中國機械工程學刊第四十卷第五期第 449~459 頁(民國一百零八年)
Journal of the Chinese Society of Mechanical Engineers, Vol.40, No.5, pp 449~459 (2019)

Type Synthesis of Five Degree-of-Freedom Hybrid Mechanism

Shuyi Ge*, Yi Cao**, Kai Liu* and Rui Ding*

Keywords：hybrid mechanism, type synthesis, fractal theory, topological graph.

ABSTRACT

With the introduction of fractal theory and topological graph, this paper deals with the type synthesis and kinematic analysis of closed-loop 5-DoF (five degree-of-freedom) HMs (hybrid mechanisms). Firstly, the concept of fractal theory and topological graph are introduced. Then, theoretical grounds which consist of motional feature and algorithm for the type synthesis of HMs are presented in detail. The motional feature is composed of route, branch and kinematic pairs represented by a 8-bit binary string, while the algorithm is composed of route rule for dealing with the topological graph and calculation rule of union-intersection-preserving. Meanwhile, a mathematic model for both fractal and HMs is developed considering the interaction and correlation between them. Furthermore, the concrete procedure of type synthesis of 5-DoF HMs can be structured base on a selected topological graph and a flowchart is recommended in detail. Finally, the analysis of the motional characteristic for the end-effector is proposed and applied to an example chosen from type synthesis to demonstrate the validity of the methodology in this paper.

INTRODUCTION

Currently, HMs attract attention from plenty of researchers widely due to their excellent performances which contain the advantages of serial and parallel mechanism and avoid the weakness of both. Serial mechanism features large workspaces, high dexterity and easy to control but suffers from lack of stiffness and relatively large positioning errors due to their cantilever type of kinematic arrangement. As for the parallel mechanism, it is distinguished by parallel mechanisms consist of a series of parallel kinematic chains with active and passive joints which accuracy, rigidity, ability to manipulate large loads, actuators close to base and high speed but has to undergo small workspace, complex structure and expensive cost.

And a lower-mobility HM is suitable for many tasks requiring less than 6-DoF. Generally, a HM with 5-DoF is a lower-mobility HM, and has been wide-known due to its potential applications for machine tool. Therefore, HMs with 5-DoF have drawn extensive interests. At the same time, the design of HMs is a challenging problem in the machinery products design process, consisting of structure design and mechanism design. The mechanism design contains three important parts: evaluating characteristics, type synthesis and dimension synthesis, among which type synthesis is the most original and the most inventive one.

Dating back to the late 20th century, the design of HMs began to rise up, but very few works exist on the topic of type synthesis for lower-mobility HMs while more attention has been paid to the investigation of structure and analysis in hybrid mechanism [1-10].

According to the discrimination in mobility kind, that is, rotational or translational, the lower mobility PMs can be classified. When referring to mobility, R denotes a rotational DOF, and T denotes a translational DoF. The 5-DoF HMs are sorted in two categories, namely 3R2T and 2R3T. One category 3R2T has three rotational DoFs and two translational DoFs; the other category 2R3T has two rotational DOFs and three translational DOFs.

Meanwhile, it has been a significant and challenging issue to complete the type synthesis of 5-DoF hybrid robot. Despite the years of study, the relevant theoretical knowledge and practical method to obtain the structural synthesis of mechanism are scarce in the domestic and foreign research. Gao proposed several types of composite pairs and new kinds of sub-chains with specific degree-of-freedom based on the special Plücker coordinates [11, 12]. Tsai synthesized four degree-of-freedom (4-DoF) and 5-DoF overconstrained parallel manipulators with

Paper Received June, 2016, Revised February, 2018, Accepted February, 2018, Author for Correspondence: Yi Cao.

* *Master Candidate, School of Mechanical Engineering, Jiangnan University, Wuxi, Jiangsu 214122, China.*

** *Professor, School of Mechanical Engineering, Jiangnan University, Wuxi, Jiangsu 214122, China.*
** *Professor, Jiangsu Key Laboratory of Advanced Food Manufacturing Equipment and Technology, Wuxi, 214122, China.*

identical serial limbs [13]. Zhu presented a simple method for all 18 5-DoF 3R2T parallel manipulators to avoid singularity with base-mounted actuator structure [14]. Huang and Li proposed two novel decoupled symmetrical 5-DoF parallel mechanisms， which can perform two translations and three rotations in space [15]. Wang et al. conducted the experimental study of a redundantly actuated parallel manipulator for a 5-DoF hybrid machine tool [16, 17].

As the authors are aware, there have been some literature in the field of type synthesis based on the general Chebyshev–Grubler-Kutzbach (CGK) mobility formula [18], configuration evolution approach [19] based on the classical mechanism makeup, the constraint synthesis approach based on the reciprocal screw theory [20-22], the synthesis approach based on Lie group theory [23-25], the synthesis approach based on single-open-chain theory [26, 27], the linear transformations theory [28], and the synthesis approach based on GF set [29-31]. However, the types of hybrid mechanisms are deficient and far from the high demanding in practice. The essential reason for the situation is that a very simple yet effective theoretical guidance is lacking. Hence, it is essential to put forward a novel methodology to deal with the above problems.

The organization of the paper is as follows. In Section 2, some relevant information about fractal theory and topological graph is recalled. Section 3 devotes to the basic concept of kinematic pairs with position description, route and route rule, and branch with its calculation rule. After a short recommendation of the type synthesis principle, Section 4 focuses on a concrete approach and general procedure performed by block diagram to synthesize HMs with the aid of fractal and topological graph. Meanwhile according to this method, the type synthesis of 3T2R HMs is presented by some tabulation in Section 5. In Section 6, the above procedure is applied to the type synthesis and one instance is illustrated to demonstrate the validity of the proposed method. In Section 7, the conclusions are drawn.

