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Transient Elastohy Drodynamic Lubrication of Artificial Hip Joints with Non-Newtonian Synovial Fluids

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Keywords：Artificial hip joints,EHL, non-Newtonian Synovial fluids

ABSTRACT

This paper analyzes that the ball (femoral head) squeeze to the ultra-high molecular weight polyethylene (UHMWPE) acetabulum component gradually under constant load condition with non-Newtonian synovial fluid by using the constrained elastic deformation mode. This phenomenon belongs to hard-on-soft transient elastohydrodynamic lubrication (EHL) problem. The Newton-Raphson method (NRM) and the Gauss-Seidel iteration method are used to solve the transient modified Reynolds equation, the elasticity deformation equation, and the load balance equation simultaneously in spherical coordinates to obtain the transient pressures (P) and film thicknesses (H). The simulation results reveal that the pressure becomes concentrated toward the center gradually with decreasing film thickness. In order to maintain mass conservation, a necking phenomenon of the joint synovial fluid film thickness would occur at the outlet, so the minimum film thickness occurs at the outlet instead of the central area. As the characteristic length (L) increases, the synovial fluid film thickness increases, the central pressure decreases, but pressure increases outside the central area. The necking phenomenon would increase with decreasing the characteristic length. This study not only has academic innovation but also provides information to the biomedical industry for design artificial hip joints, so it has great potential for biomedical industrial application.

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INTRODUCTION

Owing to the effects of gravity and momentum, hip joints support the force equivalent to 6-8 times of a human body's weight during running, jumping and exercising. They are one of the joints that receive the most force of the whole body. Therefore, hip joints more easily damaged than other joints. The serious pathological changes of hip joints will experience stiffness and pain in their hips, so that mobility is inconvenient, and they are unable to cope with daily life. In this circumstance, artificial joint replacement can help improve the symptoms. The structure of a traditional artificial hip joint (AHJ) is similar to a human hip joint. It consists of four components with the femoral stem inserted into the medullary cavity to provide better stability. The spherical femoral head is inserted in the acetabulum cup with the ultra-high molecular weight polyethylene (UHMWPE) spherical liner to form an artificial hip joint as shown in Fig. 1.

Since synovial fluid exists in all natural human joints and artificial joints, therefore, Saari et al. (1993) and Delecrin et al. (1994) thought the synovial fluid around the joints must be assumed in joints lubrication analysis and design. Fan et al. (2007) presented that the main components of the synovial fluid are albumin, γ-Globulin, Hyaluronic acid, Phospholipid. Almost all the synovial fluids can be treated as mixture lubrication fluids compound of a base fluid and several additives. Furthermore, Mabuchi et al. (1999) have proven that artificial synovial additives can improve lubrication characteristics and reduce friction. Therefore, the effects of additives on the fluid rheology of the synovial fluid have received considerable research attention.

In the elastohydrodynamic lubrication (EHL) analysis of artificial joints, characteristics of lubricant must be taken into account. Currently, the hydrodynamic lubrication theories based on viscous fluid mechanics indicate that the fluid and solid surfaces can be adsorbed to each other, and there is no relative slip between the contact interfaces, so the motion of each layer of fluid is able to maintain shear conservation.

The EHL theory (Hamrock, 1994) was used to design and analyze artificial joints. As to metal-on-metal or ceramic-on-ceramic artificial joints, the “hard-on-hard bearing” will generate high contact stress due to small contact deformation area and great elastic modulus. Therefore, the influences of the pressure-viscosity and the pressure-density should be considered in practical engineering application of the elastohydrodynamic lubrication problem. But the rheology should be not considered for design artificial hip joints. With regard to metal/ceramic on UHMWPE artificial joints, the elastohydrodynamic lubrication problems may occur to the “hard-on-soft bearing” due to lower elastic modulus and larger contact deformation of UHMWPE.

Yao et al. (2003) thought synovial fluid has a strong shear thinning property at a relatively low shear rate, while such a shear thinning property decreases significantly at a higher shear rate. Under steady state condition, Jin et al. (1997) thought artificial hip joints have an extremely high shear rate, so synovial fluid can be treated as Newtonian fluid. For EHL calculation, the pressure-viscosity relationship of lubricant is a very important factor. But the experimental results show that the viscosity of synovial fluid is almost constant for the pressure less than 100MPa. Wang and Jin (2005) thought artificial hip joints made up of different materials, such as metal-on-UHMWPE, metal-on-metal and ceramic-on-ceramic artificial joints, the synovial film pressure can’t exceed 100MPa, so the influence of pressure on the synovial fluid viscosity can be ignored in analysis EHL artificial joints problem. In recent years, many scholars (Myant and Cann, 2014; Liu et al., 2006; Mattei et al., 2011 and Tian et al., 2017) conducted more comprehensive and systematic researches on lubrication characteristics with respect to hip joints of different materials and structures by using different numerical simulation methods. These theoretical researches were carried out on the design of artificial hip joints by using different motion parameters and anatomical features.

