

Workspace Analysis and Optimal Design of 3-PRR Planar Parallel Manipulators

M.R. Shivakumar, R. Srikrishna, R. Sriram, SreeSailesh, Manoj Kumar Reddy, Teja Prakash and Anjan Kumar Dash*

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ABSTRACT

In the optimum design of parallel manipulators, workspace of the manipulator is of greater importance. The shape and area of the workspace are the main parameters under this. In this paper, a new geometrical approach is presented to determine the shape and size of the constant orientation workspace for the 3-PRR planar parallel manipulators. All possibilities of shapes of workspaces are determined with variation of different parameters. For each shape of workspace corresponding geometrical conditions are also put forth. Closed form area expression of workspace is derived by geometrical approach for each shape. Such closed form expression of area is not possible with non-dimensional approach. This becomes extremely useful during optimal design procedure. A *look-up table* is also presented seeing which the designer can choose geometrical conditions between different parameters which will ensure a void free workspace. A case study is presented wherein a user gives his required workspace area and an algorithm is presented which gives all possible combinations of geometrical parameters satisfying the workspace area requirement. Then based on various considerations including singularity analysis an optimal parallel manipulator is offered for the task which does not have any void within the workspace having least/nil singularities.

INTRODUCTION

A parallel manipulator is a closed-loop kinematic mechanism whose end effector is linked to

the base by several independent kinematic chains (Merlet, 2006). A parallel robot is defined as a robot which is made up of an end-effector with n degrees of freedom, and a fixed base, linked together by at least two independent kinematic chains. Actuation takes place through n simple actuators. Parallel robots for which the number of chains is strictly restricted to the number of DOF of the end effector are called fully parallel manipulators (Chablat and Wenger, 2007). There are two main cases in the analysis of fully parallel manipulators. They are: planar manipulators and spatial manipulators. In the present paper we consider 3-PRR planar manipulator. The closed kinematic chain has potential applications where maneuverability and requirement of workspace is low but dynamic loading is severe and high speed and precision motions are of paramount importance (Yang et al., 2019). The applications of parallel manipulators include force-torque sensor, machining center, aircraft simulator, earthquake simulator, pointing device, mechanism design, path planning, conformational trajectory planning of proteins (Madden et al., 2009) and in many other cases (Yang et al., 2017) etc. The 3-PRR planar parallel manipulator, due to its inherent stiffness and accuracy, and less inertia of moving links is a suitable mechanism for high-speed and high-accuracy robotic applications as a planar positioning and orientation device (Ren, 2005; Wang, 2005 and Xu et al., 2017).

Workspace determination of parallel manipulators has been described in many papers. In Bohigas et al. (2012), a complete method for determination of workspace boundary is described on general structure manipulators by using a branch and prune technique. Study of singular configurations of a three DOF planar parallel mechanisms with three identical legs is described in Bonev et al. (2003). Dynamics and vibration control of a 3-PRR parallel manipulator with three flexible links is analyzed by generating dynamic equations (Zhang, 2009). It also studies the buckling effect on links and developing an active vibration control strategy. A methodology was described in Binaud et al. (2010) to obtain sensitivity coefficients of the pose of the moving platform of the manipulator to variation in geometric parameters and actuated variables. A methodology to enlarge the workspace of parallel manipulators by using non-singular transitions has been

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* Professor, School of Mechanical Engineering, SASTRA Deemed University, Thanjavur, Tamil Nadu-613401, India

described in Arana (2012). Dynamics of a 3-PRR manipulator has been explained in many papers as in Staicu (2008).

In case of 3-DOF planar parallel manipulators, it is extremely difficult to determine the workspace area with the help of non-dimensional parameters especially when the shape of workspace is very complex as shown in (Merlet, 2006; Arockia, 2014; Bonev, 2000; Merlet, 2006; Merlet et al., 1998 and Gao et al., 2001). Especially in case of 3-PRR planar parallel manipulator, the workspace shape becomes very complicated with combinations of straight lines, circles and arcs and consequently determination of area also becomes very difficult. Sometimes, researchers adopt to approximate algebraic or numerical method to determine the workspace area in the absence of such closed form expression of workspace area (Hrishi, 2010). One disadvantage of such approach is that it would be computationally tasking and involves error.