FRACTAL THEORY AND TOPOLOGICAL GRAPH

Notion of Fractal Theory

Fractal theory was proposed by B. B. Mandelbrot in the 1970s firstly which defined the fractal as the object whose component is similar to the entirety in the exact direction. The fact that the basic fractals are dimensionally discordant can serve to transform the concept of fractal from an intuitive to a mathematical one. On intuitive and formal grounds two definitions need to be mentioned which strongly deserves to be called its dimension, each of which assigns to every set R in Euclidean space, the more intuitive is denoted by D_T. And the second dimension was formulated in Hausdorff 1919 and put in final form by Besicovith which is denoted by D.

Whenever it is operated in the Euclidean span R, both D_T and D are at least 0 and at most E. But the resemblance ends here. The dimension D_T is always an integer, but D need not to be an integer. And the two dimensions need not coincide; they only satisfy the Szpilrajn inequality:

$$D \geq D_T, \tag{1}$$

For all of Euclid, $D=D_T$. But nearly all sets in this essay satisfy $D>D_T$. There was no term to denote such sets, which led to coin the term fractal, and to define it as follows:

A fractal is by definition on a set for which the Hausdorff Besicovith dimension strictly exceeds the topological dimension.

Every set with a non-integer D is a fractal. For example, the original Cantor set is a fractal because of the following deriving:

$$D = \log 2/\log 3 \approx 0.6309 > 0, \text{ while } D_T = 0$$

The fractal theory is utilized to develop the configuration in order from the view of fractal dimensions and mathematics which builds it more important feature like reality and intuition compared with other dimensional space. This unique characteristic is capable to apply in mechanism which displays the mechanical feature directly and handle the problem efficiently to decrease the workload. Hence, the concept of fractal theory plays a vital role not only in this paper, but in diverse subjects.

Concept of Topological Graph and Introduction of Fractal Modes

One of the critical issues in the kinematic analysis is the work of the topological graph. It is composed of a fixed base which is equal to frame section, a moving platform that means the end-effector and branches. As for the branch, its concrete definition is that according to specific direction the branch connects the fixed base, moving platform and transitive platform.

The main topic of topological graph is divided into 3 parts:

(a) Transform the HMs to the link mechanism through simplification procedure which cut the kinematic pairs off.

(b) Much concern is given to the design of branch and route in the wake of the confirmed topological graph of HMs.

(c) Combine the first and second step served to realize the holistic architecture design of HMs.

The fractal theory is applied to the topological graph in order to generate diverse topological graphs effectively which derives three fractal modes: branch fractal, platform fractal and branch-platform fractal (BP fractal) shown in Figure 1

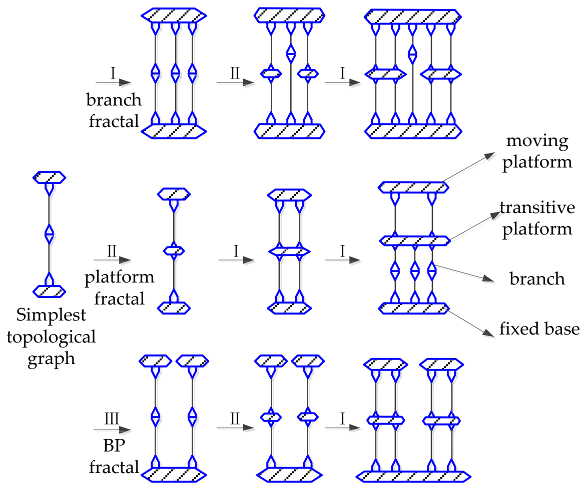


Fig. 1. Fractal modes of the topological graph

In Fig.1, Ⅰ represents branch fractal. The basic component of branch fractal is branch which consists of one or more kinematic pairs and could be divided into horizontal branch fractal and vertical one.

Ⅱ stands for platform fractal which breaks one of the branches of the topological graph and equips with a new transitive platform and two more branches to ensure the connection between platforms. What deserves to be noticed is that the requisite condition for platform fractal is the existence of that branch broken by the platform fractal.

Ⅲ signifies BP fractal which is the combination of two fractal modes above. The accurate definition is that a branch and a platform are added to any existed platform which leads to another unexpected output motion without a little affiliation about the output of mechanism. So the BP fractal is utilized for the design of multi-output structure.

It is noteworthy that the BP fractal isn't the key subject of the investigation for the reason that this paper focus on the close-loop topological graph on one hand. And branch fractal and platform fractal just maintain one output motion assigned to close-loop while BP belonging to open-loop topological graph has more end-effectors. On another hand, open-loop configuration is easy which can be treated as the aggregation of actuators, so the analysis of that does not make any sense.

DESCRIPTION OF MOTIONAL FEATURE AND ALGORITHM

The core content of the design of configuration of 5-DoF HMs based on the topological graph tends to be conducted into 3 sections. The first section is transition from topological graph to the route, second section is identifying the branches according to the route obtained in the first section and the last section is defining the kinematic pairs of each branch in terms of terminal characteristics.

Introduction of Route and Route Rule

There is no doubt that the route for its own sake is one of the most vital elements in the structural design of 5-DoF HMs which turn out to make the research of route great significance.

Route is comprised of branches with which connect some transitive platforms, the fixed base and the end-effector in accordance with a certain combined form. It ensures that the route depends on the fixed base and end-effector, and has no affiliation with any transitive platform on the contrary.

The essential idea of route rule on the basis of the topological graph is acquisition of entire satisfied routes with their branches in terms of the expected output of the end-effector. In this rule, the relationship between the branches assigned to one route is regulated as serial connection, while the relation between each route is regulated as parallel connection. In order to obtain the set of output displacement of the moving platform, the ways of derivation is replaced by counting the union and intersect set of branches as the following explicit expression.