As mentioned above, almost all the synovial fluids can be treated as mixture lubrication fluids compound of a base fluid and several additives. Oliver (1988) and Spike (1994) have shown clear evidence of load enhancement and friction reduction effects due to the presence of additives. Therefore, Mongkolwongrojn et al. (2010) and Gao et al. (2016) though the actual lubricant in human joints synovial fluid is known to exhibit non-Newtonian behavior. The effects of additives on the fluid rheology of a lubricant received great research attention. Since the flow behaviour of a Newtonian lubricant blended with various additives could not be described accurately by the classical continuum theory, therefore, many microcontinuum theories have been proposed by Ariman et al. (1973), Stokes (1966), and Ramanaiah (1979). Among these theories, Stokes theory (1966) is the simplest theory which accounts for the effects of couple stresses, body couple, and asymmetric tensors. This couple stress model aims to examine the effects of particle sizes. This model is important for the applications of pumping flow, i.e., liquid crystal, polymer-enhanced oil, animal blood, and artificial fluid. Several investigators have used the Stokes couple stress fluid theory to analyze the performance characteristics of various bearing systems, such as journal bearings (Lin, 2001), hydrostatic bearings (Lin, 1999), squeeze films (Naduvinamani et al., 2001; Nabhani et al., 2013), and elastohydrodynamic lubrication line contacts (Dac, 1997; Chippa and Sarangi, 2013). So far, no attempt has been made to study the squeeze film characteristics of elastohydrodynamic lubrication on artificial hip joints with couple stress synovial fluid under constant load condition.

In this paper, squeeze artificial hip joints EHL motion of spherical contacts with ultra-high molecular weight polyethylene (UHMWPE) using the constrained elastic deformation mode is explored under constant load condition. This study assumed that the synovial fluid is non-Newtonian synovial fluid. The Newton-Raphson method (NRM), finite difference method (FDM), and the Gauss-Seidel iteration (GSI) method are used to solve the transient modified Reynolds equation, the elasticity deformation equation, and the load balance equation simultaneously. The transient pressure profiles and film shapes of the synovial fluid, and normal squeeze velocities during the pure squeeze process under various operating conditions in the EHL regime are discussed. This study differs from the other studies in the past and possesses academic innovation. The non-Newtonian fluid model is used to describe actual human joints synovial fluid to obtain accurate analysis. It also has a great value in biomedical industrial applications.

THEORETICAL ANALYSIS

An anatomical illustration of artificial hip joint under 3D loading and rotation is shown in Fig. 2. In addition, in calculation of EHL of the artificial hip joints, it always assumes that the acetabulum is fixed, and hip’s motion is a composition of motions of femoral head in all directions relative to the acetabulum. Usually, only the vertical load and flexion/extension motion of the coronal plane are taken into account. If the contact surface is fixed within the acetabulum and away from the boundary, the acetabulum can be rotated to a horizontal position to form a simplified lubrication model as shown in Fig. 3.

The calculation of EHL considers only the main hip flexion/extension motion. The influence of some minor motions, such as adduction-abduction motion and internal rotation-external rotation motion can be ignored. In the process of lower limb motion, the lubricating fluid between the contact surfaces generates EHL phenomenon due to interaction and relative motion between the acetabulum and the femoral head.

During human movements, loads sustained by joints and speed of relative motion will change with time. Owing to the diversities and changes of human movements, an unsteady Reynolds equation will be required to obtain the variable speed and variable load for EHL analysis of hip joints. This study assumed that the synovial fluid was couple stress fluid. Besides, the thickness of the synovial fluid was much smaller than the size of the ball (femoral head), and the flow of lubricant belonged to laminar flow at an isothermal state, no slip occurred at the boundary of the lubricant and the solid.

According to the Stokes microcontinuum theory (1966), the field equations of an incompressible couple stress fluid in the absence of body forces and body couples are expressed as follows:

$$\nabla \cdot \mathbf{V} = 0 \tag{1}$$

$$\rho \frac{D\mathbf{V}}{Dt} = -\nabla p + (\mu - \eta \nabla^2)\nabla^2 \mathbf{V} \tag{2}$$

where $\mathbf{V}$ is the velocity vector, ρ is the density, p is the pressure, μ is the classical viscosity coefficient, and η is a new material constant with the dimension of momentum responsible for the couple stress fluid property. Since the ratio η/μ has the dimensions of length squared, the dimension of $l = (\eta/\mu)^{1/2}$ characterizes the material length of the couple stress fluids, and l is assumed to be a material constant in the present analysis.

