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Type Synthesis of Fully-isotropic 2T3R 5-DoF Hybrid Mechanisms

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ABSTRACT

In mechanical design, it is necessary to synthesize the mechanism，especially for the fully-isotropic hybrid mechanisms (HMs). This article proposes a simple but very effective approach for fully-isotropic HMs by means of G_F set theory. firstly, a type synthesis method for HMs is simply presented, especially, the option standard of the input motion joint and the steps for the integration of the branches are given which provide the theoretical foundations for the integration of the type structure of completely isotropic HMs. Secondly, the process of type synthesis for fully-isotropic HMs is described in detail. To corroborate the usefulness of the proposed method for fully-isotropic HMs, type synthesis of fully-isotropic 2T3R 5-Degree-0f-Freedom (DoF) HMs is illustrated with example, where T denotes translation and R denotes rotation. Finally, the forward displacement analysis of a synthesized 2T3R fully-isotropic HM is carried out. The expression of the Jacobian matrix is deduced, it shows that the Jacobian matrix of the synthesized fully-isotropic HM is a 5×5 identity matrix throughout the entire workspace validating the fully-isotropic feature of the HM and the effectiveness of the type of composite approach for fully-isotropic HMs.

INTRODUCTION

With the continuous development of the mechanisms, more and more scholars (Tsai et al, 2002; Chung et al, 2004) begin to pay attention to the hybrid structure. Because it has the advantages of both series and parallel mechanism (Tanev, 2005; Gómezbravo et al, 2012), hence it is also widely used in industrial production.

Jacobian matrix of the mechanism can be received by kinematics analysis of the deployment of the mechanism. The conditional number of Jacobian matrix determines the isotropy of the mechanism, and the mechanism with isotropic property is easier to manipulate (Carricato et al, 2002). Therefore, it is necessary to design mechanisms with isotropic properties, which is also an important subject in the design of hybrid mechanisms.

Carricato (2005) designed a 4-DoF parallel mechanism with isotropic properties by helix theory. By applying the linear transformation theory and combining with the morphological theory, Gogu et al. (2004, 2007, 2012) puts forward a multibody system with isotropy characteristic, which can realize spatial operation. Zhang et al. (2009, 2013) provided a 2T1R structure with the characteristic of fully-isotropic through a systemic methodology of the reciprocal screw theory. Zeng et al. (2009, 2010, 2011) introduced an useful approach by utilizing the screw principle to design the robot with the advantage of decoupling. Kong and Gosselin (2004, 2007) studied the 3-DoF decoupled parallel mechanism by using the geometrical description method. Yu and Zhang et al (2010, 2011) adopt different configuration methods to design multi-body mechanism with certain application value and the theory proposed has certain guiding significance.

However, very few works has been performed with regard to the type integration of completely isotropic HMs. In last two decades, a lot of research attention has been paid to the analysis and modeling for hybrid robots (Liu et al, 2007; Takuma et al, 2010; Zhao et al, 2012; Hu, 2014; Joubair et al, 2015), The design of some hybrid mechanisms has already been developed and applied in some area, and more and more scholars are also doing scientific research on it (Huang et al, 2010; Li et al, 2012; Pisla et al, 2013; Campos et al, 2008; Shen et al, 2011; Zeng et al, 2014; Zhou et al, 2016, 2017). Especially, Cao et al.

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(2017) proposed a quite convenient and useful approach for structure integration of HMs by means of G_F method and structure integration of 3T2R and 2T3R 5-DoF HMs was given. Nevertheless, the types of fully-isotropic HMs are deficient and far from the high demanding in practice. The method presented in this paper solves the shortcomings mentioned above and this approach provides an effective way for the design of hybrid mechanism.

The organization of the paper is listed like this: Firstly the he option standard of the input motion joint and the standard of the integration of the branches is proposed, the process of type synthesis for fully-isotropic HMs is described in detail in Section 2. By using the method described above, the type integration of the structure of 2T3R completely isotropic HMs is performed in Section 3. Meanwhile, the forward displacement analysis of a synthesized 2T3R fully-isotropic HM is carried out in Section 4 to demonstrate the fully-isotropic feature of the HM. The summing-up and prospective works of the paper are concluded in Section 5.