There are mainly two types of mathematical operations on G_F sets, i.e., the union and intersection. The union mathematical operation is generally exploited in the kinematic characteristics analysis of serial mechanisms defined by:

$$R_j = \#_{i=1}^{I} B_{ij} \text{ , } P = \&_{j=1}^{J} R_j \text{ ,} \tag{2}$$

where B_{ij} expresses the set of output displacement of the ith branch in the jth route. R_j represents the set of output displacement of route. P represents set of output displacement of end-effector of the HMs. I expresses the amount of the jth route. And J represents the sum of routes. What is more, the condition that several routes share common branches is acceptable.

It is suggested that the amount of routes derived from Equation (2) should be analysed according to various concrete statuses. And the statuses can be divided into 2 groups in general.

(a) Normal topological graph (without horizontal branches)

Due to the public platform, the way to obtain routes increases according to different combinations of branches shown as followed:

$$l = \sum_{i=1}^{n} (a_{i1} \times a_{i2}) , \qquad (3)$$

where l expresses the amount of routes, n expresses the amount of public platform, and a_{i1} expresses the amount of branches placed under the public platform in the ith public platform.

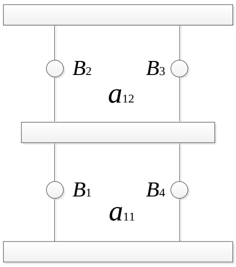


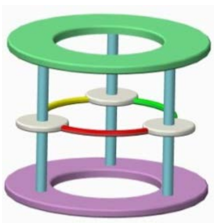
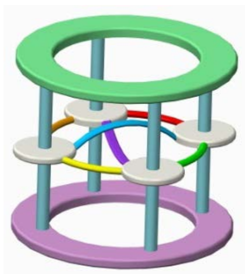
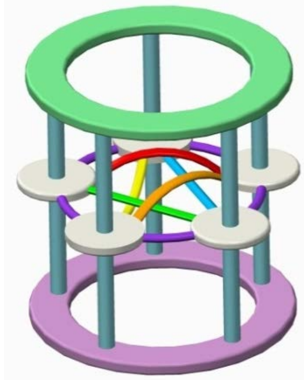
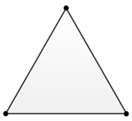
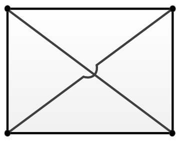
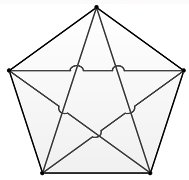
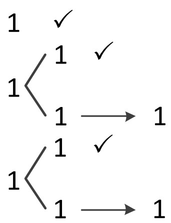
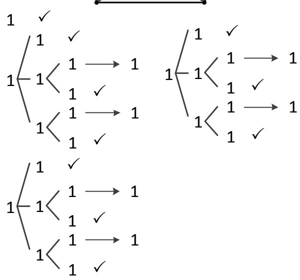
Fig. 2. Normal topological graph

(b) Atypical topological graph (with horizontal branches)

The detailed conditions are concluded in Table 1. After complicated deriving, the formula to calculate the amount of routes in in atypical topological graph is shown as following:

$$a_n = \sum_{j=1}^{n-2} [\prod_{i=1}^{j} (n-i)] , \qquad c_n = n(n+1)a_n \qquad (4)$$

Table 1. Conclusion of Route Rule in Atypical Topological Graph

Number	1	2	3	4
Topological graph				
Sketch of horizontal branches				
Analysis of routes	1 ✓	1 ✓ 1 ⟶ 1		
Amount of route a_n from point to point	1	2	5	16
Accumulated amount of routes c_n	2	12	60	320

To be clear, a tree structure is built to analysis the route in Table 1. The number 1 represents starting point and the symbol tick means there is no transitional point in this route from starting point to end point.

Details about Branch and Calculation Rule of Union-intersection-preserving

Correlation between branches and fractal

The branch plays an important role both in topological graph and fractal procedure, hence, there is no difficult at all in figuring out the correlation between branches and fractal which is illustrated in the followed Figure 3.

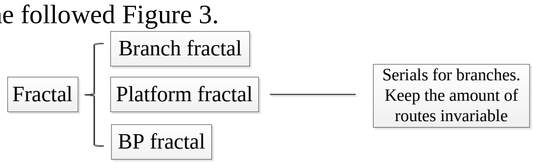


Fig. 3. Relationship between platform fractal and branches

Particularly worth mentioning is the horizontal branch fractal which complicates the above correlation and makes it necessary to be analyzed in detail.

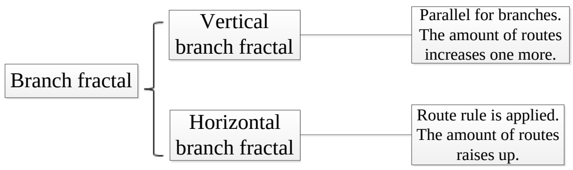


Fig. 4. The relationship between branch fractal and branches

As an important symbol of transverse development for HMs, the horizontal branch fractal is comprised of horizontal branch which serves to connect two parallel transitive platforms. Not all of the topological graph can form the multipath atypical HMs, but the horizontal branch fractal ones. There is no difficult at all in concluding the relation between branch fractal and fractal as supplement in Figure 4.

Calculation rule of union-intersection-preserving

From the view of configuration, the mechanism is composed by the parallel routes which are consisted of branches in serial. If another view is substituted to observe this problem like mathematical logic, the problem will transform into the derivation of the set of output displacement for the end-effector which is also conducted by the interaction of sets of output displacement for each route. Meanwhile, the routes are derived from the union of sets of output displacement for each branch.