A typical optimum design cycle for a parallel manipulator involves following steps: 1). specification of workspace area by the user and other specifications; 2). determination of possible parallel manipulators (combination of link lengths) satisfying this area and other constraints; 3). determination of singularity points in each possible parallel manipulator; 4). select the one with least singularity points.

Singularity analysis of parallel manipulators in general and specifically that of 3-PRR planar parallel manipulator has been addressed by many researchers (Bonev and Gosselin, 2001; Firmani and Podhorodeski, 2005; Huijing et al., 2018; Paskevich et al., 2006; Liu et al., 2004; Yang et al., 2018; Masouleh and Gosselin, 2006). Workspace analysis of this manipulator has also been addressed (Xu et al., 2017; Bonev et al., 2003 and Merlet et al., 1998), however shapes of the workspace with variation in the value of the parameters has not been reported. Also closed form expression of workspace area of 3-PRR planar parallel manipulators has not been reported so far. Such closed form expression of workspace area becomes very useful for parallel manipulators because they are mostly proposed for high accuracy applications.

In this paper, effect of all possible variation of link lengths on the shape and area of workspace (constant orientation) of a 3-PRR planar parallel manipulators is studied and specific mathematical conditions for each workspace shape is determined. For each shape of the workspace, workspace area is also calculated in a geometrical way in closed form expression. Such geometrical expressions for determination of workspace area are very helpful for optimal design of such manipulators. A flow chart is presented to determine optimal parallel manipulator(s) for a given workspace area which free of singularities. A case study is given to verify this.

KINEMATICS OF A 3-PRR MANIPULATOR

As shown in Fig. 1, each kinematic chain of such a manipulator is of PRR type. The triangular shaped platform $C_1C_2C_3$ is to be oriented by the manipulator. The triangle $A_1A_2A_3$ forms the base.

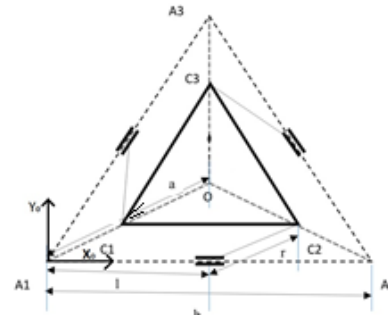


Figure 1: 3-PRR planar parallel manipulator

Generally, the workspace is determined on different parameters as follows.

l =maximum length that each slider can move

r = radius of the revolute link

b = side length of the base triangle

a = distance from the centroid of the platform to its vertex of the mobile platform

Determination of constant orientation workspace is shown in a step by step method in Fig. 2 using Vertex space method (Bonev and Gosselin, 2002). Figure 2(a) shows the line diagram of the same manipulator as in Fig. 1 and Fig.2(b) shows the individual workspace of each PRR chain i.e. WS_1 is the workspace of 1st serial chain (A_1 -slider- C_2); WS_2 is the workspace of 2nd serial chain (A_2 -slider- C_3) and WS_3 is the workspace of 3rd serial chain (A_3 -slider- C_1). Then, these workspace are moved in a direction determined by the orientation of the mobile platform by a distance 'a' as shown in Fig. 2(c). Consequently, the intersection of the three workspaces make up the total workspace of the manipulator as shown in Fig. 2(d). This procedure is followed in all the cases and please note that in all the cases, the location of the coordinate system is at A_1 i.e. at the left vertex of the base triangle. Also it is to be observed that to determine the actual area of the workspace and have a comparison amongst them, all the figures are to be drawn to a fixed scale which is not the case here; however the numerical value of area of workspace can be determined from the algebraic expression given for area for each workspace.

Each parameter has its own effect on the shape and, hence, area of workspace. The variation of parameter is classified as follows:

Case 1: $l=b$

Case 2: $l < b$

Case 3: $l > b$

Among all the three cases, $l=b$ is the simple one because two parameters are taking only one value. The third case of $l > b$ is physically impractical to fabricate; however, this case is also investigated for academic interest and, as it would be found out, there exists

definite workspace even when $l > b$. In general, for every set of links on the PRR, maximum workspace is drawn and void area is subtracted from it. The intersection of workspaces of each link gives rise to total workspace of the manipulator. This is done for each and every individual case. The intersection area is

determined by the integration of intersecting areas. In all the subsequent figures, workspace is generated without showing the step by step procedure and in each case, coordinate system is chosen at left vertex of the base platform.