TYPE SYNTHESIS APPROACH FOR FULLY-ISOTROPIC HMS BASED ON G_F SETS

Document (Zhou et al, 2016; Gao et al, 2011) gives a detailed presentation about the notion of G_F theory and its arithmetic rules, which will not be described here owing to the limitations of the space.

Jacobian matrix can be used to describe the fully-isotropy of mechanism. For a fully-isotropic mechanisms, its driving joint speed corresponds to its motion speed. The type and distribution of branched chains and the choice of kinematic pairs at branched chain joints are all related to the fully-isotropic design of the mechanisms. To ensure the driven branch possesses decoupled input in a certain direction, both three rotation axes R_j ($j=a$, β, γ) and three translation axes T_i ($i=a$, b, c) should be perpendicular to each other.

Type Synthesis Principle for HMs

Based on the contents stated above, there are two types of G_F sets of hybrid mechanisms, which are given in Fig.1 and Fig. 2.

Through the method of G_F set, the terminal output characteristics at the end of the mechanism and the relationship between the characteristic size of the end of the mechanism and its structural parameters are as follows:

$$G_F = G_{FS1} \cup G_{FS2} \ldots \cup G_{FSi} \cup G_{FP1} \cup G_{FP2} \ldots \cup G_{FPj} \, , \quad (1)$$

$$\begin{cases} F = F_p + F_s \\ F - \sum_{i=1}^{n} q_i = 0 \\ N = F - \sum_{i=1}^{n} (q_i - 1) + p \\ n = N - p \quad (n \le F) \end{cases} , \quad (2)$$

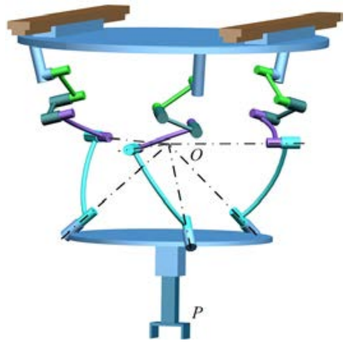

Figure. 1 G_F^{I} hybrid mechanism

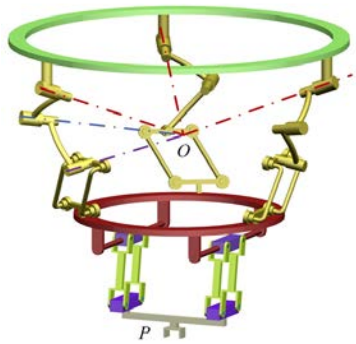

Figure. 2 G_F^{II} hybrid mechanism

The description of the physical significance of relevant symbols in Eq. (1) and Eq. (2) is detailed in reference (Zhou et al, 2016), which is not further elaborated here.

Selection Criterion of the Input Kinematic Pair

For the purpose of simplifying the selection of the input motion pair, the number synthesis of HMs denoted by Eq. (2) can be set to p=0, q_i=1, F=N=n. In this situation, the selection basis of the input motion pair is:

(1) For the serial mechanism module G_{FSi}, all elements of G_{FSi} are input kinematic characteristics. If the input kinematic characteristic is a translational characteristic, the driving joint must be a prismatic pair. When it is a rotational characteristic, the driving pair shall be the rotating pair.

(2) For the parallel mechanism module G_{FPj}, each element should be an input kinematic characteristic of each branched chain which constitutes a parallel mechanism module. If the input

kinematic characteristic of a branched chain is a translational characteristic, the driving joint shall be a prismatic pair or a cylindrical pair. While it is a rotational characteristic, the driving joint shall be a rotating pair or a cylindrical pair.

(3) Each branched chain constituting a parallel mechanism module should only has one actuated joint, that is q_i=1 and the number of passive limbs p=0. Hence, the number of driving arms of the mechanism is equal to the size of characteristics of the mechanism, $n=F_p$.

Criterion for Type Synthesis of the Branches

For a serial mechanism module, the design of decoupled chains for G_{FSi} can be carried out based on the fact that the element T_i, R_j and prismatic pair, revolute pair is a one-to-one relationship. For the parallel mechanism module G_{FPj}, the input kinematic characteristic of each branched chain should be determined based on the selection criterion of the input kinematic pair described in the previous section. For purpose of ensuring the decoupling feature of each branch, the configuration synthesis method of branched chains is as follows:

(1) If the input kinematic characteristic is T_i, the driving joint is a prismatic pair or a cylindrical pair, the axial directions of the remaining prismatic pairs and other cylindrical pairs should be vertical to T_i, two or three revolute pairs with parallel axis in the branch should have one unique axis which is parallel to T_i.