In order to simplify the above explanation, calculation rule of union-intersection-preserving is defined in the Equation (5).

$$\begin{cases} \text{serial: } A\#B = A\cup B - A\cap B \\ \text{parallel: } A\&B = A\cap B \\ \text{calculation formula: } A\#\left(B\&C\right) = \left(A\#B\right)\&\left(B\#C\right) \end{cases} \tag{5}$$

where # is operative symbol represented serial algorithm and & represents parallel algorithm. A and B means the sets of output displacement. $\cup$ serves to obtain the union, while $\cap$ is for interaction. By the way, the position relationship will not be able to be brought into the computational process because of its irregularity (it will be introduced in chapter3.3.2).

Kinematic Pairs

Transformation of kinematic pairs

In this paper, a systematic study on the type synthesis of 5-DoF HMs is performed involving R (rotation) and T (translation) joints which are the foundational elements of kinematic pairs. The HMs tend to relate with every kinematic pairs involving C (cylindrical), U (universal), S (spherical), G (planar) joints, and so on. They are shown by following operators:

(a) Substitute a combination of one R and one T joints with common axe with a C joint;

(b) Substitute a combination of two successive R joints with intersecting axes with a U joint;

(c) Substitute a combination of three successive R joints with concurrent axes with an S joint;

(d) Substitute a combination of one R and two T joints which the axe of R joint is perpendicular to the plane composed by the two T joints with a G joint;

(e) Substitute a combination of two successive T joints with intersecting axes with a TP joint;

(f) Substitute a combination of three successive T joints with concurrent axes with an TX joint;

(g) Substitute a combination of three successive T joints with concurrent axes and one R joint with an X joint.

There is no difficulty in finding out that all of the kinematic pairs can be transformed as the combination of R joints and T joints. So it is the evidence to demonstrate that the structural design of HMs can be completed by the R and T joints.

Expression of R/T joints

The 8-bit binary string is utilized to express the R and T joints in order to make it concise. There is no term to denote such joints which leads to coin the expression. In addition, the 8-bit binary string not only expounds the type of joints and position relation with fixed coordinate, but also states position relation between axes of the adjacent joints and branch connection. What deserves to be mentioned is that this expression would be beneficial to the subsequent digital design.

First of all, the 8-bit binary string is divided into two parts that the former 3 bits are taken use of performing the position relation between axes of the adjacent joints, while the latter 5 bits serve to signify a single kinematic pair.

(a) Description for single kinematic pair is shown in Table 1 in detail according to the type and position relation with fixed coordinate, as well as illustrated in Figure 5.

Table 2. Binary Expression of Single Kinematic Pair

P0.4	P0.3	P0.2	P0.1	P0.0
R	*T*	*X*	*Y*	*Z*

From Table 2, two messages are conveyed for a deeper comprehension to analyze the meaning of the description. The first one: P0.4 and P0.3 are the bits to distinguish the type of joint R and T, where P0.3=1 is rotation joint, P0.4=1 is translation joint, while the two bits can't be equal to 1 at the same time.

The second message: P0.2, P0.1 and P0.0 are the bits of the axial vector X, Y, Z which share the identical direction as the fixed coordinate. If an arbitrary bit is 1, the axial vector of joint must parallel with the vector bit. And there are only one bit is 1 at any time.

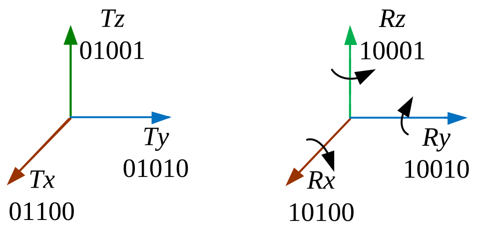


Fig. 5. Description of single kinematic pair

(b) Because the position relationship is only concluded as coaxial, parallel and perpendicular by the latter 5 bits with limitation for its own sake, a necessary supplement has to be prepared for a comprehensive expression of position relation.

Table 3. Expression of Position Relationship of Adjacent Kinematic Pair

Number	Axial position	P0.7	P0.6	P0.5
1	Coaxial	0	0	1
2	Parallel	0	1	0
3	Perpendicular	0	1	1
4	Angularity	1	0	0
5	Non-coplanar perpendicular	1	0	1
6	Non-coplanar	1	1	0

Hence, the former 3 binary bits are put forward to complete the function comprehensively shown in Table 3. Particularly mention that if any axes of the adjacent joints are in relation of angularity, the axes must be intersected in one point. Please note that most of the position relationship has the characteristic of transitivity, for example parallel and coaxial, which are named of regular position relation, otherwise the others are called irregular position relation.

TYPE SYNTHESIS METHOD OF 5-DOF HMS BASED ON FRACTAL THEORY

The outline of the configuration design for HMs primarily focuses on two points: choosing the suitable kinematic pairs based on the topological graph and estimating if the layout of kinematic pairs is satisfied by the design process. Anyway, the configuration design is on the foundation of topological graph which is displayed based on fractal theory. In another words, fractal theory is applied to the development of topological graph, furthermore, hybrid mechanism is transformed by topological graph.

Type Synthesis Principle for HMs

As stated in Section 3 that horizontal branch fractal is the symbol of transverse structure for HMs, the hybrid configuration can be categorized into two types as well, one is the topological graph which has horizontal branch and ability to generate the three-dimensional fractal hybrid robot shown in Figure 6(a), and other is contrary in Figure 6(b).

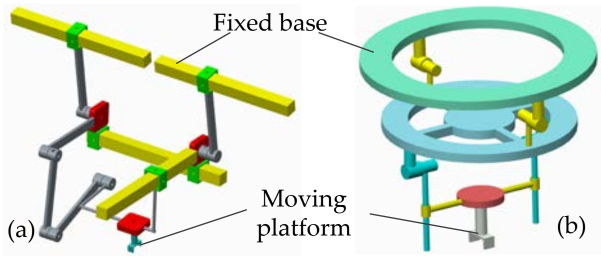


Fig. 6. The classification of hybrid mechanism

From the view of the displacement set point, configuration of HMs is constituted by routes according to the parallel rules while the route consists of branches which are gained through the distribution of degree-of-freedom. The composition relationship of the whole process is based on the Eq. (2) and the algorithm is on the Eq. (5).