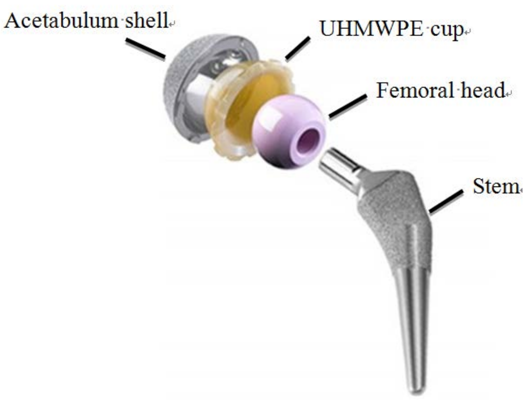

Fig. 1 Geometry of an artificial hip joint (Mattei et al., 2011)

The transient Reynolds equation under the spherical coordinate system was as follows:

$$\sin\theta \frac{\partial}{\partial \theta}(h^3 \sin\theta \psi \frac{\partial p}{\partial \theta}) + \frac{\partial}{\partial \phi}(h^3 \psi \frac{\partial p}{\partial \phi}) = 6\mu R_2{}^2 \sin\theta$$

$$[(-\omega_x \sin\phi + \omega_y \cos\phi)\sin\theta \frac{\partial h}{\partial \theta} + (-\omega_x \cos\phi \cos\theta$$

$$-\omega_y \sin\phi \cos\theta + \omega_z \sin\theta)\frac{\partial h}{\partial \theta} + 2\sin\theta \frac{\partial h}{\partial t}] \tag{3}$$

where

$$\psi = 1 - 12(\frac{l}{h})^2 + 24(\frac{l}{h})^3 \tanh(\frac{h}{2l})$$

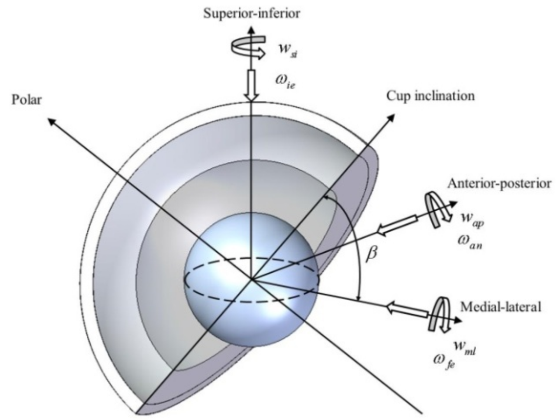

Fig. 2 An anatomical illustration of artificial hip joint under 3D loading and rotation

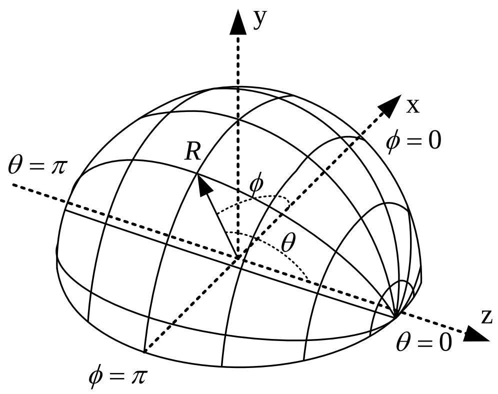

Fig. 3 Hip joint arrangement under study (Wang and Jin, 2005)

The subscript x, y, z is the coordinate axis under the Cartesian coordinate system, and z-axis coincided with the polar axis of the spherical coordinate system, and ψ was the correction factor of non-Newtonian synovial fluid. If only the main flexion-extension motion in the z-axis direction and the main load component in the vertical direction were considered, then $\omega_z = \omega$, $\omega_x = \omega_y = 0$. The equation (3) could be simplified as:

$$\sin\theta \frac{\partial}{\partial \theta}(h^3 \sin\theta \psi \frac{\partial p}{\partial \theta}) + \frac{\partial}{\partial \phi}(h^3 \psi \frac{\partial p}{\partial \phi})$$

$$= 6\mu R_2{}^2 \sin^2\theta(\omega \frac{\partial h}{\partial \phi} + 2\frac{\partial h}{\partial t}) \tag{4}$$

The two items in the right bracket represent entrainment motion and squeeze motion respectively. Sasada, et al. (1990) indicated that, when people stands, the hip rotation range and the angular velocity are smaller, and the hip rotation has less influences on the formation of lubricant film. The formation of hydrodynamic film relies mainly on squeeze motion of the femoral head. Besides, many theoretical and experimental studies (Jin and Dowson, 1999; Jin et al., 1993) indicate that the natural joints and artificial joints will experience heavy load and low speed stage in the stance phase of the walk cycle, and squeeze action is the dominant lubrication mechanism, especially during the starting and stopping phases of the walking process because squeeze action is the only lubrication mechanism, which postpones the direct contact between the joint bearing surfaces.