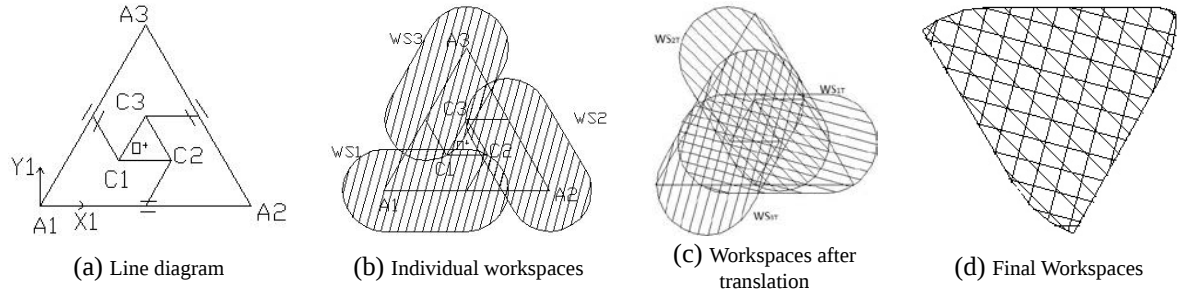


Figure 2: Generation of constant orientation workspace for 3-PRR manipulator

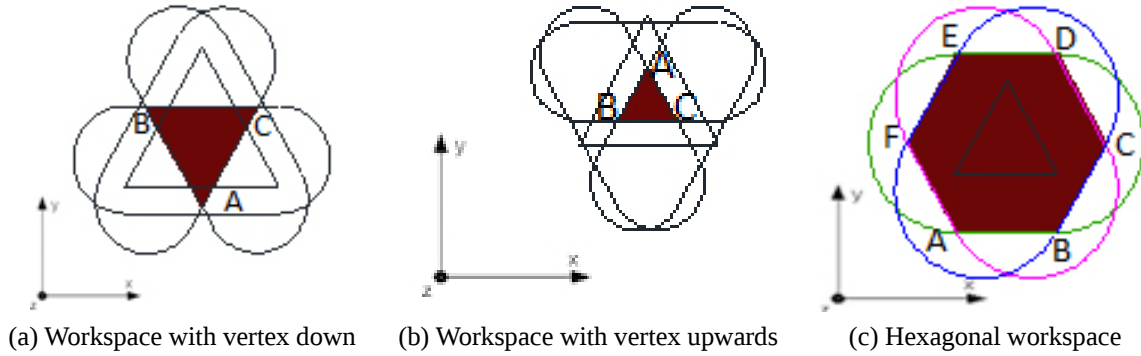
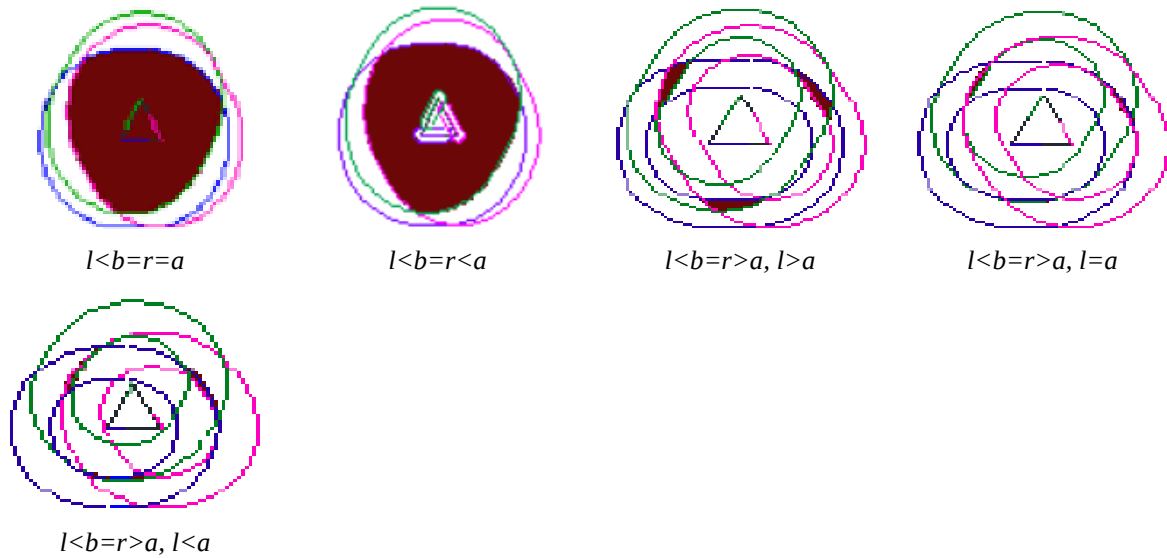


