

Working Performance Evaluation of Rolling Bearings Using Modern Statistics

Yongzhi Xu^{*}, Xintao Xia^{**} and Xiang Nan^{***}

Keywords: robust theory; significance level; variation ratio; intrinsic interval; working state

rolling bearing; the fusion method combining the median estimate and Huber (M) estimate of the

ABSTRACT

The performance assessment of time series with unknown distributions, which belongs to the category of problems with poor information, is a key challenge for modern statistics. On the basis of modern statistics, the fusion method of histograms and a normality test to judge the robustness and the direction of unsteady data of time series, and the fusion method combines the median estimate and Huber (M) estimate obtains robust data, unsteady data and the significance level of the time series. These methods are used in the vibration analysis of rolling bearings to verify their effectiveness, and the results show that unsteady data exist in time series at both ends of the order statistics. The reliability reflects the significance level of the rolling bearing vibration data and avoids error due to artificial factors. The intrinsic interval and the variation ratio accurately represent the working performance of rolling bearings, even in cases of complex and diverse running states. Additionally, the above fusion method provides a valuable solution to robustness problem for unknown distribution, the significance level test data and the boundary value of the Huber (M) estimate in modern statistical methods.

Paper Received November, 2017. Revised August, 2018. Accepted August, 2018. Author for Correspondence: Xintao Xia

**. PhD student, School of Mechanical Engineering, Northwestern Polytechnical University, Xi'an 710072, China*

*** Professor and Doctoral Supervisor, School of Mechatronics Engineering, Henan University of Science and Technology, Luoyang 471003, China; Collaborative Innovation Center of Machinery Equipment advanced Manufacturing of Henan Province, Luoyang 471003, China, E-mail: xiact1957@163.com.*

****. PhD student, School of Mechanical Engineering, Northwestern Polytechnical University, Xi'an 710072, China*

of the histogram and normality test provides rules to judge the performance variation of the

Introduction

Research Status

The working performance of a mechanical system has an important influence on the safe and reliable operation of equipment (Chen C, et al., 2007; Sun S H, et al., 2007). The working performance of a rolling bearing, a key part of a mechanical system, may directly influence the reliability and lifetime of a mechanical product. Vibration can reflect errors in bearing manufacturing, installation, and lubrication, affect the bearing's noise, dynamic characteristics, time-life and reliability and represents a topic of wide concern to modern engineering and theory (Xia X, et al., 2007). Many studies have proposed various methods for assessing the vibration performance of rolling bearings. Time domain features, neural networks and spectral analysis (Srividya A, et al., 2009; Castejon C, et al., 2010; Antoni J, 2007) of vibration signals are used in bearing fault diagnosis. The grey bootstrap method is proposed and used in the dynamic assessment and diagnosis of bearing vibration data (Xia X, et al., 2007; Tzu-Li Tien, 2005; Efron B, 1979; Reeves J J, 2005; Yatracos Y, 2002; Deng J, 1989). The Hilbert Huang method is used in the analysis of the vibration properties of rolling bearings (Xiong W, et al., 2006). The phase space method is proposed to analyse the characteristic parameters of rolling bearing vibration (Xia X, et al., 2007). The relationship of vibration and noise and measurements of decreasing vibration and reducing noise are also studied (Banda N, 2003; Wang P, et al., 2005; Arenas J P, 2005; Estocq P, et al., 2006; Xia X, et al., 2005).

The above methods show that the vibration performance of rolling bearings is complex and diverse; does not follow a single distribution function and thus belongs to the category of problems with poor information. However, these methods focus on attaining information from

vibration data and do not consider the robust performance of vibration data. In modern statistics, robust performance of test data is a premise of data analysis, and not considering the robustness of time series can lead to serious results. Therefore, this paper provides a fusion method to analyse the working performance of time series with unknown distributions based on robust theory and fusion theory.

Fusion Theory

Fusion theory originates from the tale of the blind man and the elephant. The tale shows that different conclusions result from different methods, that is, different methods provide

different ways of observing an object. Therefore, the combination of results of multiple methods can more fully represent the characteristics of an object. This theory is named fusion theory, the principle of fusion theory is shown in Fig. 1. However, in practice application, due to the limitations of the adopted method, obtaining the characteristic of the object can lead to error in the data analysis results. A fusion method that alleviates the limitations of some methods is used to process time series according to the characteristics of various methods. The principle of the fusion method is shown in Fig. 2.

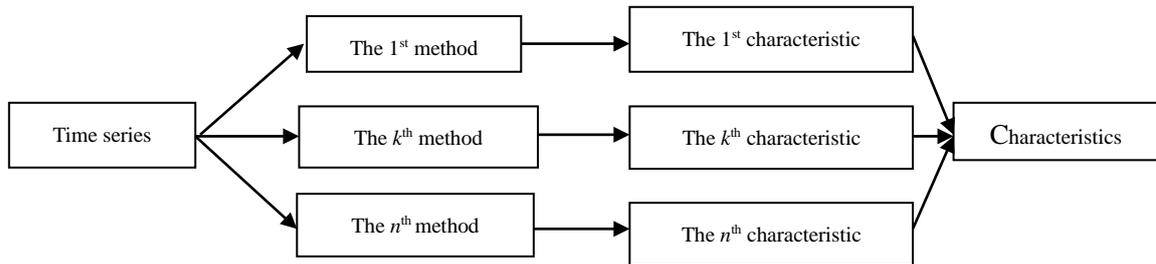


Fig. 1. Principle of fusion theory

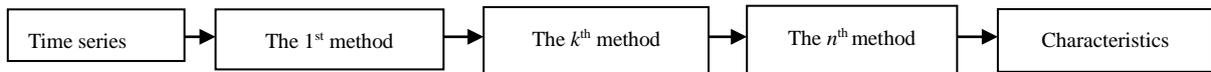


Fig. 2. Principle of the fusion method

Selection of the Fusion Method

Based on modern statistics theory, Fusion Method of Histogram and Normality Test and