The Reynolds equation considering only squeeze motion in the y direction:

$$\sin\theta \frac{\partial}{\partial\theta}(h^3 \sin\theta\psi \frac{\partial p}{\partial\theta}) + \frac{\partial}{\partial\phi}(h^3\psi \frac{\partial p}{\partial\phi})$$

$$= 12\mu R_2^{\,2} \sin^2\theta \frac{\partial h}{\partial t} \tag{5}$$

The boundary conditions for Eq. (5) are:

$p = 0$, inner surface of the acetabular edge (6a)

$\frac{\partial p}{\partial\theta} = \frac{\partial p}{\partial\phi} = 0$, cavitation caused by rupture of lubricant film (6b)

Equation (5) can be expressed in dimensionless form as:

$$\sin\theta \frac{\partial}{\partial\theta}\left(H^3 \sin\theta\,\Psi \frac{\partial P}{\partial\theta}\right) + \frac{\partial}{\partial\phi}\left(H^3\,\Psi \frac{\partial P}{\partial\phi}\right)$$

$$= \lambda \sin^2\theta \frac{\partial \mathrm{H}}{\partial \mathrm{T}} \tag{7}$$

where

$$\lambda = \frac{12R_2^{\,2}}{c^2}, \quad \Psi = 1 - 12(\frac{L}{H})^2 + 24(\frac{L}{H})^3 \tanh(\frac{H}{2L})$$

The boundary conditions for Eq. (7) are:

$$P = 0 \tag{8a}$$

$$\frac{\partial P}{\partial\theta} = \frac{\partial P}{\partial\phi} = 0 \tag{8b}$$

For the constant load, the instantaneous load balance equations are:

$$R_2^2 \int_{\phi_1}^{\phi_2}\int_{\theta_1}^{\theta_2} p\sin^2\theta\cos\phi\, d\theta d\phi = 0 \tag{9}$$

$$R_2^2 \int_{\phi_1}^{\phi_2}\int_{\theta_1}^{\theta_2} p\sin^2\theta\sin\phi d\theta d\phi = w \tag{10}$$

$$R_2^2 \int_{\phi_1}^{\phi_2}\int_{\theta_1}^{\theta_2} p\cos\theta\sin\theta\, d\theta d\phi = 0 \tag{11}$$

Equation (10) in dimensionless form as:

$$W = \int_{\phi_1}^{\phi_2} \int_{\theta_1}^{\theta_2} P\sin^2\theta\sin\phi\, d\theta\, d\phi \tag{12}$$

The film thickness equation consists of geometric distance and elastic deformation. If UHMWPE is used as the material of acetabulum, only elastic deformation of UHMWPE acetabulum should be calculated because the elastic modulus of UHMWPE is much smaller than that of a metal or ceramic material, and the UHMWPE acetabulum should be rigidly fixed on a metal substrate or a layer of bone cement. The elastic deformation of UHMWPE can be calculated in a restrained column mode under the conditions mentioned above by using the following simple formula (Bartel et al.,1985):

$$h(\theta,\phi) = s(\theta,\phi) + \delta(\theta,\phi) \tag{13}$$

$$s(\theta,\phi) = c(1 - \varepsilon_y \sin\theta\sin\phi) \tag{14}$$

$$\delta(\theta,\phi) = \frac{R_2\,[\,(\frac{R_3}{R_2})^3 - 1]}{E\,[\frac{1}{1-2\nu} + \frac{2}{1+\nu}(\frac{R_3}{R_2})^3\,]}\; p(\theta,\phi) \tag{15}$$

Equation (13) in dimensionless form as:

$$H = S + \Delta = 1 - \varepsilon_y \sin\theta\sin\phi + \Delta \tag{16}$$

where

$$\Delta = \frac{R_2\,[\,(\frac{R_3}{R_2})^3 - 1]}{c\,[\frac{1}{1-2\nu} + \frac{2}{1+\nu}(\frac{R_3}{R_2})^3\,]} P \tag{17}$$

Such an elastic deformation calculation mode is shown in Fig. 4. For the sizes of UHMWPE acetabula in current use, the reasonable results obtained from the experiments (Jin et al., 1999) showed a good consistency with the results obtained from the finite element method (Jagatia et al., 2001).

