each factor can be:

1) Continuous: it can adopt all the actual numerical values in the defined interval; In our case, the geometric dimensions of the disk are continuous factors (Thickness, diameter, etc.).
2) Discrete: the set of values that the factor can take is finished; These values are within a specified interval; For example, the number of notches in a brake disc is a discrete factor.
3) Qualitative: the set of values that the factor can take is finished: we speak here of modalities. The type of discs or the characteristic of the material used are examples of qualitative factors.

Geometrical Model

The aircraft brake disc selected for this study is shown in Figure 1. The aim geometrical dimensions of the brake disc and the pad are respectively shown in Figure 2 and Figure 3 Moravan (1988). The variables chosen to establish the experimental design are the hole number, the slit number, the disc thickness and the external face of the disc. The table 1 shows the levels of each variable. In Figure 4 are presented the different studied disc configurations.

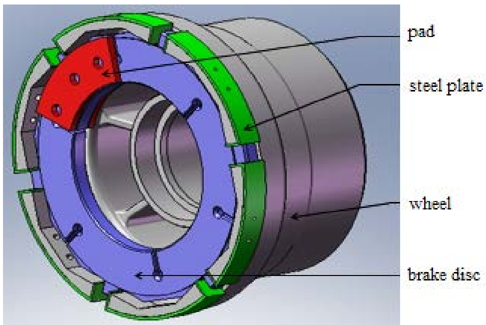

Fig. 1. Aircraft brake disc.

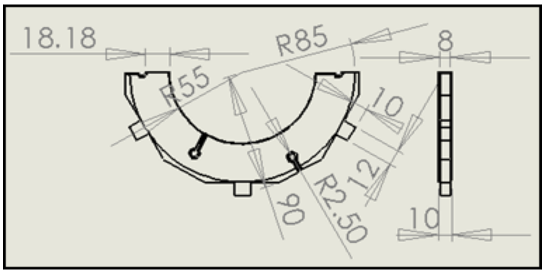

Fig.2. Brake disc dimensions.

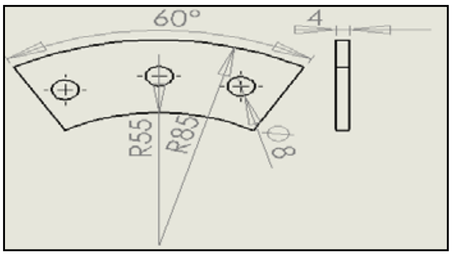

Fig. 3. Pad dimensions.

Table 1. Design modifications of the brake disc

Factors / Levels	x_1	x_2	x_3	x_4	x_5
Low (-)	8 mm	3	3	no	yes
High (+)	12 mm	6	6	yes	no
x_1: thickness, x_2: slit number, x_3: hole number x_4: hole number in the disk, x_5: external shape					

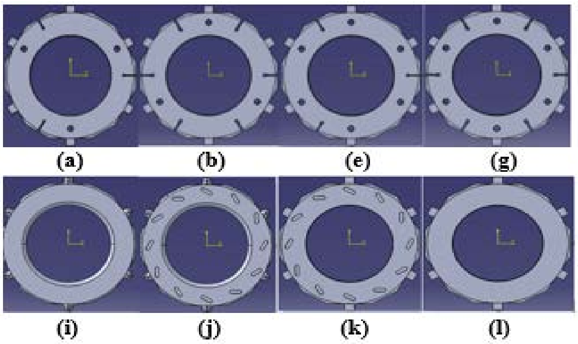

Figure 4. Disc configurations.

(a) Disc (3 holes, 3 slits, thickness 8 mm)
(b) Disc (3 holes, 3 slits, thickness 12 mm)
(e) Disc (6 holes, 3 slits, thickness 8 mm)
(g) Disc (6 holes 6 slits, thickness 8 mm)
(i) Full disc with modified external shape
(j) Disc slots and modified external shape
(k) Disc with slots and real external shape
(l) Full disc, thickness 10 mm.

THERMAL MODELING

Thermal Flux as Function of Braking Time

Table 2 gives the values of the parameters required to calculate the brake speed V_h, the initial angular velocity of brake disc ω_0, the applied force on the disk F_d and the heat flux $q_{inst}(t)$ Mckin (2002).