combining median estimate and Huber (M) estimate were proposed to analyse working performance of rolling bearings, the flow chart of the working performance of time series are shown in Fig. 3.

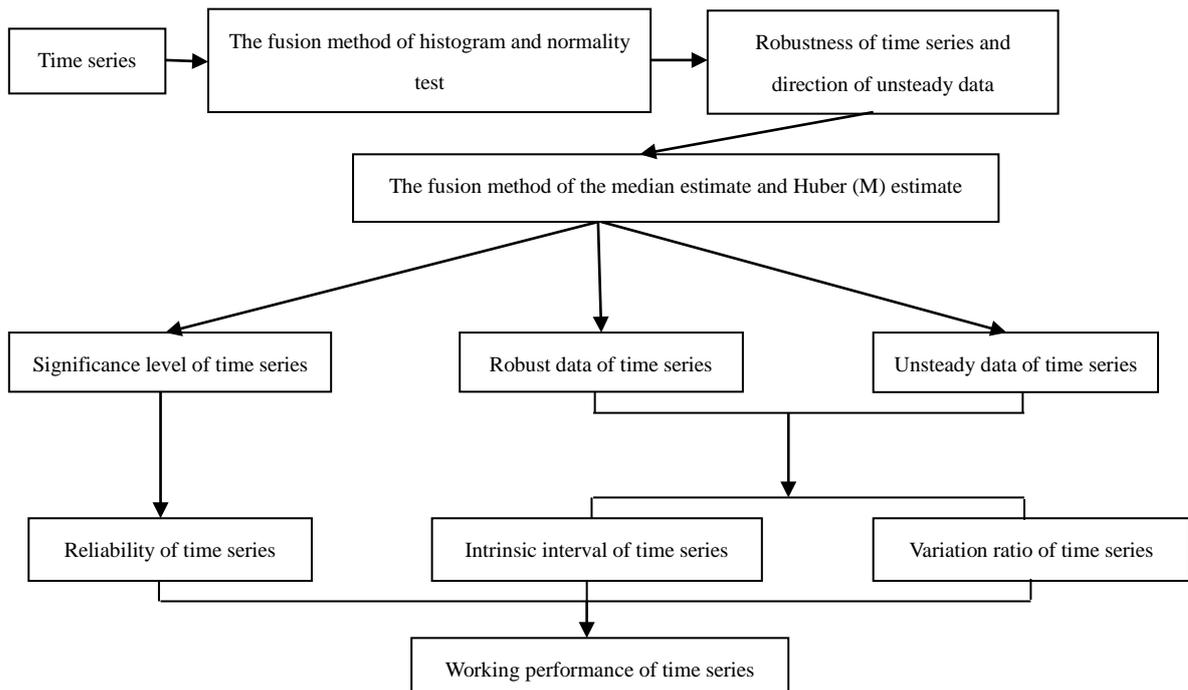


Fig. 3. Flow chart of the working performance of time series

Mathematical Model

Basic Definition

According to the above method analysis, the mathematical model of the fusion method is as follows:

(1) In the real-time monitoring process, suppose that the original time series, which can be represented as a vector \mathbf{X} , output by the measurement system is given by

$$\mathbf{X} = \{x(t)\}; t = 1, 2, \dots, N \quad (1)$$

where \mathbf{X} is the vector of the time series, $x(t)$ is the vibration value at time interval t , t is the time order, and N is the total number of time intervals.

(2) The data of the i^{th} set in \mathbf{X} constitute the vector at moment t , and \mathbf{X}_i is given by

$$\mathbf{X}_i = \{x_i(t)\}; t = 1, 2, \dots, N; i = 1, 2, \dots, m \quad (2)$$

where i is the order number of bearings, \mathbf{X}_i is the vector of the i^{th} set, $x_i(t)$ is the vibration data of time interval t in set i , i is the order of the bearing set, and m is the total number of bearings, and m is the total number of sets.

(3) According to the order from the smallest to the largest data of time series \mathbf{X}_i , order statistic \mathbf{Y}_i is given by

$$\mathbf{Y}_i = \{y_i(n)\}; i = 1, 2, \dots, m, n = 1, 2, \dots, N \quad (3)$$

where \mathbf{Y}_i is the order statistics of \mathbf{X}_i , $y_i(n)$ is the n^{th} data of the i^{th} set.

Mathematical Model of the Fusion Method of Histogram and Normality Test

Frequency of data series \mathbf{X}_i

(1) Calculating poor d_i of data series \mathbf{X}_i , according to order statistics \mathbf{Y}_i .

$$d_i = y_i(N) - y_i(1); i = 1, 2, \dots, m \quad (4)$$

where d_i is poor of data series \mathbf{X}_i , $y_i(N)$ is the N^{th} value of the order statistic of the i^{th} set of rolling bearings, $y_i(1)$ is the 1st value of order statistic of the i^{th} set of rolling bearings.