Research proves that, the components in the approach have a close connection with each other which has been interconnected and interlocked. As concluded, the structural element of HMs is topological graph, while the central strategy is fractal theory. Therefore, it is necessary to establish synthesis formulas, and generate a query table explaining the relationships between the dimensions of end-effector characteristics and the structural parameters of mechanisms. The relationship among the degree-of-freedom of HMs (F_{dof}), number of actuated kinematic pairs in the corresponding branch (Bi), number of fractal (N), the number of passive branches (b), the number of active branches (n), the number of branch fractal (L), the number of platform fractal (P), the number of platform fractal in the shortest route ($P_{\min}$) is as follows:

$$\begin{cases} F_{dof} - \sum_{i=1}^{n} B_i = 0 \\ N+1 = F_{dof} - \sum_{i=1}^{n} (B_i - 1) + b \\ N = n + b - 1 \\ N = L + P \\ P_{\min} \le F_{dof} - 1 \end{cases} \tag{6}$$

Based on the fractal theory, the principle for the structure design of HMs can be as followed:

(a) The main thought of the proposed methodology takes the shortest route as the center to develop from the topological graph. This point serves to solve with the instantaneity due to the fact that the displacement output of other routes is the superset of the shortest route's set.

(b) For the reason that structural foundation for HMs is the topological graph which achieves its diversity depending on the fractal theory, hence, the methodology proposed in this paper is capable to accomplish the diversification of configuration for HMs.

(c) By means of alternative unit, bring a suitable configuration into branches to satisfy the expected motion which is called substitution design for branch. In addition, the number of branches in substitutive configuration is less than that of HMs.

(d) Dealing with the relationship between superset and subset, it can be categorized in 2 groups in total. The first one majors in routes that the set of output displacement in the shortest route is the subset, while that of other routes are superset. The second one is for branches that the branch assigned to the shortest route compared to the parallel branch is the subset one, while the corresponding one is superset.

The Concrete Procedure of Type Synthesis for HMs

As stated above, type synthesis of HMs from the point of fractal theory and topological graph can be achieved by the following steps:

(a) Determine the number of the branch fractal and platform fractal in the simplest topological graph (shown in Fig. 1) according to the expected displacement set of the end-effector and complete the layout of the topological graph of HMs.

(b) The number of routes and calculation algorithm of displacement sets of topological graph is derived by the route rule. Meanwhile, the position of the shortest route should be confirmed.

(c) According to the algorithm mentioned in Eq. (2), the satisfied degree-of-freedom for routes and the set of output displacement for branches have been obtained through software matlab or enumeration which the serial-parallel relationship between each branch can be involved.

(d) In terms of the Eq. (5), the sets of output displacement of the above branches and routes can be derived which are the basic calculative elements for the displacement sets of the end-effector for HMs. What is essential to be known is the result of comparison between the expected output and computational output. The identical result is the only evidence of the validity of the methodology proposed in this paper.

(e) Deal with the three-dimensional model after the configuration of HMs is reaped.

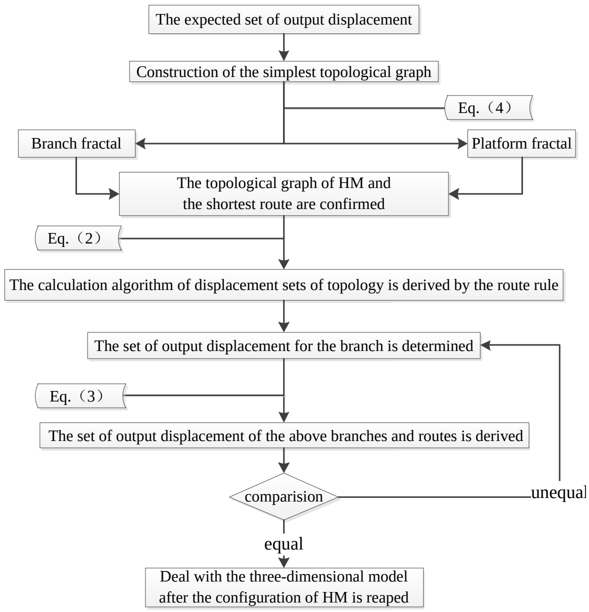


Fig. 7. The flowchart of the concrete procedure of type synthesis for HMs

For an explicit expression of the concrete procedure of type synthesis for HMs, a flow chart is utilized to reflect the process in Fig. 7.

TYPE SYNTHESIS OF CONFIGURATION FOR 5-DOF HMS

In order to demonstrate the feasibility of the aforementioned method of type synthesis for HMs based on the fractal theory, type synthesis of 3T2R (three-rotational and two-translational) HMs will be discussed in the following. Obviously, diverse topological graphs can be gained in accordance with fractal theory. The type synthesis in particular refers to a designated topological graph for the reason that the number of fractal is not limited. The topological graph is shown in the Figure 8.

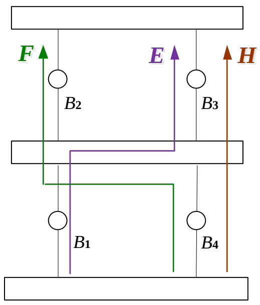


Fig. 8. The appointed topological graph

Based on the route rule, it can be derived that in the topological graph there are 4 routes in total which are revealed as followed:

$$\{B_1B_2,\ B_3B_4,\ B_2B_4,\ B_1B_3\} \tag{7}$$

The route B1B2 is defined as the shortest route and endowed by the output of 3T2R. As for other routes introduced above, their displacement set should be the superset of the shortest route which results in the output of 3T2R and 3T3R. The consequence of the permutation and combination of displace set of other routes is enumerated in the following table.