Brake Speed:

$\sigma = 32.54 - 5.31X_1 - 8.42X_2 + 3.54X_3 - 3.44X_1X_2 - 1.19X_1X_3 + 2.68X_2X_3$

$V_b = V_i - a_d t$ (1)

Applied force on the disk: $F_d = 1512$ [N],

Heat flux: $qinst?t) = 681902 - 75630.48 * t$ (2)

Initial angular velocity of brake disc $\omega_b = \frac{V_b}{r}$.

Table.2. Values of main simulation parameters

Total braking time t_b [s]	9.00
Time step Δt [s]	0.01
Initial time t_i [s]	0
Aircraft weight *m* [kg]	1050
Initial speed of landing V_i [m/s]	28.00
Aircraft deceleration *dec* [m/s^2]	3.00
Braking distance L_b [m]	130
Wheel radius *r* [m]	0.42
Flux distribution rate *k*	0.04
Contact surface (disc/pad) A_d [mm2]	13194.68

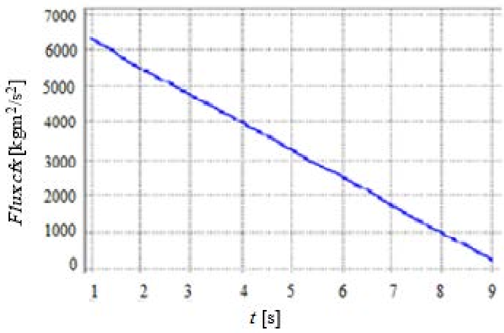

Fig. 5. Thermal flux versus braking time.

The linear decreasing form of the thermal flux versus the braking time illustrates the conversion process of the mechanical energy to heat.

Convective Heat Transfer Coefficient (h = h (t))

In the modeling of the thermal behavior, the convective heat transfer coefficient as a function of time (h = h (t)) is first determined. This parameter will then be used to calculate, using the commercial software ANSYS, the disk temperature and to visualize its three-dimensional distribution. Figure 6 and 7 show respectively the half of the full disc with 8 convective heat exchange surfaces and the faces of the air domain. For computing the convective heat transfer coefficient h = h (t) on each free surface of the disc using ANSYS CFX, we consider the following configuration:

1) The fluid domain: ambient air at 25 ° C
2) Reference pressure of 1 atm with a variation of speed
3) Turbulent flow of shear stress transport type.
4) The solid domain: brake disc with a variable angular speed.

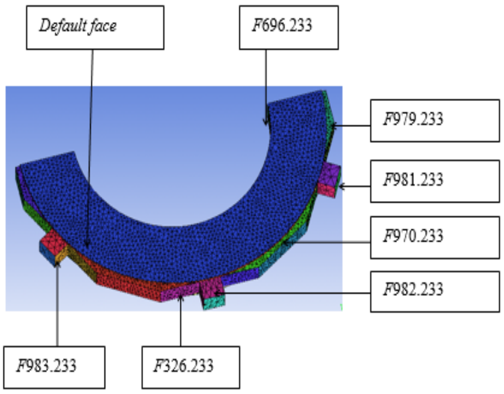

Fig. 6. Faces nomenclature of half full brake disc (*F*981.233) fin lateral faces, (*F*983.233) fin top faces, (*F*982.233) fin lateral faces, (*F*979.233) small lateral faces of the disc, (*F*970.233) disc contour faces, (*F*696.233) inner side of the disc,*(Default face)* friction zone ,(*F*326) external faces of the disc.

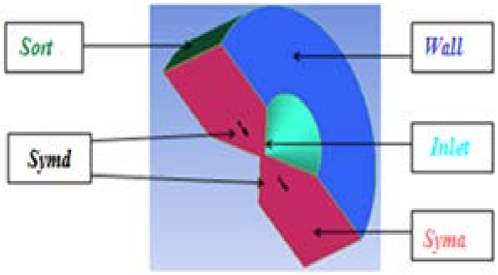

Fig. 7. Air domain representation

INLET: air Inlet, *SYMA*: symmetrical faces of air domain *SYMD*: symmetrical face of the disc., *WALL*: lateral symmetrical face of the air domain. *SORT*: air outlet.

The distribution of the computed convective heat transfer coefficient of the disc is visualised in Fig. 8.