(2) Group number k of data series \mathbf{X}_i

Based on the N value of data series \mathbf{X}_i , the group number k of \mathbf{X}_i is determined according to N .

(3) Group width p_i of data series \mathbf{X}_i

The group width p_i of data series \mathbf{X}_i is as follows:

$$p_i = \frac{d_i}{k} = \frac{y_i(N) - y_i(1)}{k}; i = 1, 2, \dots, m \quad (5)$$

(4) Group interval of data series \mathbf{X}_i

a is the lower boundary value, which is slightly smaller than $y_i(1)$, and b is the upper boundary value, which is slightly larger than $y_i(N)$; thus, $[a, b]$ of data series \mathbf{X}_i is attained according to a, b .

According to statistics, the group interval of data series \mathbf{X}_i is $(a, a + p_i], (a + p_i, a + 2p_i], \dots, (a + (k-1)p_i, a + kp_i]$.

(5) The frequency ω_i of data series \mathbf{X}_i is given by

$$\omega_i = \{\omega_i(l)\}; i = 1, 2, \dots, m; l = 1, 2, \dots, k \quad (6)$$

Where, ω_i is the frequency of the data series, $\omega_i(l)$ is the frequency of the l^{th} group interval of the data series, i is the order number of bearings, m is the total number of bearings, l is the order of the group interval of the data series, and k is the total number of group intervals.

(6) Histogram of data series \mathbf{X}_i

According to the frequency ω_i of data series \mathbf{X}_i , a histogram of data series \mathbf{X}_i can be obtained.

The robust of data series \mathbf{X}_i can be judged according to whether the histogram follows a normal distribution. If the histogram of data series \mathbf{X}_i follows a normal distribution, then \mathbf{X}_i is robust; otherwise, \mathbf{X}_i is not robust, and a normality test is needed.

A normality test can identify data that do not follow a normal distribution; these data are named non-robust data. The normality test of data series \mathbf{X}_i can judge the direction of non-robust data.

Mathematical Model of the Fusion Method Combining the Median Estimate and Huber (M) Estimate

(1) Median β_i of \mathbf{Y}_i

$$\beta_i = \begin{cases} y_i\left(\frac{N+1}{2}\right), N \text{ is odd} & ; \\ \frac{1}{2}\left(y_i\left(\frac{N}{2}\right) + y_i\left(\frac{N}{2} + 1\right)\right), N \text{ is even} & \end{cases}$$

$$i = 1, 2, \dots, m \quad (7)$$

where β_i is the median of the i^{th} set.

(2) New data series $\mathbf{Z}_i(n_1, n_2)$

Suppose that $y_i(b)$ and $y_i(e)$ are two data points of \mathbf{Y}_i ,

$$y_i(b) \leq \beta_i; i = 1, 2, \dots, m \quad (8)$$

$$\beta_i \leq y_i(e); i = 1, 2, \dots, m \quad (9)$$

The number of data points from $y_i(b)$ to β_i is n_1 ,

and the number of data points from β_i to $y_i(e)$ is n_2 ;

If $y_i(n) \leq y_i(b)$, then

$$y_i(b) = y_i(n); \quad i = 1, 2, \dots, m \quad (10)$$

If $y_i(n) \geq y_i(e)$, then

$$y_i(e) = y_i(n); \quad i = 1, 2, \dots, m \quad (11)$$

Then, new data series $Z_i(n_1, n_2)$ is constituted by

$$Z_i(n_1, n_2) = \{z_i(n; n_1, n_2)\}; \quad i = 1, 2, \dots, m$$

$$n = 1, 2, \dots, N \quad (12)$$

(3) Mean $\eta(n_1, n_2)$ of robust data series $Z_i(n_1, n_2)$

$$\eta_i(n_1, n_2) = \frac{1}{N} \sum_{n=1}^N z_i(n; n_1, n_2); \quad i = 1, 2, \dots, m$$

$$n = 1, 2, \dots, N \quad (13)$$

(4) The absolute value $D_i(n_1, n_2)$ of the difference between median β_i and mean $\eta(n_1, n_2)$

$$D_i(n_1, n_2) = |\beta_i - \eta_i(n_1, n_2)|; \quad i = 1, 2, \dots, m \quad (14)$$

n_1 and n_2 can be chosen according to the specific requirements; in this paper,

$$n_1 = n_2 = 1, 2, \dots, \frac{N}{2} \text{ or } \frac{N+1}{2}, n \text{ is even or odd} \quad (15)$$

(5) Boundary values $K_{i,1}$ and $K_{i,2}$

$K_{i,1}$ and $K_{i,2}$ are the boundary values of the series absolute difference $D_i(n_1, n_2)$ obtained according to the absolute value of the difference between the median β_i and mean $\eta(n_1, n_2)$ of equation (13). $D_{i,min}(n_1, n_2)$ is the min value of $D_i(n_1, n_2)$, and the corresponding $y_i(b)$ and $y_i(e)$ are $K_{i,1}$ and $K_{i,2}$:

$$K_{i,1} = y_i(b); \quad i = 1, 2, \dots, m \quad (16)$$

$$K_{i,2} = y_i(e); \quad i = 1, 2, \dots, m \quad (17)$$

Therefore, n_1 and n_2 are determined according to $D_{i,min}(n_1, n_2)$.