(2) If the input kinematic characteristic is R_j, the actuated joint is a rotating pair or a cylindrical pair, the axes of the remaining revolute or cylindrical pairs in the branch should be perpendicular to the axis of R_j. Due to the fact that decoupling design of the mechanism weakens the stability of the mechanism, the driving joint of the branched chain is preferably located on a fixed platform or a mobile platform.

(3) The axial directions of the input kinematic characteristics T_i connected to the base platform should be perpendicular each other and the axial directions of the input kinematic characteristics R_j connected to the moving platform should be perpendicular each other as well.

Process of the Type Synthesis for Fully-isotropy HMs

From what has been discussed above，type synthesis of full-isotropy HMs via G_F method will be realized, that is:

(1) G_F sets can describe the moving features of HMs. Firstly, through the Eq. (1), the motion features of the series and the parallel mechanism are determined, meanwhile, the formation parameters of HMs will be established according to the Eq. (2). Then, complete type synthesis of series mechanism.

(2) For the G_{FPj} set of the parallel mechanisms to be obtained currently, the moving features of each limb should be determined while the amount of actuated limbs and limb actuators can be achieved by Eq. (2). Furthermore, the type synthesis of each limb can be realized and the branch synthesis method proposed in the previous section, and the actuated joint of each branch chain can be determined on the basis of the selection method of the moving input pair in the previous section.

(3) Based on the decoupling design of the mechanism in steps 1-2, synthesized fully-isotropic mechanism module are connected conforming to Eq. (1) strictly. Type synthesis of the fully- isotropic HMs can be made successfully.

TYPE SYNTHESIS OF 2T3R FULLY-ISOTROPIC HMS

For the purpose of demonstrating the validity of the above theory of type synthesis for fully-isotropic HMs, 2T3R fully-isotropic HMs will be discussed in the following. By the G_F theory, the 2T3R 5-DoF HMs can be described by:

$$G_F^{\mathrm{I}} = \left(T_a\, T_b\, 0; R_{\square} R_\beta\, R_\gamma\right) \tag{3}$$

The Eq. (3) can be simplified as G_{F}^{2T3R}. In accordance with Eq. (1), type synthesis of 2T3R HM based on G_F^{I} can be written in Table 1:

Table 1. Type of compositions for G^{2T3R} fully-isotropic HMs

N	The Combination Form	Type	N	The Combination Form	Type
1	$G_{\mathrm{FP}}^{2T2R} \cup G_{\mathrm{FS}}^{1R}$	P+S	8	$G_{\mathrm{FP}}^{2T1R} \cup G_{\mathrm{FP}}^{2R}$	P+P
2	$G_{\mathrm{FP}}^{2T1R} \cup G_{\mathrm{FS}}^{2R}$		9	$G_{\mathrm{FS}}^{1T} \cup G_{\mathrm{FP}}^{1T1R} \cup G_{\mathrm{FS}}^{2R}$	S+P+S
3	$G_{\mathrm{FP}}^{2T} \cup G_{\mathrm{FS}}^{3R}$		10	$G_{\mathrm{FS}}^{1T} \cup G_{\mathrm{FP}}^{1T2R} \cup G_{\mathrm{FS}}^{1R}$	
4	$G_{\mathrm{FS}}^{1T} \cup G_{\mathrm{FP}}^{1T3R}$	S+P	11	$G_{\mathrm{FS}}^{2T} \cup G_{\mathrm{FP}}^{2R} \cup G_{\mathrm{FS}}^{1R}$	
5	$G_{\mathrm{FS}}^{2T} \cup G_{\mathrm{FP}}^{3R}$		12	$G_{\mathrm{FP}}^{2T} \cup G_{\mathrm{FP}}^{2R} \cup G_{\mathrm{FS}}^{1R}$	P+P+S
6	$G_{\mathrm{FS}}^{2T1R} \cup G_{\mathrm{FP}}^{2R}$		13	$G_{\mathrm{FS}}^{1T} \cup G_{\mathrm{FP}}^{1T1R} \cup G_{\mathrm{FP}}^{2R}$	S+P+P
7	$G_{\mathrm{FP}}^{2T} \cup G_{\mathrm{FP}}^{3R}$	P+P	14	$G_{\mathrm{FP}}^{2T} \cup G_{\mathrm{FS}}^{1R} \cup G_{\mathrm{FP}}^{2R}$	P+S+P