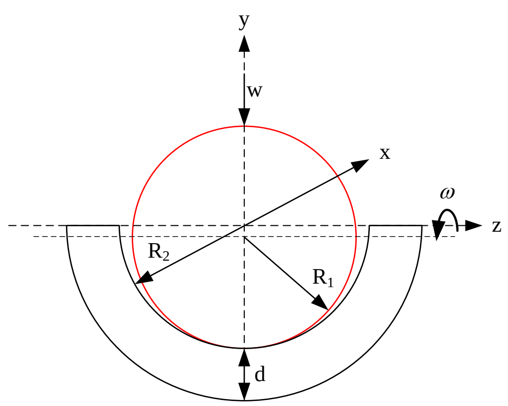

Fig. 4 Ball in socket model lubrication models for artificial hip joint (Wang and Jin, 2005)

The finite difference method was employed to discrete the Reynolds equation in spherical coordinates. The numerical iterative method was adopted to obtain the pressure distribution and the film thickness shape of the non-Newtonian synovial fluid on the squeeze motion at different time step under constant load condition.

RESULTS AND DISCUSSION

This paper analyzed that the ball (femoral head) squeeze to the ultra-high molecular weight polyethylene (UHMWPE) acetabulum component gradually under constant load condition in the spherical coordinates. Therefore, the purpose of this study explored the transient elastohydrodynamic lubrication problems caused by the pure squeeze motion of point contact between elastomer. This study conducted the elastohydrodynamic lubrication analysis of the metal/UHMWPE artificial joints in the non-Newtonian synovial fluid by using the constrained elastic deformation mode. The contact deformation is great due to UHMWPE possesses lower elastic modulus. This phenomenon belongs to hard-on-soft EHL problem.

The finite difference method and the Newton-Raphson method are used to solve the transient modified Reynolds equation, the elasticity deformation equation, the rheology equation, and the load balance equation simultaneously in spherical coordinates. In terms of the initial conditions, the synovial fluid film pressure was zero because of the initial eccentricity was zero. Numerical solutions of film thickness (H) and pressure (P) in pure squeeze motion are calculated using the various input parameters presented in Table 1.

Table 1 Computational data used in this paper.

Viscosity of Synovial fluid, Pa-s	0.01
Radius of artificial femoral head, mm	14.0
Radius of artificial acetabulum, mm	14.1
Thickness of UHMWPE cup, mm	7.0
Elastic modulus of UHMWPE, Pa	1.0×10^{9}
Poisson's ratio of UHMWPE	0.4
Load, N	2500.0

Owing to the pure squeeze motion, the analytical values in the spherical coordinate system were symmetrical with the ϕ axis *and* θ axis. Therefore, the computational domain was set to be $\pi/2\times\pi/2$. A grid size of 121 × 121 grids in the quarter domain is used for the evaluation of pressure and film thickness at each time step. A typical problem for $W=1.26\times10^{-2}$, $E_{UHMWPE}=1.0$ GPa, $\nu_{UHMWPE}=0.4$ $R_{femoral}=14.0mm$, $R_{\mathrm{Polyethylene}}=14.1mm$, and d=7.0mm is solved.

Fig. 5 shows the relative change in the pressure distribution and film thickness for a femoral head approaching a lubricated UHMWPE acetabulum with a Newtonian synovial fluid at $\phi=\pi/2$ in θ-direction under constant load condition. It can be seen that the pressure profile is quite flat at relatively large film thicknesses at initial squeeze stage. Meanwhile, the elastic deformation is small. Furthermore, the pressure becomes steeper and concentrates toward the center gradually with decreasing film thickness. As the loading is constant, the integration of the pressure distribution over the loading area is a constant. Therefore, the pressure distribution is found reverse outside the central region. The film thickness showed a flat distribution due to elastic deformation in the central area. In the final stage of the squeeze motion, the eccentricity increases, and the joint synovial fluid film thickness is smaller. The pressure distribution is more concentrated toward the center, the greater the central pressure is, and the greater the elastic deformation is. In order to maintain mass conservation, a necking phenomenon of the joint synovial fluid film thickness would occur at the outlet, so the minimum film thickness occurred at the outlet instead of the central area.