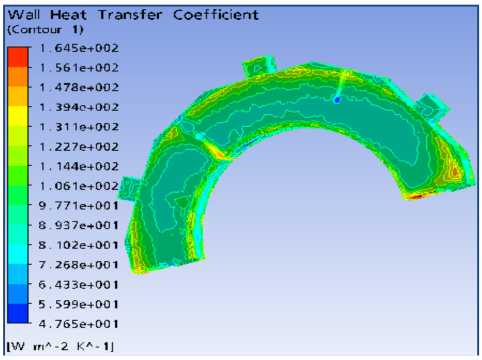

Fig. 8. Visualisation of heat transfer coefficient.

Temperature Calculation

The simulation is carried out for all disc variants presented previously in Figure 4, while respecting the boundary conditions and using the convection coefficients obtained for each variant and the heat flux, Figure 5. For each simulation, the maximum value of the temperature is recorded. As example, the figures 9, 10, 11 and 12 show respectively the temperature distribution at t=6.67s and the temperature evolution versus the time for the disc variant **(j)** and (**k**).

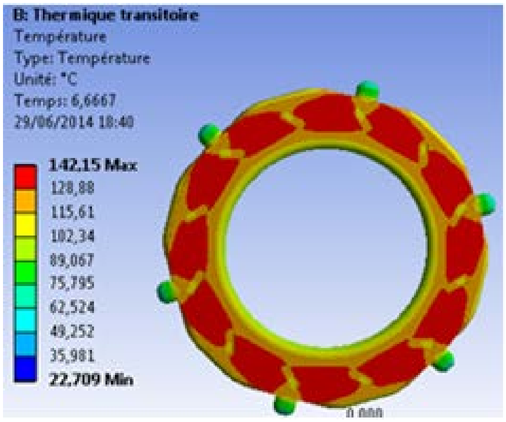

Fig. 9. Temperature distribution of disc **(j)** at t=6.67s.

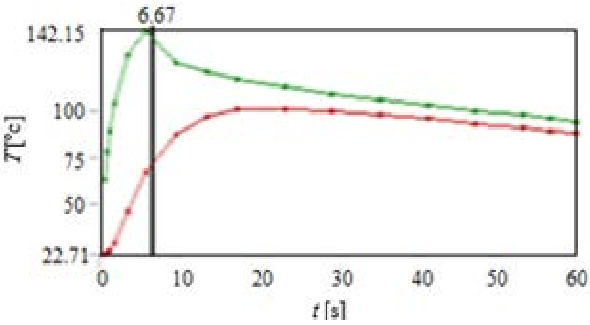

Fig. 10. Temperature of disc **(j)** versus braking time

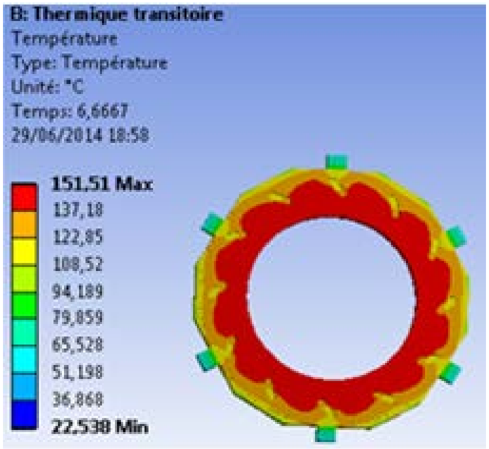

Fig. 11. Temperature distribution of disc **(k)** at t=6.67s.

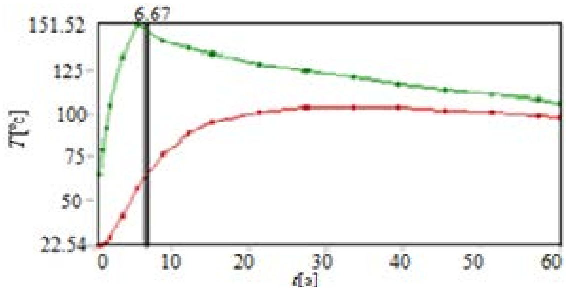

Fig. 12. Temperature of disc **(k)** versus braking time.

MECHANICAL MODELING

In this part, we determine the mechanical stress distribution of the previous two disc variants indicating their maximum values under the following boundary conditions:

1) Right pad is fixed.
2) Angular velocity of the disc $\omega_t = 132$ rad/s.
3) Pressure applied to the pad is $p = 1.72$ MPa.
4) The fins considered as a fixed support.