(6) Intrinsic interval $[K_{i,1}, K_{i,2}]$

The intrinsic interval $[K_{i,1}, K_{i,2}]$ can be obtained according to equations (13) and (14). $[K_{i,1}, K_{i,2}]$ is the intrinsic interval of the time series and reflects the working performance of the time series. A smaller value indicates better working performance of the time series. $K_{i,1}$ and $K_{i,2}$ are the boundary values of the intrinsic interval of the i^{th} rolling bearing, i is the order of the rolling bearing, and m is the total number of rolling bearings.

Thus, $[K_1, K_2]$, n_1 and n_2 are determined according to the above $D_{i,min}(n_1, n_2)$.

$$\alpha_i = (1 - \frac{n_1 + n_2}{N})\%; \quad i = 1, 2, \dots, m \quad (18)$$

The range of significance level α_i is $[0, 0.1]$, where α_i is the significance level of the i^{th} bearing.

The mathematical model of the fusion method of the median estimate and Huber (M) estimate is given as follows:

$$P_i = \{p_i(t)\} = \begin{cases} K_{i,1} & x_i(t) < K_{i,1} \\ x_i(t) & K_{i,1} \leq x_i(t) \leq K_{i,2} \\ K_{i,2} & x_i(t) > K_{i,2} \end{cases}$$

$$i = 1, 2, \dots, m, \quad t = 1, 2, \dots, N \quad (19)$$

Index of Robust Rolling Bearing Data

According to modern statistical theory, unsteady test data lead to an increase in the absolute value of the difference between the largest and smallest value, the variance and the absolute value of the difference between the mean and the median of a data series. Therefore, the absolute value of the difference between the largest and smallest value, the variance and the absolute value of the difference between the mean and the median of a data series can reflect the robustness of a data series.

(1) The absolute value W_i of the difference between $\max x_i(t)$ and $\min x_i(t)$ of data series X_i is determined as follows:

$$W_i = |\max x_i(t) - \min x_i(t)|, \quad i = 1, 2, \dots, m \quad (20)$$

The absolute value W_{Ri} of the difference between $K_{i,2}$ and $K_{i,1}$ of robust data series Y_i is determined as follows:

$$W_{Ri} = |K_{i,2} - K_{i,1}|, \quad i = 1, 2, \dots, m \quad (21)$$

(2) The variance S_i of data series X_i is given by

$$S_i^2 = \frac{1}{N} \sum_{t=1}^N (x_i(t) - A_i)^2, \quad i = 1, 2, \dots, m, \quad t = 1, 2, \dots, N$$

$$(22)$$

$$S_i = \sqrt{S_i^2}, \quad i = 1, 2, \dots, m, \quad t = 1, 2, \dots, N \quad (23)$$

$$A_i = \frac{1}{N} \sum_{t=1}^N x_i(t), \quad i = 1, 2, \dots, m, \quad t = 1, 2, \dots, N \quad (24)$$

The variance S_{Ri} of robust data series X_i is given by

$$S_{Ri}^2 = \frac{1}{N} \sum_{t=1}^N (p_i(t) - B_i)^2, \quad i = 1, 2, \dots, m, \quad t = 1, 2, \dots, N \quad (25)$$

$$S_{Ri} = \sqrt{S_{Ri}^2}, \quad i = 1, 2, \dots, m, \quad t = 1, 2, \dots, N \quad (26)$$

$$B_i = \frac{1}{N} \sum_{t=1}^N p_i(t), \quad i = 1, 2, \dots, m, \quad t = 1, 2, \dots, N \quad (27)$$

T (3) The absolute value U_i of the difference between the mean A_i and median β_i of

data series X_i is as follows:

$$U_i = |\beta_i - A_i|, i = 1, 2, \dots, m \quad (28)$$

The absolute value U_{Ri} of the difference between the mean B_i and median β_i of robust data series X_i is as follows:

$$U_{Ri} = |\beta_i - B_i|, i = 1, 2, \dots, m \quad (29)$$

Assessment of the Working Performance of Time Series with Unknown Distributions

(1) The variation ratio V_i of a time series can be obtained according to equation (30):

$$V_i = \frac{|\max x_i(t) - K_{i,2}| + |K_{i,1} - \min x_i(t)|}{B_i}$$

$$i = 1, 2, \dots, m \quad (30)$$

A larger V_i indicates larger performance variation of the time series. $\max x_i(t)$ is the largest value of data series X_i in the i^{th} set of rolling bearings, and $\min x_i(t)$ is the smallest value of data series X_i in the i^{th} set of rolling bearings.

(2) The reliability U_i of time series X_i can be obtained according to equation (31):

$$U_i = (1 - \alpha_i)\% \quad i = 1, 2, \dots, m \quad (31)$$

U_i is the reliability of the vibration data of a rolling bearing and reflects the degree of confidence of time series X_i : a larger U_i indicates higher confidence.

(3) The intrinsic interval $[K_{i,1}, K_{i,2}]$ of the time series can be obtained according to equation (6) in section 1.3:

where $[K_{i,1}, K_{i,2}]$ is the intrinsic interval of the time series and reflects the working performance of time series X_i :

Test

To test the above assessment results of the fusion method for the working performance of rolling bearings, the vibrations of 3 sets of tapered roller bearings (30204) are measured with special test equipment (designed by Henan University of Science and Technology). The objective of the test is to assess the working performance of rolling bearings.