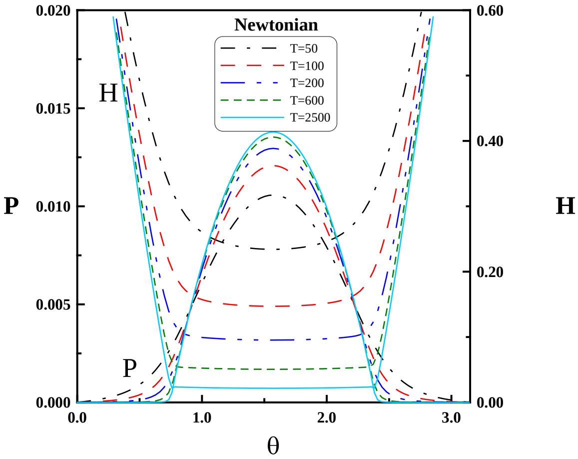

Fig. 5 Pressure and film thickness distributions versus time using Newtonian lubricant

Fig. 6 shows the relative change in the pressure distribution and film thickness for a femoral head approaching a lubricated UHMWPE acetabulum with a couple stress synovial fluid at $\phi=\pi/2$ in θ-direction under constant load condition. Since the viscosity of the couple stress lubricating fluid was higher than that of Newtonian lubricating fluid, therefore, the oil film thickness with couple stress synovial fluid is greater than that with Newtonian synovial fluid at the same time step. The pressure with Newtonian synovial fluid is greater than that with couple stress synovial fluid at the central region at the same time step. However, the pressure distribution is found reverse outside the central region at the same time step in order to maintain a constant load.

Fig. 7 shows the central pressure and film thickness versus time with different couple stress synovial fluid (L=0.01, L=0.02) under constant load condition at ϕ=0 and θ=0. The central pressure increases rapidly with time in the initial stage. The pressure increased as the squeeze time increased, and the oil film thickness decreased as the squeeze time increased. In the initial stage of the squeeze motion, the minimum film thickness is the central film thickness. As shown, the greater the pressure is, the greater the elastic deformation is. When the dimensionless time reached to 1326, a necking phenomenon of the joint synovial fluid film thickness would occur at the outlet with couple stress synovial fluid (L=0.01). When the dimensionless time reached to 1442, a necking phenomenon of the joint synovial fluid film thickness would occur at the outlet with couple stress synovial fluid (L=0.02). Therefore, the difference would produce between the minimum film thickness and the central oil film thickness. As shown, the smaller the characteristic length (*L*), the earlier the necking phenomenon formed.

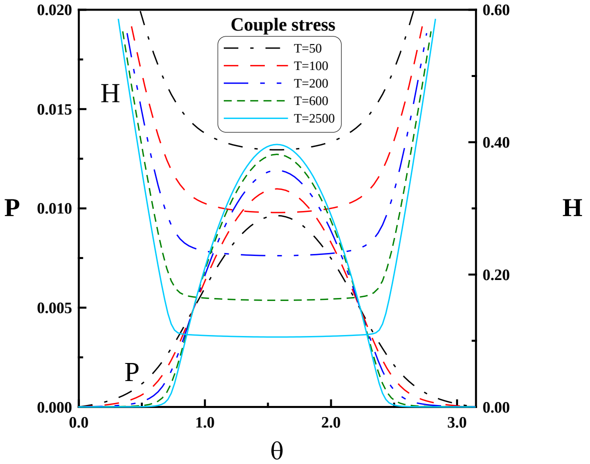

Fig. 6 Pressure and film thickness distributions versus time using couple stress lubricant

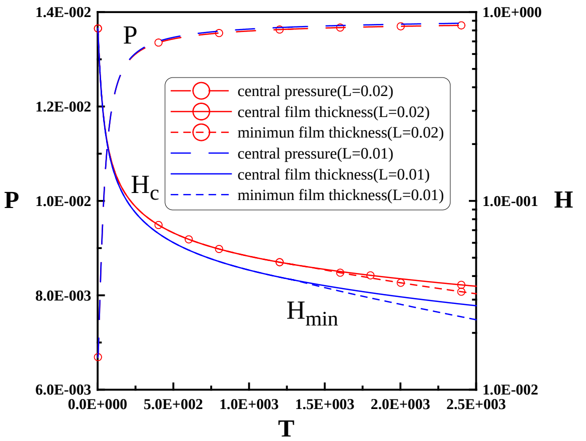

Fig. 7 P and H versus T using different lubricants

Fig. 8 shows the change of central pressure and oil film thickness versus time with different characteristic length (*L*) during the squeeze motion under constant load condition. The central pressure increased with time. The central pressure increased with decreasing the characteristic length (*L*). The central film thickness decreased with time. The central film thickness increased with increasing the characteristic length (*L*).

Fig. 9 shows the change of pressure and oil film thickness versus time at quarter position ($\phi=\pi/4, \theta=\pi/4$) with different characteristic length (*L*) during the squeeze motion under constant load condition. As shown in Fig. 9, the quartered pressure decreased with time. The quartered pressure decreased with decreasing the characteristic length (*L*). The quartered film thickness decreased with time. The quartered film thickness increased with increasing the characteristic length (*L*).