Without loss of generality, a combination form $G_{FS}^{1T} \cup G_{FP}^{1T2R} \cup G_{FS}^{1R}$ in Table 1-(10) is elected as an example, the series connection is G_{FS}^{1T}=(T_a 0 0; 0 0 0) and G_{FS}^{1R}=(0 0 0; 0 R_β 0), respectively, while the parallel mechanism module is G_{FP}^{1T2R}=(0 T_b 0; R_α 0 R_γ). For the serial mechanism module G_{FS}^{1T} and G_{FS}^{1R}, G_{FS}^{1T} is a prismatic pair and G_{FS}^{1R} represent a rotating joint whose axial in line with *Y*-direction, respectively. For the parallel mechanism module G_{FP}^{1T2R}, according Eq. (1), when p=0, n=N=3, the G_{Fi} sets of the three limbs can be constructed as followings:

$$G_{Fi}^{\mathrm{I}} = \left(0\ T_b\ 0\ ;\ R_\alpha\ 0\ R_\gamma\right), \tag{4}$$

$$G_{Fi}^{\mathrm{I}} = \left(0\ T_b\ 0\ ;\ R_\alpha\ R_\beta\ R_\gamma\right), \tag{5}$$

$$G_{Fi}^{\mathrm{I}} = \left(T_a\ T_b\ 0\ ;\ R_\alpha\ 0\ R_\gamma\right), \tag{6}$$

$$G_{Fi}^{\mathrm{I}} = \left(T_a\ T_b\ 0\ ;\ R_\alpha\ R_\beta\ R_\gamma\right), \tag{7}$$

$$G_{Fi}^{\mathrm{I}} = \left(T_a\ T_b\ T_c\ ;\ R_\alpha\ 0\ R_\gamma\right), \tag{8}$$

$$G_{Fi}^{\mathrm{I}} = \left(T_a\ T_b\ T_c\ ;\ R_\alpha\ R_\beta\ R_\gamma\right), \tag{9}$$

Expressions (4)~(5) can be simplified as G_F^{1T2R}, G_F^{1T3R}, G_F^{2T2R}, G_F^{2T3R}, G_F^{3T2R}, G_F^{3T3R}, respectively. Hence, the combination forms of these three limbs constituting a parallel mechanism module G_{FP}^{1T2R}=(0 T_b 0; R_α 0 R_γ), are shown in Table 2. Especially the intersection operation of G_F set should in accordance with the exchange law (Gao et al, 2011), so the sequence of the limbs in Table 2 can be exchanged.

Table 2. Type of compositions for G_{FP}^{1T2R} parallel mechanisms

N	The Combination Form	N	The Combination Form
1	$G_{F1}^{1T2R} \cap G_{F2}^{2T2R} \cap G_{F3}^{3T2R}$	7	$G_{F1}^{1T2R} \cap G_{F2}^{3T3R} \cap G_{F3}^{3T3R}$
2	$G_{F1}^{1T2R} \cap G_{F2}^{2T2R} \cap G_{F3}^{3T3R}$	8	$G_{F1}^{1T3R} \cap G_{F2}^{2T2R} \cap G_{F3}^{3T2R}$
3	$G_{F1}^{1T2R} \cap G_{F2}^{2T3R} \cap G_{F3}^{3T2R}$	9	$G_{F1}^{1T3R} \cap G_{F2}^{2T2R} \cap G_{F3}^{3T3R}$
4	$G_{F1}^{1T2R} \cap G_{F2}^{2T3R} \cap G_{F3}^{3T3R}$	10	$G_{F1}^{1T3R} \cap G_{F2}^{2T3R} \cap G_{F3}^{3T2R}$
5	$G_{F1}^{1T2R} \cap G_{F2}^{3T2R} \cap G_{F3}^{3T2R}$	11	$G_{F1}^{1T3R} \cap G_{F2}^{3T2R} \cap G_{F3}^{3T2R}$
6	$G_{F1}^{1T2R} \cap G_{F2}^{3T2R} \cap G_{F3}^{3T3R}$	12	$G_{F1}^{1T3R} \cap G_{F2}^{3T2R} \cap G_{F3}^{3T3R}$