Test Conditions

The testing environment is clear, and the foundation of the test equipment lacks vibration. The environmental temperature is 20°C, and the relative humidity is < 70%.

Test Theory

As shown in Fig. 4, in the measuring device, a tapered 30204 roller bearing is installed on the drive shaft, which is installed in the inner ring, is driven by the motor. The axial load is loaded by the force of the end face of the outer ring, and in the experiment, the vibration data are acquired by means of the speed of the outer ring. The speed signal of the vibration of the rolling bearing is collected by a B1010 acceleration

sensor located in the bearing's outer ring radius. The inspection standards are JB/T10236-2001, and the testing method follows JB/T5313-2001.

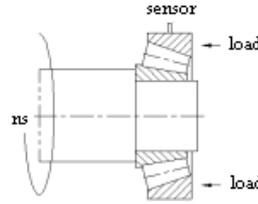


Fig. 4. Measurement principle for rolling bearing vibration

Test Scheme

The test parameters are as follows:

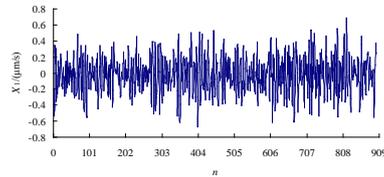
(1) The rotating speed of the test equipment was 1800 r/min. The axial load was 60 N.

(2) The time interval of the test data was 0.03 s. A total of 901 vibration data points of a tapered 30204 roller bearing were collected according to the test requirements.

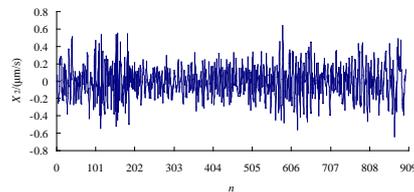
Analysis of the Vibration Data of Rolling Bearings

Robustness of the Test Data

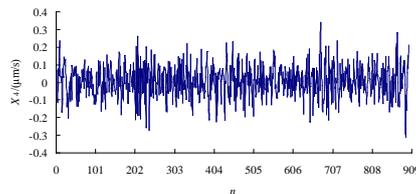
The vibration of 3 sets of tapered 30204 roller bearings is tested over time using the above test scheme. The vibration data are shown in Fig. 5.



(a) First rolling bearing



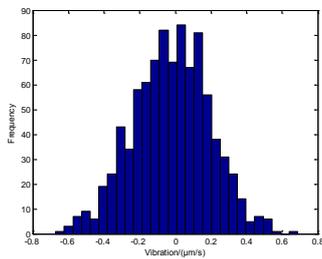
(b) Second rolling bearing



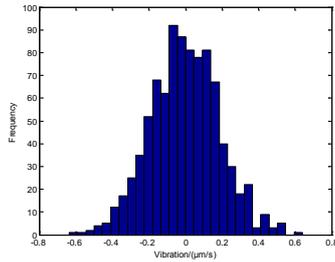
(c) Third rolling bearing

Fig. 5. Rolling bearing vibration data

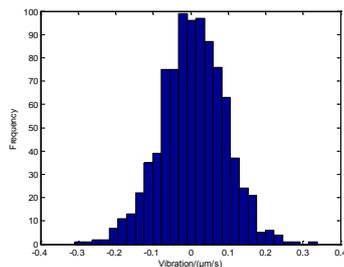
The vibration of the 30204 rolling bearings is shown in Fig. 5. Vibration data of the first and second rolling bearings are in the interval [-0.7, 0.7], and the data of the third rolling bearing are in the interval [-0.4,0.4]. As time passes, the vibration of the first, second and third bearings increases slightly, indicating a change in the performance of the tapered 30204 roller bearings. The type of change is not easy to identify. To study the performance of the vibration data of a tapered 30204 roller bearing, the data distribution of the test data is studied via histogram. The results are shown in Fig. 6.



(a) First rolling bearing



(b) Second rolling bearing

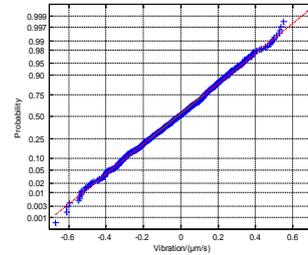


(c) Third rolling bearing

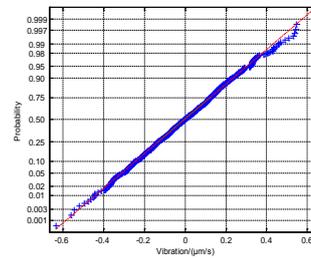
Fig. 6. Histograms of the rolling bearing vibration data

A total of 901 vibration data points are collected, which surpasses the 800 required according to modern statistics, so the vibration data can be considered to follow a normal distribution. If the vibration data do not follow a normal distribution, then the vibration data are not robust, that is, the performance of the rolling bearing varies. According to the results in Fig. 6,

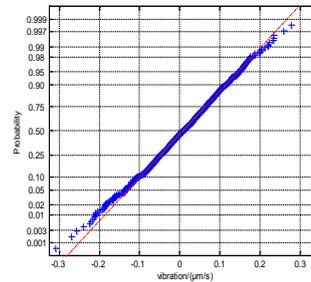
the histogram of the rolling bearing vibration does not agree with the normal distribution; thus the vibration is non-robust. To assess the performance variation of the rolling bearing, the vibration data must be tested for normality to determine the extent of non-robust data. The results of the normality test are shown in Fig. 7.