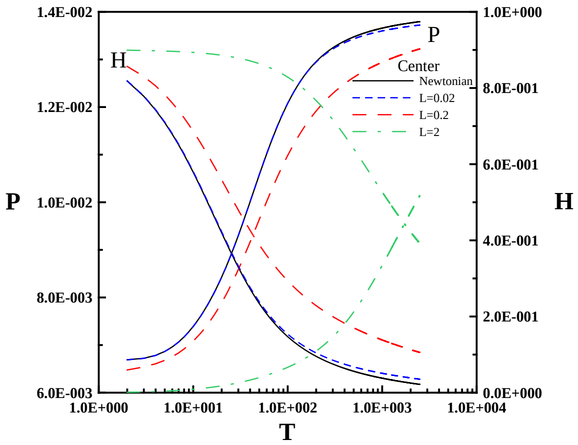

Fig. 8 P and H versus T using different L at center

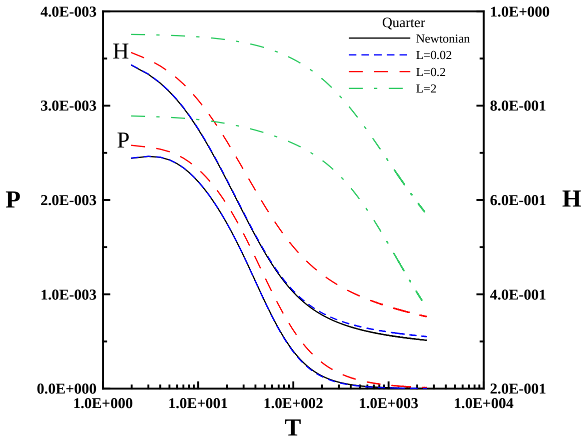

Fig. 9 P and H versus T using L at quarter

Fig. 10 shows the distribution of pressure and film thickness for a femoral head approaching a lubricated UHMWPE acetabulum at T=2500 with couple stress synovial fluid at $\phi=\pi/2$ in the θ direction under constant load condition. Since the viscosity of the couple stress synovial fluid was higher than that of Newtonian synovial fluid, therefore, the synovial fluid film thickness would increase with increasing the characteristic length (*L*). The necking phenomenon would increase with decreasing the characteristic length (*L*). In the central area, the pressure would decrease with increasing the characteristic length (*L*), while the pressure outside the central area would increase with increasing the characteristic length (*L*).

Fig. 11 shows the relationship of the central normal squeeze velocity and the central film thickness for different characteristic length (L) under constant load. The central normal squeeze velocity decreased rapidly with decreasing central film thickness in the initial stage. As shown, the greater the characteristic length, the smaller the normal squeeze velocity under the same central film thickness.

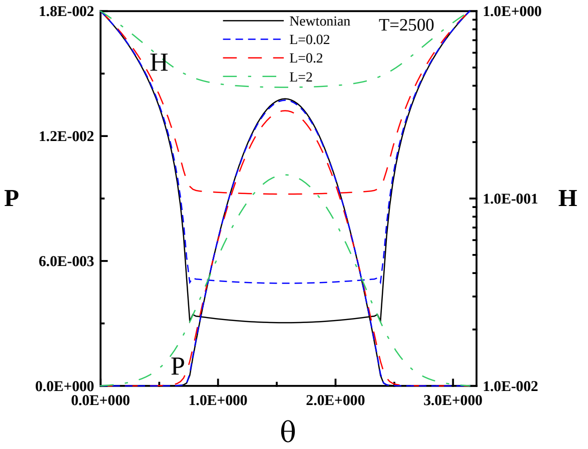

Fig. 10 P and H distributions versus L at T=2500

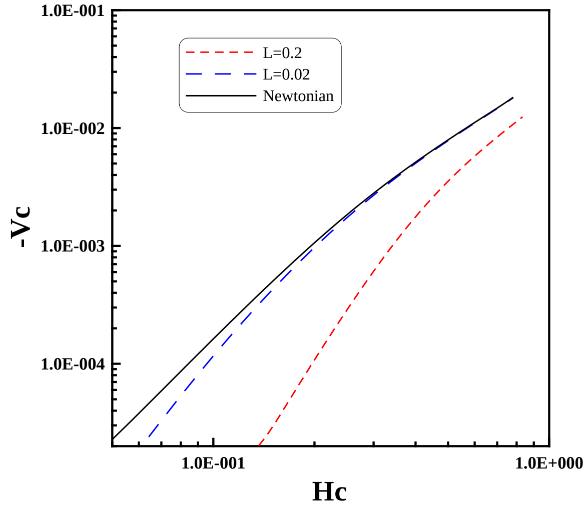

Fig. 11 Variation of central normal squeeze velocity with central film thickness

CONCLUSIONS

This study conducted the transient squeeze elastohydrodynamic lubrication analysis of the metal/UHMWPE artificial joints with couple stress synovial fluid by using the constrained elastic deformation mode. The non-Newtonian fluid model is used to describe actual human joints synovial fluid to obtain accurate analysis. The finite difference method and the Newton-Raphson method are used to solve the pressure distribution, velocity, and the synovial fluid film thickness at different time on squeeze motion. In the light of the micro continuum theory, the effects of couple stress synovial fluid and elastic deformation on the performance of squeeze motion are proposed and discussed under the conditions of constant load. The conclusions from the main results can be summarized as follows:

1. As the squeeze time increases, the pressure distribution is more concentrated toward the center, the greater the central pressure is, the greater the elastic deformation is, and the flatter the synovial fluid film thickness is.
2. As the squeeze time increases, a necking phenomenon of the synovial fluid film thickness would occur at the outlet rather than at the center.
3. The central pressure increased with time. The central synovial fluid film thickness decreased with time. As the characteristic length (L) increases, the central pressure decreases, and the central synovial fluid film thickness increases.
4. The pressure and synovial fluid film thickness decrease with time at $\square = \pi/4$ and $\square = \pi/4$. As the characteristic length (L) increases, the pressure increases, and the synovial fluid film thickness increases.
5. At the same time, as the characteristic length (L) increases, the synovial fluid film thickness shape increases, the central pressure distribution decreases, and pressure distribution increases outside the central area.
6. The necking phenomenon would increase with decreasing the characteristic length (L). The smaller the characteristic length, the earlier the necking phenomenon formed.
7. The central normal squeeze velocity decreased rapidly with decreasing central film thickness in the initial stage. The greater the characteristic length, the smaller the normal squeeze velocity under the same central film thickness.

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NOMENCLATURE

c	*radial clearance, R_2-R_1 (m)*
d	*thickness of UHMWPE cup, R_3-R_2 (m)*
e	*eccentricity between the center of the femoral head and the acetabulum socket*
E	*elastic modulus of UHMWPE cup (Pa)*
h	*synovial fluid film thickness (m)*
h_c	*central synovial fluid film thickness (m)*
h_{min}	*minimum synovial fluid film thickness (m)*
H	*dimensionless film thickness, h/c*
l	*characteristic length of the couple stress fluids,* $l=(\eta/\mu)^{1/2}$
L	*dimensionless characteristic length of the couple stress fluids,* l/c
p	*pressure (Pa)*
p_c	*central pressure (Pa)*
P	*dimensionless pressure, p/E*
R_1	*femoral head radius (m)*
R_2	*inner radius of UHMWPE cup (m)*
R_3	*outer radius of UHMWPE cup (m)*
s	*rigid synovial fluid film thickness defined in equation (14) (m)*
t	*time (sec)*
T	*dimensionless time,* tE/μ
Vc	*dimensionless normal velocity of the ball's center,* $v_c\mu/Ec$
w	*load (N)*
W	*dimensionless load,* w/ER_2^2
x,y,z	*Cartesian coordinates*
ϕ, θ	*angular coordinates in the entraining and side-leakage directions, respectively*
η	*material constant responsible for couple stress parameter*
μ	*viscosity of synovial fluid lubricant (Pa-s)*
δ	*elastic deformation of UHMWPE cup (m)*
Δ	*dimensionless elastic deformation of UHMWPE cup*
ε	*dimensionless eccentricity ratio, e/c*
λ	*dimensionless normal velocity of the ball's center,* $12R_2^2/c^2$
ω	*angular velocity*
ν	*Poisson's ratio*

以非牛頓關節液模式研究人工髖關節之暫態彈動潤滑問題

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摘要

本文分析了球形(股骨頭)在非牛頓潤滑液的定負荷條件下，採用約束彈性變形模式逐漸擠壓超高分子量聚乙烯(UHMWPE)髖臼組分。這種現象屬於軟-軟暫態彈流潤滑(EHL)問題。採用 Newton-Raphson 方法(NRM)和 Gauss-Seidel 迭代法在球坐標系下同時求解暫態修正雷諾方程，彈性變形方程和負荷平衡方程，得到暫態壓力和液膜厚度。模擬結果顯示，隨著液膜厚度的減小，壓力逐漸向中心集中。為了保持質量守恆，在出口處會出現關節潤滑液膜厚度之頸縮現象，因此最小的膜厚度出現在出口區而不是中心區域。隨著特徵長度（L）增加，關節液膜厚度增加，壓力在中心區域減小，但壓力在中心區域外卻增加。頸縮現象會隨著特徵長度的減小而增強。該研究不僅具有學術創新，而且為生醫界設計人工髖關節提供了重要依據，因此極具生醫產業應用潛力。