Table 4. Permutation and Combination of Displace Set of Routes

Route	The set of output displacement for the routes			
B_3B_4	3T2R	3T2R	3T2R	3T2R
B_2B_4	3T2R	3T2R	3T3R	3T3R
B_1B_3	3T2R	3T3R	3T2R	3T3R
B_3B_4	3T3R	3T3R	3T3R	3T3R
B_2B_4	3T2R	3T2R	3T3R	3T3R
B_1B_3	3T2R	3T3R	3T2R	3T3R

It is assumed that the branch B1 and B3 compose the route E, branch B2 and B4 compose the route F, and branch B3 and B4 compose the route H. The displacement set of the rest branches can be derived from the algorithm listed blew.

$$\begin{cases} B_3 \cup B_4 = \{H\} \\ B_1 \cup B_3 = \{E\} \\ B_2 \cup B_4 = \{F\} \end{cases} \tag{8}$$

From Table 4, a random selection of distribution for the degree-of-freedom of branches is determined (in this paper the data in the last column is chosen). Because of the large work load of the derivation, the matlab project is applied to solve the problem for a comprehensive consequence shown as followed:

```
Derivation for the degree-of-freedom of the branches provided by the Equation (8):
    A2=[Rx Ry Rz];
    A1=[Tx Ty Tz];
    a=length(A2);
    for i=0:a
        S=nchoosek(A2,i);
        [m1,n1]=size(S);
        for j=1:m1
            A3=S(j,:);
              B4=union(A1,A3)
              c1=[Rx Ry];
            c2=[Tx Ty Tz Rz];
              b=length(c2);
              for k=0:b
                  S1=nchoosek(c2,k);
                  [m2,n2]=size(S1);
                  for l=1:m2
                      c3=S1 (l,:);
                      B3=union(c1,c3)
                  end
              end
          end
      end
```

From the consequence of the above program, it is convenient to obtain the sets of output displacement for the whole branches by choosing one set of data. And from the data, the sets of output displacement of branches can be fetched for equipping the HMs on the basic of the topological graph displayed in Fig. 8. As statistics, there are 127 kinds of results to achieve the demands of the Eq. (8) after necessary optimization, for example, avoidance of repetition through the program. Meanwhile, due to the extent of the paper, other configuration can't be listed in detail.

ANALYSIS AND VERIFICATION OF CONFIGURATION

The Analysis of the Motional Characteristic for the End-effector in HMs

In order to demonstrate the validity of the 5-DoF HMs synthesized in chapter 5, the analysis of the motional characteristic for the end-effector deserves to be proposed by the following steps.

(a) Extract the topological graph and branches from the configuration of the HMs.
(b) Change the branches into kinematic pairs and 8-bit binary strings could be gained through the application of the chapter three.
(c) The set of output displacement of branches can be derived by the formula of serial calculation of the above kinematic pairs in accordance with Eq. (5).
(d) The topological graph is analysed by the route rule seeking for all of the routes and based on the part (b) it is convenient to reap the set of output displacement for the above routes through mathematical logic rules.
(e) In terms of the parallel calculation of the routes, the motional characteristic for the end-effector could be gained which should be equal to the set of output displacement for the HMs overall.

The Analysis of Output Degree of Freedom

A group of data chosen from the results of program is in Table 5 to demonstrate the validity of the proposed methodology in this paper.

Table 5. Distribution for the Degree-of-freedom of Branches

Branch number	Distribution for the degree of freedom of branches
B_1	$T_{(U)}T_{(V)}$
B_2	$R_{(U)}R_{(V)}T_{(V)}T_{(W)}$
B_3	$R_{(U)}R_{(V)}\,T_{(U)}T_{(W)}$
B_4	$T_{(U)}T_{(V)}$

In accordance with the above table and selected topological graph, the adaptive kinematic pairs is assembled to the configuration of the HM shown in the Figure 9.

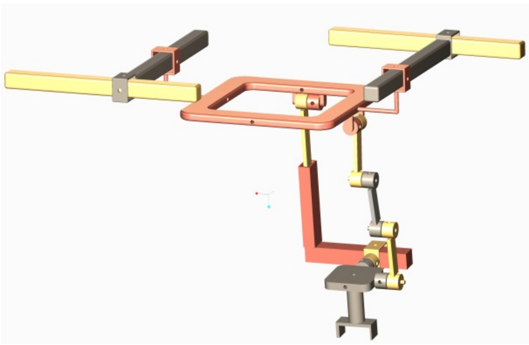


Fig. 9. The model of 5-DoF hybrid mechanism

The verification of the above HMs is requisite to compare the displacement set of output between the HM in Fig.9 and expected one. And the calculation procedure is as followed.

$$\begin{aligned} B_1 &= T_{(U)}T_{(V)} \\ &= [04\ 0C\ 02]_H^T \# [04\ 0A\,02]_H^T, \\ &= [04\ 0A\ 0C\ 02]_H^T \end{aligned} \quad (9)$$

$$\begin{aligned} B_2 &= R_{(U)}R_{(V)}T_{(V)}T_{(W)} \\ &= [06\ 14\ 04]_H^T \# [06\ 12\ 04]_H^T \# \\ &\quad [04\ 0A\,02]_H^T \# [04\ 09\ 02]_H^T, \\ &= [06\ 09\ 0A\,12\ 14\ 04]_H^T \end{aligned} \quad (10)$$

$$\begin{aligned} B_3 &= R_{(U)}R_{(V)}T_{(U)}T_{(W)} \\ &= [06\ 14\ 04]_H^T \# [06\ 12\ 04]_H^T \# \\ &\quad [04\ 0C\,02]_H^T \# [04\ 09\ 02]_H^T, \\ &= [06\,09\ 0C\ 12\ 14\ 04]_H^T \end{aligned} \quad (11)$$

$$\begin{aligned} B_4 &= T_{(U)}T_{(V)} \\ &= [04\ 0C\ 02]_H^T \# [04\ 0A\,02]_H^T, \\ &= [04\ 0A\ 0C\ 02]_H^T \end{aligned} \quad (12)$$