(a) First rolling bearing



(b) Second rolling bearing



(c) Third rolling bearing

Fig. 7. Normality tests of the rolling bearing vibration

If the probability of vibration data deviates from the line representing a normal distribution, then that part of the vibration data is not robust. According to the results in Fig. 7, non-robust data are observed at both ends of the distribution. On the basis of the extent of non-robust data, the rolling bearing vibration data are robustly processed via the fusion method of the median estimate and the Huber (M) estimate. The parameter $D_i(n_1, n_2)$ represents the robustness of the vibration data: and smaller $D_i(n_1, n_2)$ indicates more robust vibration data. The results of $D_i(n_1, n_2)$ are shown in Fig. 8.

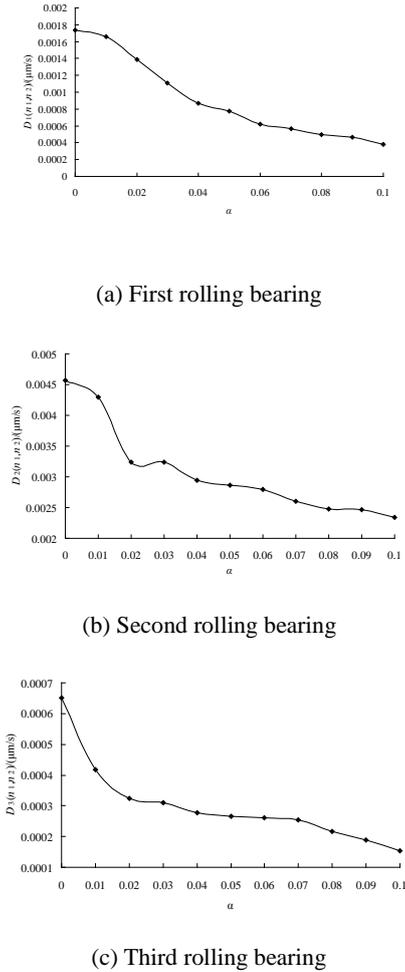


Fig. 8. Comparison of $D_i(n_1, n_2)$ of the rolling bearing vibration data

As the significance level increases, $D_i(n_1, n_2)$ for the first, second and third bearings decreases, showing that the robustness of the vibration data increases with increasing significance level. According to the fusion method combining the median estimate and the Huber (M) estimate, a smaller $D_i(n_1, n_2)$ value corresponds to more robust vibration data, so the significance level of the first, second and third bearings is 0.1.

According to the 0.1 significance level of the first, second and third bearings, based on the fusion method that combines the median estimate and the Huber (M) estimate, the boundary value is calculated according to equations (10)-(11) and (16)-(17). Robust rolling bearing vibration data can be obtained according to the boundary

values. The results of the robustness of the test data are shown in Fig. 9.

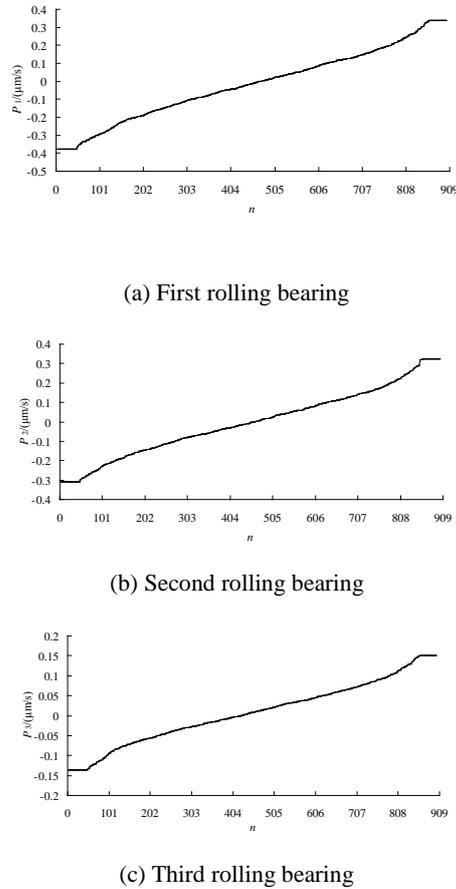


Fig. 9. Distribution functions of the rolling bearings under the 0-0.1 significance level

At a significance level of 0.1, the results in Fig. 9 show the distribution functions of the first, second and third bearings, and the robust test data have a median at the centre, exhibit continuity, and are not decreasing, which meets the requirement of the fusion method combining median estimate and Huber (M) estimate.

Characteristics of Robust Data

To characterize the robust data of the vibration data of a rolling bearing, according to equations (20)-(29), the discreteness, stability and robust parameters of the robust data are listed in Table 1.