Material specifications Choi, Lee (2004):

a) Disc in cast iron: FG 25 AL.
b) Pads material characteristics:
   - ✓ Young's modulus: $E = 1000$ MPa.
   - ✓ Density: $\rho = 1400$ kg/m^3.
   - ✓ Poisson coefficient: $\nu = 0.25$.

     Friction coefficient: $\mu = 0.2$.

Figures 13 and 14 show respectively the Von-Mises stress distribution for the disc (J) and (K).

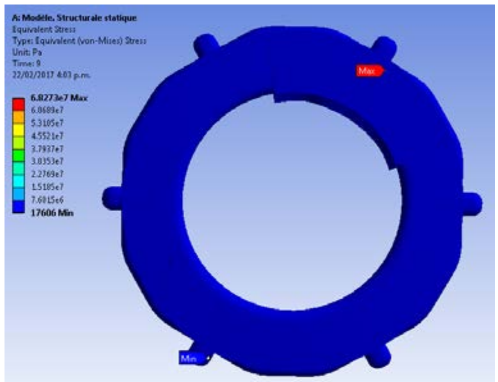

Fig. 13. Von-Mises stress distribution disc (J)

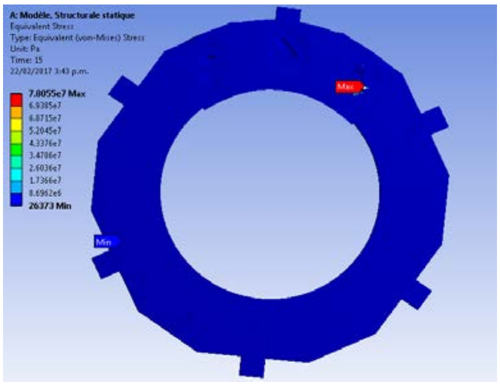

Fig. 14. Von-Mises stress distribution disc (K)

DESIGN OF EXPERIMENTS

In the full experimental design, $\mathbf{2^k}$ experiments should be performed for ***k***-selected factors. The main disadvantage of full factorial designs is the number of experiments, especially when the number of factors is high. The fractional factorial design method is a more practical alternative. It introduces the notion of confusion of effects and reduces considerably the number of experiments: each calculated effect is in fact the sum of the simple effects. In order to evaluate the gap between the behavior models to be developed we apply the method of the full and fractional designs. Since the objective of this study is the thermal and mechanical behavior optimization of the brake discs, the responses are respectively the maximum temperature y = T and the maximum stress y = σ. Table 1 gives the selected factors and their levels.

In this paper, we applied two types of DOE, the full factorial design and fractional factorial design taking the temperature and the mechanical stress respectively as response.

DOE for the factors (X_1, X_2, X_3)

a) 2^3 full factorial design for the factors (X_1, X_2, X_3) Table 3 and 4.

b) 2^{3-1} fractional design for the factors X_1, X_2 and X_3, Table 6 and 7.

DOE for the factors (X_4, X_5)

2^2 full factorial design for the factors X_4 and X_5, Table 8.

The factors (x_i) are generally of different nature, therefore the DoE using dimensionless coded values (X_i):

$$X_i = \frac{x_i - x_0}{\Delta x_i}$$

x_0: Value of experimental center
x_i: Value of input variable
Δx_i: Value of the range of variation

From Table 3, we note that the effect of the factor X_1 (thickness) on the response (temperature) is about three times larger than that of factor X_2 (slit number). The factor X_3 (hole number) and the interaction effects $X_1 X_2$, $X_1 X_3$, $X_2 X_3$ and $X_1 X_2 X_3$ are negligible.

From Table 3, the analytical model of the disc temperature can be written as follow:

$$T = 98 - 9.86\,X_1 - 2.6X_2 \quad (3)$$

According to this model, the maximum temperature T_{max} = 110.46 ºC is obtained at the low levels X (-1, -1) and the minimum temperature T_{min}= 85.54 ºC at the high levels X (1, 1). This means that with the increase in the thickness and the number of slits, it is possible to improve the thermal behavior of the brake disc.

In Table 4, the effect of the factor X_2 (slits number) is the largest compared to others factors; its impact on the response (Von Mises stress) is nearly double the effect of X_1 and X_3. Less stress in this design is recorded in the 6th experience. Note that there is a high stress concentration in the slit areas, which means that the increase in the slit number favours the appearance of rupture zones.