Expressions (9-12) can be the calculative elements for derivation of the set of output displacement in each route in terms of Eq. (5) as followed:

$$\begin{aligned} &B_1 \# B_2 = B_4 \# B_2 = T_{(U)}T_{(V)} \# R_{(U)}R_{(V)}T_{(V)}T_{(W)} \\ &= [04\ 0A\ 0C\ 02]_H^T \# [06\ 09\ 0A\,12\ 14\ 04]_H^T \\ &= [06\ 09\ 0A\,12\ 14\ 04\,0C\ 02]_H^T \end{aligned} \quad (13)$$

$$\begin{aligned} &B_1 \# B_3 = B_4 \# B_3 = T_{(U)}T_{(V)} \# R_{(U)}R_{(V)}T_{(U)}T_{(W)} \\ &= [04\ 0A\ 0C\ 02]_H^T \# [06\,09\ 0C\ 12\ 14\ 04]_H^T \\ &= [06\,09\ 0C\ 12\ 14\ 04\ 0A\ 02]_H^T \end{aligned} \quad (14)$$

The set of output displacement in the end-effector of the HM could ultimately be derived blew by the expressions (13) and (14) shown in following.

$$\begin{aligned} P &= (B_1 \# B_2) \& (B_3 \# B_4) \& (B_1 \# B_3) \& (B_2 \# B_4) \\ &= [06\ 09\ 0A\,12\ 14\ 04\,0C\ 02]_H^T \& [06\,09\ 0C\ 12\ 14\ 04\,0A\ 02]_H^T \\ &\& [06\,09\ 0C\ 12\ 14\ 04\ 0A\ 02]_H^T \& [06\ 09\ 0A\,12\ 14\ 04\,0C\ 02]_H^T \\ &= [06\ 09\ 0A\,12\ 14\ 04\,0C\ 02]_H^T \\ &= T_{(U)}T_{(V)}T_{(W)}R_{(U)}R_{(V)} \end{aligned}$$

In summary, the set of output displacement for the moving platform is as same as the expected one and also equal to that of route B1B2 which proves the correctness of the methodology for the 5-DoF HMs based on the fractal theory.

CONCLUSION

This paper presents a simple enough yet effective type synthesis approach for 5-DoF HMs from the point of fractal theory and topological graph. The theme is combined by two sections, one is about the topological graph that is the structural foundation of HMs and links the branches in order, and another is motional characteristic included kinematic pairs, branches and routes which mechanics are equipping with according to the above algorithms.

(a) Fractal theory is utilized for transformation of the topological graph with highly ordered. According to the permutation and combination of the sets of output displacement and distribution of degree-of-freedom, the diversity of configuration of hybrid robots is achieved mathematically.

(b) As a novel concept, route is presented for developing the horizontal structure in HMs.

Meanwhile, the route rule is provided to deal with output of multi-route in multi-loop topological graph.

(c) The algorithm is proposed to express the operation rule of union-intersection-preserving which contains the parallel operation between the routes and serial operation between the branches and kinematic pairs. In addition, the relationship of fractal and calculation rule is revealed.

(d) The matlab program is adopted for the consequence of the whole adaptive kinematic pairs of corresponding branch to synthesize the 5-DoF HMs. Particularly one of the synthesized HMs is modeled and analyzed to demonstrate the validity of the proposed methodology.

ACKNOWLEDGMENT

This work reported here was supported by the National Natural Science Foundation of China (Grant No. 50905075, 51505190), Six talent peaks project in Jiangsu Province (Grant No. ZBZZ-012), the 111 Project (Grant No. B18027).

REFERENCES

R. Alizade, F C Can, E Gezgin. "Structural synthesis of Euclidean platform robot manipulators with variable general constraints," *Mech Mach Theory*, Vol. 43, No.11, pp.143l-1449 (2008).

A Campos, C Budde, J Hesselbach. "A type synthesis method for hybrid robot structures," *Mech Mach Theory*, Vol. 8, No. 8, pp.984-995 (2008).

L Romdhane. "Design and analysis of a hybrid serial-parallel manipulator, " *Mech Mach Theory*, Vol. 34, No.7, pp.1037-1055 (1999).

K T Tanev. "Kinematics of a hybrid (parallel-serial) robot manipulator," *Mech Mach Theory*, Vol. 35, No. 9, pp.1183-1196 (2000).

L W Tsai, S Joshi. "Kinematic analysis of 3-DoF position mechanisms for use in hybrid kinematic machines," *J Mech Design*, Vol. 124, No.2, pp.245-253 (2002).

A Ramadan, K Inoue, T Arai, et al. "New design of a compact parallel micro-Nano two-fingered manipulator hand," *IEEE International Conference on Nano/Micro Engineered and Molecular Systems*, Bangkok, Thailand, pp.515-519 (2007).

H. Shen, H. Zhao, J. Deng, et al. "Type design method and the application for hybrid robot based on freedom distribution and position and orientation characteristic set," *J Mech Eng-en* (in Chinese), Vol. 47, No. 23, pp.56-64 (2011).

S. Bandyopadhyay, A. Ghosal. "Analytical determination of principal twists in serial, parallel and hybrid manipulators using dual vectors and matrices," *Mech Mach Theory*, Vol. 39, No. 12, pp.1289-1305 (2004).