Table 1. Parameters of the robust rolling bearing vibration data

| No | Discreteness parameter | | Stability parameter | | Robustness parameter | |
|----|--------------------------|-----------------------|--------------------------|-----------------------|--------------------------|-----------------------|
| | $W_{Ri}/(\mu\text{m/s})$ | $W_i/(\mu\text{m/s})$ | $S_{Ri}/(\mu\text{m/s})$ | $S_i/(\mu\text{m/s})$ | $U_{Ri}/(\mu\text{m/s})$ | $U_i/(\mu\text{m/s})$ |
| 1 | 0.714156 | 1.360751 | 0.196915 | 0.214754 | 0.000379 | 0.001734 |
| 2 | 0.629924 | 1.276149 | 0.170684 | 0.188037 | 0.002336 | 0.004569 |
| 3 | 0.284889 | 0.647092 | 0.077466 | 0.086610 | 0.000154 | 0.000652 |

According to the results in Table 1, W_{Ri} , S_{Ri} and S_{Ri} of the robust data are smaller than W_i , S_i and S_i of the vibration data, indicating that the discreteness, stability and robustness of the robust data are better than those of the vibration data, according to equations (20)-(29). For the robust vibration data, the intrinsic interval and variation ratio of the rolling bearings are shown in Table 2. The reliability of the 2 bearings is 90%, indicating that the test data are high quality and reflect the characteristics of rolling bearings.

The variation ratio of the first bearing is smaller than that of the other two bearings, showing that the variation performance of the first bearing is the best. The working performance of the first bearing does not change easily. The intrinsic interval of the third bearing is smaller than that of the other two bearings, indicating that the running performance of the third bearing is the best. Therefore, the third bearing is suitable for high-precision equipment, and the first bearing is suitable for equipment with high reliability.

Table 2. Assessment parameters of the rolling bearing vibration data

| No | Reliability $U_i(\%)$ | Intrinsic interval [$K_{i,1}$, $K_{i,2}$] ($\mu\text{m/s}$) | Variation ratio $V_i (\%)$ |
|----|-----------------------|--|----------------------------|
| 1 | 90 | [-0.37909, 0.336541] | 30.72014 |
| 2 | 90 | [-0.31033, 0.325981] | 123.0462 |
| 3 | 90 | [-0.13672, 0.151494] | 52.01116 |

Discussion

The fusion method of histogram and the normality test of modern statistics provide a solution to assess the robustness of test data with an unknown distribution. Because the total number of data points is 901, according to statistics, the distribution of the test data can be assumed to follow a normal distribution. The results of the vibration data in Fig. 6 and Fig. 7 show confirm this assumption. Furthermore, the results of the normality test in Fig. 7 show that both ends of the order statistics of the vibration data deviate from the normal distribution line, and these data are unsteady data.

A fusion method combining the median estimate and Huber (M) estimate, which do not require information about the distribution and trend of the test data, is proposed to solve robustness problems of rolling bearing vibration data. According to robust theory, the median is robust to the data, so the median can be regarded as an indicator of the robustness. According to the test results of the first, second and third bearings in Fig. 10, the means of the vibration data do not equal the medians. Therefore, the test data contain unsteady data or discrete values that need a robust analysis process.

In addition, the results of the first, second and third bearings in Fig. 8 show that the fusion method combining median estimate and Huber (M) estimate is correct. To reflect the boundary value of the robust test data, the significance level is introduced using the fusion method that combines the median estimate and the Huber (M) estimate. The range of the significance level is determined according to the data requirements. In this paper, the range of the significance level is typically 0-0.1. According to the robust

principal of the mean being close to the median, $D_i(n_1, n_2)$, which is the absolute value of the difference between the mean and median of the data series in the 0-0.1 significance level range, is a robust standard for rolling bearing vibration data. According to the min $D_i(n_1, n_2)$ value, the significance level and boundary value can be determined by equations (10)-(11) and (16)-(17). Robust vibration data of a rolling bearing are attained by equation (19), and the results of the robust data are shown in Fig. 9. The discreteness, stability and robustness of the robust data are better than those of the vibration data according to equations (20)-(29), as shown in Table 1. Therefore, the fusion method can obtain robust test data.

According to the analysis results of the fusion method combining the median estimate and the Huber (M) estimate, the intrinsic interval of the test data is obtained, and the intrinsic interval reflects the working characteristics of the rolling bearing. The variation ratio, which reflects unsteady data and robust data, can represent the variation characteristics of the working performance of the rolling bearing. Both reflect that excellent working characteristics and reliability are expected for the working state of a rolling bearing; however, in practice, both values are not usually available. Studies show the presence of complexity and diversity. The working characteristics of some rolling bearings are good according to the intrinsic interval. However, the performance variation is very serious, according to the variation ratio, and the performance of other rolling bearings is opposite. The working characteristics and variation characteristics commonly reflect the working state of a rolling bearing.

Conclusion

Fusion theory and the fusion method can be used to obtain additional information to compensate for a lack of information by making full use of the data series information, thereby providing a solution for poor information systems.

Based on modern statistics, the fusion method of histogram and normality test is proposed to judge the robustness and the direction of unsteady data of data series, as shown in Figs. 8 and 9, when the number of samples reaches a certain value.

The fusion method combining the median estimate and Huber (M) estimate obtains robust

data, unsteady data and the significance level of data series and provides a valuable solution for robustness problems of test data with unknown distributions, significance levels and boundary values of the Huber (m) estimate in modern statistical methods.

The reliability, intrinsic interval and variation ratio constitute an assessment system of the working state and reflect the reliability characteristics of the test data and the working performance of rolling bearings.