Table 3. Full factorial design 23 (Temperature as a response)

Treatments	**Average**	**Factors**			**Interactions**				**Response**
N^0	Y_0	X_1	X_2	X_3	X_1X_2	X_1X_3	X_2X_3	$X_1X_2X_3$	T [^{0}C]
1	+	-	-	-	+	+	+	-	110.83
2	+	+	-	-	-	-	+	+	88.94
3	+	-	+	-	-	+	-	+	103,59
4	+	+	+	-	+	-	-	-	84,67
5	+	-	-	+	+	-	-	+	109.09
6	+	+	-	+	-	+	-	-	89.67
7	+	-	+	+	-	-	+	-	104.06
8	+	+	+	+	+	+	+	+	85.38
Effects	98	-9.86	-2.6	-0.02	0.47	0.34	0.27	-0.27	

Table 4. Full factorial design 2^3 (Stress as response)

Treatments	**Average**	**Factors**			**Interactions**				**Response**
N^0	Y_0	X_1	X_2	X_3	X_1X_2	X_1X_3	X_2X_3	$X_1X_2X_3$	σ [MPa]
1	+	-	-	-	+	+	+	-	23.90
2	+	+	-	-	-	-	+	+	22.61
3	+	-	+	-	-	+	-	+	42.34
4	+	+	+	-	+	-	-	-	27.15
5	+	-	-	+	+	-	-	+	28.03
6	+	+	-	+	-	+	-	-	21.94
7	+	-	+	+	-	-	+	-	57.13
8	+	+	+	+	+	+	+	+	37.24
Effects	32.54	-5.31	8.9	4.49	-2.49	-0.22	2.67	0	

By referring to the full factorial design, Table 4, the analytical stress model can be written as follow:

$$\sigma = 32.54 - 5.31X_1 - 8.42X_2 + 3.54X_3 - 3.44X_1X_2 - 1.19X_1X_3 + 2.68X_2X_3 \quad (4)$$

According to the effect matrix of the fractional design for the factors X_1, X_2, X_3, Table 5, we consider only the first four experiences 5, 2, 3 and 8. It results the following fractional designs, table 5 and 6. These Tables do not allow the calculation of the interaction between the main factors.

The effects calculated in fractional factorial designs are aliased:

$$1? = 1 + 23 \ ?2 = 2 + 13 ?3 = 3 + 12? \quad (5)$$

This means that they do not directly reflect the effect of the factors taken individually but in groups of factors and interactions. It is sometimes impossible to conclude on the effect of a factor, since in contrast each term can be influential.

Polynomials for both fractional factorial designs are almost the same as full factorial designs:

$$T = 96.75 - 9.59X_1 - 2.6X_2 \quad (6)$$

$$\sigma = 32.55 - 2.63X_1 + 7.23X_2 \quad (7)$$

A fractional design with a reduced number of experiments gives an analytical model of the thermal behaviour comparable to that obtained with a Full design. However, in the case of mechanical behaviour, one notes that there is a certain deviation between the both experimental designs.

Let us now consider the full design with the factors X_4 and X_5, Table 7. From this design, we note that the impact of the external face (factor X_5) is predominant; the effects of the factor X_4 (hole number) and the interaction $X_4X_5$are non-significant. The analytical thermal model for this design is

$$T = 99.58 - 7.47X_5 \quad (8)$$

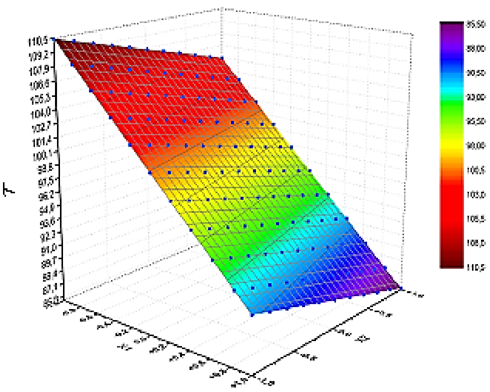

Fig. 15. Surface response of the thermal model for the design 2^3.

Table 5. Fractional design 2^{3-1} (I=123) for y=T.