It is supposed that the first branch is moving forward along *Y* axis, the second and the third turn about the *X* and *Z* axes, respectively, the actuated joint and possible architectures of the branches can be determined according to the method of the input moving joint and the method for type synthesis of the branch. The results are shown in Table 3~8.

Table 3. Enumeration of the branches with the type $G_{F_i}^{1T2R}$

Input	Actuated joint	Type of branched chain		
T_b	P_Y	$P_YR_XR_Z$	$P_YR_ZR_X$	P_YU_{XZ}
R_α	R_X	$P_XR_ZR_X$	$P_a^ZR_ZR_X$	
R_γ	R_Z	$P_XR_XR_Z$	$P_a^ZR_XR_Z$	C_XR_Z

In Table 3, the subscript indicates the direction of motion of the pair, for example, P_Y denotes a prismatic pairs with the axial direction along *Y* axis, R_X a revolute pair around *X* axis, U_{XZ} is a universal joint pair whose two vertical axes are *X* axis and *Z* axis, P_a^Z represent a parallelogram hinge with the axial direction around *Z* axis, and C_X represent a cylindrical pair around *X* axis, the meaning of other symbols is elaborated in document (Yang et al, 2011; Gao et al, 2002,2010).

Table 4. Enumeration of the branches with the type G_{Fi}^{1T3R}

Input	Actuated joint	Type of branched chain		
T_b	P_Y	$P_YR_XR_YR_Z$	$P_YR_YR_XR_Z$	$P_YR_ZU_{XY}$
		$P_YR_ZR_YR_X$	$P_YR_XU_{YZ}$	$P_YU_{XZ}R_Y$
	C_Y	$C_YR_YR_Z$	$C_YR_ZR_Y$	
R_γ	R_Z	$P_XR_ZR_YR_Z$	$P_XU_{YX}R_Z$	$P_a^ZR_ZR_YR_Z$
		^ZU^R_YR_Z	$P_a^ZU_{XY}R_Z$	

In Table 4, when R_α is the input kinematic characteristic, the actuated joint is R_X and the right subscripts *X* and *Z* in the type of branched chain highlighted with blue-colored text should be exchanged to satisfy the demanded input kinematic characteristic R_α.

Table 5. Enumeration of the branches with the type G_{Fi}^{2T2R}

Input	Actuated joint	Type of branched chain		
T_b	P_Y	$P_YP_XR_XR_Z$	$P_YC_XR_Z$	$P_YP_XU_{XZ}$
R_γ	R_Z	$P_XP_YR_XR_Z$	$P_a^ZP_a^ZR_XR_Z$	$U^PR_XR_Z$
R_α	R_X	$R_ZR_ZP_XR_X$	$R_ZP_XR_ZR_X$	$R_ZR_ZC_X$
		$R_ZR_ZR_ZR_X$	$P_a^ZR_ZR_ZR_X$	$P_a^ZR_ZP_a^ZR_X$
		$P_YR_ZR_ZR_X$	$P_YR_ZU_{ZX}$	$R_ZP_YR_ZR_X$

Table 6. Enumeration of the branches with the type G_{Fi}^{2T3R}

Input	Actuated joint	Type of branched chain		
T_b	P_Y	$P_YP_XR_ZR_XR_Y$	$P_YP_XR_ZU_{XY}$	$P_YP_XU_{XY}R_Z$
R_α	R_X	$P_YP_XR_ZR_YR_X$	$R_ZR_ZR_ZR_YR_X$	$R_ZP_YR_ZR_YR_X$
		$P_YP_XU_{ZY}R_X$	$R_ZR_ZU_{ZY}R_X$	$R_ZP_YU_{ZY}R_X$
		$R_ZP_YP_a^ZR_YR_X$	$P_a^ZR_ZR_ZR_YR_X$	$P_a^ZP_a^ZR_ZR_YR_X$
		$U^*R_YR_ZR_X$	$R_ZP_a^ZU_{ZY}R_X$	$P_a^ZP_a^ZU_{YZ}R_X$

In Table 6, when R_γ is selected as the input kinematic characteristic, the driving pair is R_Z, and the right subscripts in the type of branched chain highlighted with blue-colored text should also be exchanged.