The working performance of rolling bearings is complex and diverse and provides a way to select a suitable rolling bearing based on the requirements of different working states.

Conflict of interest

The authors declare that there is no conflict of interest regarding the publication of this manuscript.

Acknowledgement

This project is supported by the National Natural Science Foundation of China (Grant No. 51475144) and the Natural Science Foundation of Henan Province of China (Grant No. 162300410065).

References

- Chen C, Chen F, Huang J, Huang C, "Study of Interactive Bike Simulator in Application of Virtual Reality," *Journal of the Chinese Society of Mechanical Engineers*, 28(6): 633-640 (2007).
- Sun S H, Chen J L, "Knowledge Representation and Reasoning Methodology based on CBR Algorithm for Modular Fixture Design.," *Journal of the Chinese Society of Mechanical Engineers*, 28(6): 593-604 (2007).
- Xia X, Chen X, Zhang Y, et al, "Fuzzy prediction for vibration and noise of rolling bearing based on poor information.," *Journal of Jilin University (Engineering and Technology Edition)*, 37(6): 1341-1345 (2007).
- Srividya A, Verma A K, Sreejith B, "Automated Diagnosis of Rolling Element Bearing Defects Using Time-domain Features and Neural Networks.," *International Journal of Mining, Reclamation and Environment*, 23(3): 363-366 (2009).
- Castejon C, lara o, carcia-Prada J C, "Automated Diagnosis of Rolling Bearings Using MRA and Neural Networks.," *Mechanical Systems and Signal Processing*, 24(1): 289-199 (2010).
- Antoni J, "Cyclic Spectral Analysis of Rolling-element Bearing signal Facts and Fictions.," *Journal of Sound and Vibration*, 304: 497-529 (2007).
- Xia X, Chen X, Zhang Y, et al, "Dynamic assessment and diagnosis of vibration of rolling bearing using grey bootstrap.," *Journal of Aerospace Power*, 22(1): 156-162 (2007).
- Tzu-Li Tien, "A Research on the prediction of machining accuracy by the deterministic grey dynamic model DGDM (1, 1, 1).," *Applied Mathematics and Computation*, 161: 923-945 (2005).
- Efron B., *Bootstrap methods*, *The Annals of Statistics*. 7: 1-36 (1979).
- Reeves J J, "Bootstrap prediction intervals for ARCH models," *International Journal of Forecasting*, 21: 237-248 (2005).
- Yatracos Y, "Assessing the quality of bootstrap samples and of the bootstrap estimates obtained with finite resampling.," *Statistics and Probability Letters*, 59: 281-292 (2002).
- Deng J, "Introduction to grey system theory," *The Journal of Grey System*, 1(1): 1-24 (1989).
- Xiong W, Zhao G, *Analysis of Bearing Vibration Characteristics*, "Using Hilbert-Huang Transformation," *Chinese Journal of Sensors and Actuators*, 19(3): 655-658 (2006).
- Xia X, Chen X, Zhang Y, et al, "Maximum Entropy Probability Distribution and Bootstrap Inference of Space Bearing Friction Moment.," *Journal of Astronautics*, 28(5): 1395-1399 (2007).
- Banda N, "Noise and vibration in rolling bearings.," *Journal of Japanese Society of Tribologists*, 48(1): 43-48 (2003).

- Wang P, Liao M, "Adaptive demodulated resonance technique for the rolling bearing fault diagnosis," Journal of Aerospace Power, 20(4): 607-612 (2005).
- Arenas J P, "Enhancing the vibration signal from rolling contact bearing using an adaptive closed-loop feed-back control for wavelet de-noising.," Journal of Mechanical Engineering, 51(4): 184-193 (2005).
- Estocq P, Bolaers F, Dro J P, "Method of de-noising by spectral subtraction applied to the detection of rolling bearing defects," Journal of Vibration and Control, 12(2): 197-211 (2006).
- Xia X, Xie T, Sun L, et al, Theory and practice of noise of rolling bearings. China Machine Press, Beijing (2005).
- Shi Y, Xu Y, Zhou B, Modern Statistical Methods, vol 4. High Education Press, Beijing (2011).

用近代統計學評估滾動 軸承工作性能

徐永智 夏新濤 南翔

西北工業大學機電學院

河南科技大學機電工程學院

河南科技大學機械裝備先進製造河南省協

同創新中心

摘要

未知分佈時間序列的性能評估屬於乏資訊範疇，是近代統計學中的重要問題，一種融合方法被提出評估未知分佈時間序列的工作性能。基於近代統計學，採用長條圖與正態性檢驗相融合方法判斷時間序列的穩健性及不穩健資料方向；採用中位數估計與 Huber (M)估計相融合方法獲取時間序列的穩健資料、不穩健資料以及顯著性水準。上述方法對滾動軸承振動資料分析，結果顯示滾動軸承振動資料中存在不穩健資料並且在次序統計量兩端；在工作狀況複雜性多樣性條件下，可靠度可以避免人為誤差反映時間序列的置信水準；本征區間和變異率可以很好的評估滾動軸承的工作性能和變異性能。此外，長條圖與正態性檢驗融合方法給出滾動軸承性能變異的判斷準則，中位數估計與 Huber (M)估計相融合方法為分佈未知、置信水準未知試驗資料的穩健處理以及 Huber (M)估計的邊界值確定提供了一種方法。