	Factors				Response
Exp. N°	I	X_1	X_2	X_3	$T[^0C]$
5	+	-	-	+	109.09
2	+	+	-	-	88.84
3	+	-	+	-	103.59
8	+	+	+	+	85.38
Effects	96.75	-9.59	-2.26	0.48	

Table 6. Fractional design 2^{3-1} (I=123) for Y=σ.

	Factors				Response
Exp.N°	I	X_1	X_2	X_3	Y=σ[MPa]
5	+	-	-	+	28.03
2	+	+	-	-	22.61
3	+	-	+	-	42.34
8	+	+	+	+	37.24
Effects	32.55	-2.63	7.23	0.08	

Table 7. Full design 2^2 (temperature as response).

	Factors and interaction				Response
Exp. N°	X_4	X_5	X_4X_5	Y_0	Y=Tmax
1	-	-	+	+	93,79
2	+	-	-	+	90.44
3	-	+	-	+	105.83
4	+	+	+	+	108.27
Effects	-0,23	7.47	1.45	99,58	

ANALYSIS of VARIANVE (ANOVA)

The values of the responses obtained in the design of experiments must be analysed to measure the influence of factors and interactions on the observed variations in the response. The main method for this purpose is the analysis of variance (ANOVA). In general, the ANOVA includes the calculation of mean squares of factors and interactions, residual variance and Fisher's test Droesbeke, Fine, Saporta (1997).

Calculation of mean squares of factors and interactions

The variance of the factors is the sum of the squared deviations (SSD) divided by the number of degrees of freedom df_F associated with the considered factor F.

$$df_F = N_{nF} - 1 \quad (9)$$

N_{ni}: number of levels for the factor F

The sum of squared deviations associated with the factor F is:

$$SSD_F = \frac{N}{N_{ni}} \sum_{i=1}^{N_{ni}} (E_{F_{F=i}})^2 = \frac{N}{N_{ni}} \sum_{i=1}^{N_{ni}} (\overline{y}i + \overline{y})^2 \quad (10)$$

With$\bar{y}_i = \frac{1}{N}\sum_{i=1}^{N} y_i$: (11)

$\bar{y}$: Average response

$\bar{y}_i$: The mean of the responses observed for the experiments where the factor F takes its i th level.

For interactions involving factors A and B, the sum of the squares of the deviations is equal to:

$$SSD_{fg} = \frac{N}{N_{niA}N_{niB}} \sum_{i=1}^{N_{ni}} \sum_{j=1}^{N_{nj}} (\bar{y}_{ij} - \bar{y}_i - \bar{y}_j + \bar{y})^2 \quad (12)$$

$\bar{y}_{ij}$: The mean of the responses where the factor A takes its
i th level, and where the factor B takes its j th level.

The calculation of the degrees of freedom of an interaction is the product of the d_f of the factors involved in this interaction.

The value of the mean squares, associated with the considered factor or interaction x is calculated as follows:

$$MSDx = \frac{SSDx}{dfx} \quad (13)$$

Calculation of residual variance

The calculation of the residual variance (or residual mean squares) can then be written as:

$$MSDr = \frac{SSDr}{dfr} = \frac{\sum SSD}{\sum df} \quad (14)$$

The sums of squared deviations (SSD) and the numbers of degrees of freedom (df) related to the selected interactions.

The calculation of MSD_r used to test the significance of the factors and interactions and at the same time to assess the quality of the model obtained.

FISHER-SNEDECOR TEST

The Fisher test or F-test is a statistical hypothesis test to check the equality of two variances by taking the ratio of two variances and verifying that this ratio does not exceed certain theoretical value. We calculate the following ratio for the factor x considered:

$$F_{obs} = \frac{MSD_x}{MSD_r} \quad (15)$$

F_{obs}: calculated value of Fisher

The variance associated to the factor or interaction studied ($MSDx$) can be regarded as equal to the residual variance ($MSDr$) if the ratio Fobs is low, i.e., less than a statistical threshold value.

In inferential statistics, the term "null hypothesis" usually refers to a general statement or default position that there is no relationship between two measured phenomena, or no association among groups Everitt (1998). Rejecting or disproving the null hypothesis and thus concluding that there are grounds for believing that there is a relationship between two phenomena (e.g. that a potential treatment has a easurable effect) is a central task in the modern practice of science, and gives a precise criterion for rejecting a hypothesis.