Table 7. Enumeration of the branches with the type G_{Fi}^{3T2R}

Input	Actuated joint	Type of branched chain		
T_b	P_Y	$P_YP_XP_ZR_XR_Z$	$P_YP_ZP_XU_{ZX}$	$P_YP_a^YP_a^YR_XR_Z$
		$P_YU^PR_XR_Z$	$P_YU^*U_{ZX}$	$P_YP_a^YP_ZR_XR_Z$
R_α	R_X	$P_a^XP_a^YP_a^ZR_YR_X$	$P_a^XR_YR_YP_XR_X$	$P_a^XP_a^XR_YP_XR_X$
		$P_a^XC_YR_YR_X$	$C_YR_YR_YR_X$	$C_YP_ZR_YR_X$
		$R_YR_Y{\wedge}U^YR_X$	$P_a^XR_Y{\wedge}U^YR_X$	$R_YR_YR_Y{}^XU{\wedge}$
		$U^*P_ZR_YR_X$	$P_a^XU^*R_YR_X$	$P_YU^*R_YR_X$
		$R_XP_ZR_ZR_ZR_Z$	$R_XC_ZR_ZR_Z$	$R_XR_ZC_ZR_Z$
	C_X	$C_XP_YP_ZR_Z$	$C_XP_YC_Z$	$C_XR_ZR_ZR_Z$
		$C_XU^*R_Z$	$C_XP_a^ZP_a^ZR_Z$	$C_XR_ZP_YR_Z$

From Table 7, if the input motion characteristic of the limb G_{Fi}^{3T2R} is R_γ, the actuated joint can be R_Z or C_Z. When actuated joint are R_Z and C_Z, it needs to exchange the subscript X and Z in the type of branched chain.

Table 8. Enumeration of the branches with the type $G3_{Fi}^{T3R}$

Input	Actuated joint	Type of branched chain			
T_b	P_Y	$P_YP_XP_ZS$	$P_YP_XP_ZR_YR_YR_Y$	$P_YP_XC_ZR_YR_Y$	$P_YP_a^YP_a^YU_{XY}R_Z$
		$P_YP_XC_ZU_{YY}$	$P_YR_YR_YU_{YZ}R_X$	$P_YR_ZR_ZR_XR_XR_Y$	$P_YR_YP_a^YP_a^YU_{YX}$
		$P_YR_YR_YR_YR_ZR_X$	$P_Y{}^YU^PR_ZR_YR_X$	$P_Y{}^YU^PU_{YX}R_Z$	$P_YR_YP_Y{}^YU{\wedge}R_Z$
		$P_YP_a^YC_XR_ZR_Y$	$P_YR_YR_Y{}^YU{\wedge}R_Z$	$P_YP_a^YP_a^YS$	
	C_Y	$C_YR_YR_YR_ZR_X$	$C_YP_YR_YU_{ZX}$	$C_YR_YC_XR_Z$	$C_YP_a^YP_a^YU_{ZX}$
		$C_YR_Y{}^YU{\wedge}R_Z$	$C_YR_YU_{YX}R_Z$	$C_YP_a^YR_YR_XR_Z$	$C_Y{}^YU^PR_XR_Z$
R_γ	R_Z	$R_XR_XR_ZR_ZR_ZR_Y$	$P_XR_XR_XR_XR_ZR_Y$	$R_XR_XP_ZR_ZR_ZR_Y$	$P_a^XC_XU_{XY}R_Z$
		$R_YR_YP_XP_ZR_XR_Z$	$R_XR_XR_XR_YR_YR_Z$	$P_XP_YP_ZR_YR_XR_Z$	${}^XU{\wedge}R_XU_{XY}R_Z$
		$R_XP_YR_XR_ZR_ZR_Z$	$R_YR_YP_XR_XR_XR_Z$	$R_YR_YR_YP_YR_XR_Z$	${}^XU{\wedge}P_ZU_{XY}R_Z$
		$C_XR_XR_XR_YR_Z$	$C_XP_ZR_XR_YR_Z$	$P_ZP_YC_XR_YR_Z$	$R_XP_a^XC_XR_YR_Z$
		$C_XR_XU_{XY}R_Z$	$P_ZC_XU_{XY}R_Z$	$U_{XY}C_YR_YR_Z$	$C_XP_a^XP_a^XR_YR_Z$
		$U^*P_ZR_XR_YR_Z$	$U_{XZ}R_YU_{YX}R_Z$	$P_ZU^*U_{XY}R_Z$	$C_XP_a^XR_XR_YR_Z$
		$R_XR_XU_{XY}R_YR_Z$	$P_XR_XR_XU_{XY}R_Z$	$R_XU_{XY}R_YP_XR_Z$	$R_XP_a^XP_a^XP_XR_YR_Z$
		$P_ZU^PR_XR_YR_Z$	$P_XU^PU_{XY}R_Z$	$P_Y{}^PUU_{XY}R_Z$	$P_a^XP_a^XR_XP_XR_YR_Z$
	C_Z	$C_ZU_{XY}U_{YX}$	$R_XU_{XY}R_YC_Z$	$C_ZR_XR_YU_{YX}$	$C_ZU^PU_{XY}$
		$C_ZR_XR_YU_{YX}$	$C_ZU^*U_{XZ}$	$P_a^YP_a^YU_{XY}C_Z$	$C_ZC_YR_YR_X$