Zeng Q, Fang Y. "Structural synthesis of serial-parallel hybrid mechanisms based on representation and operation of logical matrix," *J Mech Robotics*, Vol. 1, No. 4, pp.469-484 (2009).

H. Liu, T. Huang, J. "Mei. Kinematic design of a 5-DoF hybrid robot with large workspace/limb-stroke ratio," *J Mech Design*, Vol. 129, No. 5, pp.530-537 (2007).

Gao F, Li W, Zhao X, et al. "New kinematic structures for 2-, 3-, 4-, and 5-DOF parallel manipulator designs," *Mech Mach Theory*, Vol. 37, No. 11, pp.1395-1411 (2002).

Gao F, Peng B, Li W, et al. "Design of a novel 5DOF parallel kinematic machine tool based on workspace," *Robotica*, Vol. 23, No. 1, pp.35-43 (2005).

Fang Y, Tsai L W. "Structure Synthesis of a Class of 4-DoF and 5-DoF Parallel Manipulators with Identical Limb Structures," *Int J Robot Res*, Vol. 21, No. 9, pp.799-810 (2002).

Zhu S J, Huang Z. "Eighteen fully symmetrical 5-DoF 3R2T parallel manipulators with better actuating modes," *Int J Adv Manuf Tech*, Vol. 34, No. 3-4, pp.406-412 (2007).

Z. Huang and Q.C. Li, "Two novel symmetrical 5-DOF parallel mechanisms," *Journal of Yanshan University* (in Chinese), Vol. 25, No. 4, pp.283-286 (2001).

Wang L, Wu J, Wang J, et al. "An Experimental Study of a Redundantly Actuated Parallel Manipulator for a 5-DOF Hybrid Machine Tool," *IEEE-ASME T Mech*, Vol. 14, No. 1, pp.72-81 (2009).

Wu J, Wang J, Wang L, et al. "Dynamic model and force control of the redundantly actuated parallel manipulator of a 5-dof hybrid machine tool," *Robotica*, Vol. 27, No. 1, pp.59-65 (2009).

L.W. Tsai. "The enumeration of a class of three-DOF parallel mechanisms," *In: Proceedings of the 10th World Congress on the Theory of Machine and Mechanisms*, Oulu, Finland, Vol. 2, pp.1121-1126 (1999).

J.S. Wang, L.W. Guang, L.P. Wang, et al. "Research on parallel robot configuration creative design," *Chin J Mech Eng-en*, Vol. 40, No. 11, pp.7-12 (2004).

Z. Huang, Q. C. Li. "Type synthesis of lower-mobility parallel manipulators," *In: Proceedings of the Workshop on Fundamental Issues and Future Research Directions for Parallel Mechanisms and Manipulators*, Quebec, Canada, pp.272-283 (2002).

Z. Huang, Q. C. Li. "Type synthesis of symmetrical lower-mobility parallel mechanisms using the

constraint-synthesis method," *Int J Robot Res*, Vol. 22, No. 1, pp.59-79 (2003).
X. Kong, C.M. Gosselin. *Type Synthesis of Parallel Mechanisms*, Springer Berlin Heidelberg, New York (2007).
J.M. Hervé. "The Lie group of rigid body displacements, a fundamental tool for mechanism design," *Mech Mach Theory*, Vol.34, No. 5, pp.719-730 (1999).
Q.C. Li, Z. Huang, J.M. Hervé. "Type synthesis of 3R2T 5-DOF parallel mechanisms using the lie group of displacements," *IEEE T Robotics Autom*, Vol.20, No. 2, pp.173-180 (2004).
C.C. Lee, J.M. Hervé. "Type synthesis of primitive Schoenflies-motion generators," *Mech Mach Theory*, Vol. 44, No. 10, pp.1980-1997 (2009).
Q. Jin, T.L. Yang. "Structure synthesis of parallel manipulators with 3-dimension translation and 1-dimension rotation," *In: ASME 2002 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, Montreal, Canada, pp.907-915 (2002).
Q. Jin, T.L. Yang. "Theory for topology synthesis of parallel manipulators and its application to three-dimension-translation parallel manipulators," *J Mech Design*, Vol. 126, No. 4, pp.625-639 (2004).
G. Gogu. *Structural Synthesis of Parallel Robots*, Springer Berlin Heidelberg, New York (2007).
F. Gao, J.L. Yang, Q.J. Ge. "Type synthesis of parallel mechanisms having the second class GF sets and two dimensional rotations," *J Mech Robot*, Vol. 3, No. 1, pp.1-8 (2011).
Q.S. Shi, F. Gao, F.G. Yu. "Type synthesis of 3T1R parallel mechanisms based on GF set," *Machine Design and Research* (in Chinese), Vol. 24, No. 4, pp.31-36 (2008).

五自由度混聯機構的型綜合

葛姝翌　曹毅　劉凱　丁銳
江南大學機械工程學院
江蘇省食品先進製造裝備技術重點實驗室

摘 要

隨著分形理論的引入和拓撲圖形, 本文研究了閉環五自由度混聯機構型綜合方法及其運動學分析。首先，介紹了分形理論和機構拓撲圖的概念及關係。其次，對混聯機構型綜合的理論基礎進行描述，包括機構的運動特徵和運算法則。其中，運動特徵包括路徑、支鏈和運動副(使用 8 位代碼表示)，運算法則包括為求解機構拓撲圖的路徑法則和串並聯計算規則。同時，建立了混聯機構與分形的關係模型。進一步的，通過一個選定的拓撲結構歸納出五自由度混聯機構構型綜合的具體步驟，並以流程圖的形式展示。最後，提出了機構末端運動特徵的分析方法，並以此方法，分析了所綜合出的混聯機構中選定的一個實例機構，從而證實本文所述方法的可行性。