The null hypothesis is generally assumed true until evidence indicates otherwise. In statistics, it is often denoted H_0 (read H-null or H-zero).

The H_0 hypothesis must be rejected at level α if:

$$P(F \geq F_{obs}) \leq \alpha \quad (16)$$

We have applied an analysis of variance for the thermal study Full factorial design 2^3 (maximum temperature as a response) Table.3, we obtained the following results, Table 8.

Table 8. Variance analysis of design 2^3 (response T)

X_i	df	SSDi	MSD	$Fobs$
X_1	1	777.76	777.76	810.17
X_2	1	54.08	54.08	56.33
X_3	1	0.003	0.003	0.003
Residual variation	4	3.85	0.96	
Total	7	835.7		

To analyse the results of Table 8, we have to read from Fisher Snedecor table the theoretical value F_{th} for ($n_1 = 1$, $n_2 = 4$) and $\alpha = 0.01$, where n_1 and n_2 represent respectively the degrees of freedom of each factor and the residual interactions. We get $F_{th} = 21.2$.

Applying Fisher-test to ANOVA results, Table 8, we conclude that the factors X_1 and X_2 are significant about 99%. We have only 1% risk of rejecting assumptions of equality with the residual variance. However, the factor X_3 is non- significant. We refer to Table.3 to see how these two factors affect the response T. We read that the less temperature is recorded when X_1, X_2 take their maximum values (+).The same procedure is applied to the mechanical investigation. From Table 4, we get the ANOVA results, Table 9.

Table 9. Variance analysis of design 2^3 (response σ)

X_i	df	SSDi	MSD	F_{obs}	F_{theo}
X_1	1	225.36	225.36	5.38	7.71
X_2	1	567.17	567.17	13.6	7.71
X_3	1	100	100	2.39	7.71
Residual variation	4	167.41	41.85		
Total	7	1055.94			

We read from Fisher Snedecor table $F_{th}=7.71$ for $n_1=1$, $n_2=4$ and $\alpha=0.05$. From Tab.10, we note that the factor X_2 is significant about 95%. We have only 5% risk of disproving the null hypothesis. This analysis excludes the factor X_1 from the mechanical study. As in the thermal case of the analysis of variance, the factor X_3 (number of holes) is not significant. We also observe that the results obtained by the fractional designs are approximately the same as those of full designs, Tables 5 and 6.

We made further changes on the outer shape of the disc but with a reduced number of simulations. From Table.8, we get:

SSD X_4=0.21, SSD X_5=223.2, SSDX4 X_5=8.41.

For this full factorial design 2^2 the theoretical value of the limiting effect Vivier (2002).

$$Eflim = \sqrt{(F_{th} * SSDr * dff/N * dfr)} \qquad (17)$$

F_{th} corresponding to this design (Table.7) with α = 0.05 is equal to 161.45. Hence, the value of the limiting effect $E_{f,\mathrm{lim}}$ = 18.42. We found that the effect of these changes of form would seem to be the most important, although this value has not depreciated a calculated value called limiting effect. Thereby, with a reduced number of experiments the significance of a given factor could not occur, although it is considered important.

CONCLUSION

Thanks to the simulation results of the thermal and mechanical behavior obtained by ANSYS software, it was possible to apply the experimental design (DOE) methods on the elaborate geometrical models of brake discs in order to determine the sensitivity and the effects of the responses. In order to optimize the brake disc behavior, DOE screening is performed to eliminate non-significant factors using the RSM methodology. Once the DOE is performed and the response surface equations are generated. The graphical representation of the response describing the thermal behavior, obtained from the analytical model of the temperature of the brake disc, allowed a rapid evaluation of the influence of the main parameters and of the interactions. Through this study, we reached the following conclusions:

1) The convection coefficient calculated by CFX of the braked brake disks varies according to the specified geometry and initial speed of the brake disc. In a brake disk, this coefficient depends on the side of the disk in question.

2) The distribution of the temperature was determined for each variant of the brake disc using the ANSYS Workbench. Each disk variant behaves thermally and mechanically completely differently.

3) Due to the experimental design method applied to the different brake disk configurations and the variance analysis, it was found that, among the factors selected in this study, the thickness and the slit number were the most influential factors for the thermal performance of the disk. However, for mechanical behaviour, the slit number was the most influential factor.

4) It should be noted that it would be preferable to include a larger number of factors to eliminate the less significant factors.

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