In Table 8, when the input kinematic characteristic is R_α, the actuated joint can be R_X or C_X. When actuated joint are R_X and C_X, it needs to exchange subscript X and Z in the type of branched chain as well.

Based on the research described above, firstly, in accordance with Table 2, the topology structure of the parallel mechanism module G_{FP}^{1T2R} can be selected and the type synthesis of each decoupled branch for the parallel mechanism can be fulfilled using Tables 3~8, then a closed-loop fully-isotropic parallel mechanism is obtained. Finally, the serial mechanism module G_{FS}^{1T}, G_{FS}^{1R} and the fully-isotropic parallel mechanism, G_{FP}^{1T2R}, is combined into a 2T3R fully-isotropic HM based on Table 1. One point to be noted is that the same procedure can be applied to any combination style in Table 1, The type synthesis of fully-isotropic 2T3R HMs can be accomplished.

Taking the form of $G_{FS}^{1T} \cup G1_{FP}^{T2R} \cup G_{FS}^{1R}$ in table 1 as an example, the type synthesis of the fully-isotropic 2T3R HMs. a combination form $G_{F1}^{1T2R} \cap G_{F2}^{3T2R} \cap G_{F3}^{3T2R}$ (highlighted with blue-colored text) is chosen from Table 2. On the assumption that the input characteristics of the three limbs followed by T_b, R_α, R_γ, type of branched chain $P_YR_ZR_X$ (highlighted with blue-colored text) is selected as the first limb of the parallel mechanism G_{FP}^{1T2R} from Table 3, $R_XP_ZR_ZR_ZR_Z$ and $R_ZP_XR_XR_XR_X$ in Table 7 are selected as other two limbs, then a fully-isotropic parallel mechanism, G_{FP}^{1T2R}, is achieved, finally a fully-isotropic 2T3R HM is combined whose CAD model is shown in Fig. 3.

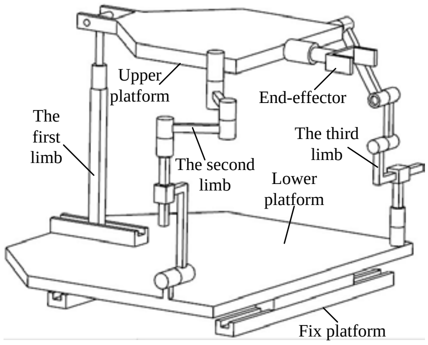

Figure.3 A fully-isotropic 2T3R hybrid mechanism

ANALYSIS OF OUTPUT VELOCITY

As mentioned above, the 2T3R fully-isotropic HMs has been proposed by G_F theory, To verify the validity of the theory, The diagram of mechanism presented in Fig. 4 is taken as the object of analysis, the global reference coordinate is O-XYZ and O is a midpoint of the fix platform, the local coordinate O_1-xyz, where O_1 is the intersecting point of the three rotation axes in the first limb and the distance between the upper platform and the fix platform is h.

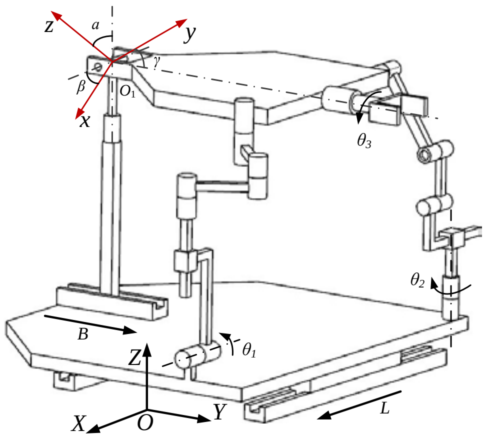

Figure.4 Diagram of mechanism motion parameters

The schematic diagram of mechanism motion parameters are present in Fig. 4. L is the displacement of a fixed platform along the X-axis, The direction indicated by the arrow B is the direction of movement of the branch chain. θ_1, θ_2, θ_3 are respectively the rotation angles of the revolute pairs of each branch shown in Fig. 4. while the output parameters of the HM is described by the end position of the mechanism denoted by (l, b, h) and direction of the end of the mechanism denoted by (α, β, γ). Initiallly, the moving reference system O_1-xyz and the fixed one O-XYZ is parallel, the position and the direction of the end of the mechanism is (l, b, h; α, β, γ)=(l_0, b_0, h; 0, 0, 0). Through the above parameters, the mechanism satisfies the following relationships:

$$\begin{cases} l = l_0 + L \\ b = b_0 + B \\ \alpha = \theta_1 \\ \beta = \theta_2 \\ \gamma = \theta_3 \end{cases}, \tag{10}$$

The velocity relationship between the input drive and the output actuator of the mechanism is:

$$\dot{\boldsymbol{X}} = \boldsymbol{J}\dot{\boldsymbol{q}}, \tag{11}$$

The Eq. (10) takes the derivation of time t:

$$\begin{cases} l' = L' \\ b' = B' \\ \alpha' = \theta_1' \\ \beta' = \theta_2' \\ \gamma' = \theta_3' \end{cases}, \tag{12}$$

Eq. (12) can be written as:

$$\begin{bmatrix} l' \\ b' \\ \alpha' \\ \beta' \\ \gamma' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} L' \\ B' \\ \theta_1' \\ \theta_2' \\ \theta_3' \end{bmatrix}, \tag{13}$$

Where, $\boldsymbol{J}=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

Obviously, the Jacobian matrix $\boldsymbol{J}$ is always an identity matrix indicating that the HM is fully isotropic and also verifying the correctness of the type synthesis theory proposed in the paper. The velocity at the driving joint is mapped to the output velocity of the mechanism, therefore, the problems of path planning and real-time control are potentially simplified.

CONCLUSIONS AND FUTURE WORKS

A very simple but very effective type synthesis approach for fully-isotropic HMs is proposed in this paper by means of G_F set, the method presented in this paper can obtain HMs structures with fully isotropic characteristics. And the velocity at the driving joint is mapped to the output velocity of the mechanism. The 2T3R 5- DoF HMs is illustrated with example to prove the validity of G_F method for fully-isotropic HMs. The forward displacement analysis of a synthesized 2T3R fully-isotropic HM is carried out and the Jacobian matrix is deduced, it shows that the Jacobian matrix is a 5×5 identity matrix which validated the fully-isotropic feature of the HM and the correctness of the G_F approach for fully-isotropic HMs.

Though a very simple but very effective type synthesis approach for fully-isotropic HMs is proposed by G_F method, two questions should be indicated **1):** type synthesis of fully-isotropic 2T3R HMs is accomplished, this work should be extended to more general cases; **2):** The type synthesis of completely isotropic HMs depends on the designers some relevant experience and intuitive perception of the mechanism. At the same time, this work will consume a lot of time and energy of designers. Therefore, relevant software shall be developed to realized the type synthesis design of completely isotropic HMs. All of these mentioned above will be further studied in future.

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