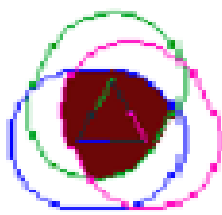
Figure 3: Workspace shapes for case 1 ($l=b$)

Table 1: Workspace shapes when $l < b$

● For $b=r$



● For $b > r$



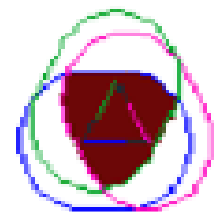
$$l < b > r = a, l = r$$



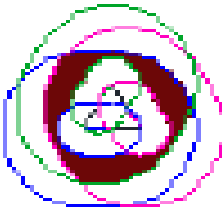
$$l < b > r < a, l < r, b > a$$



$$l < b > r > a, a = l < r$$



$$l < b > r = a, l > r$$



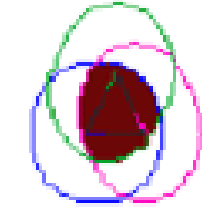
$$l < b > r < a, l = r, b = a$$



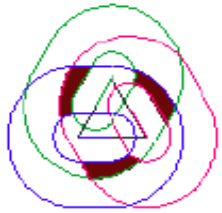
$$l < b > r < a, l = r, b > a$$



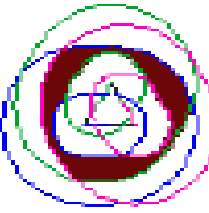
$$l < b > r > a, l < a < r$$



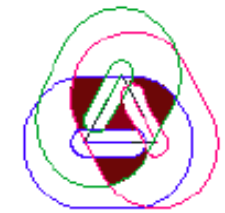
$$l < b > r = a, l < r$$



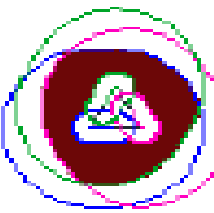
$$l < b > r > a, a < l < r$$



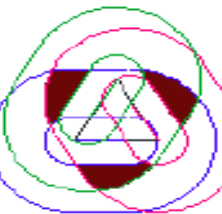
$$l < b > r < a, l = r, b < a$$



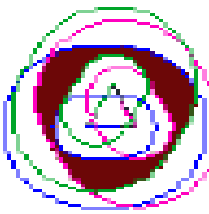
$$l < b > r > a, a < r < l$$



$$l < b > r < a, l < r, b = a$$



$$l < b > r > a, a < l = r$$



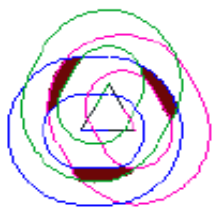
$$l < b > r < a, l = r, b < a$$



$$l < b > r < a, l > r, b > a$$



$$l < b > r < a, l > r, b < a$$



$$l < b > r < a, l > r, b = a$$

● FOR $b < r$



$$l < b < r > a, a = b > l$$



$$l < b > r = a$$



$$l < b < r > a, a > b > l$$



$$l < b < r > a, a > b > l$$



$$l < b < r > a, b > l > a$$

NOTE:

1. The colors green, pink and blue represent the workspace for each PRR chain.
2. The maroon color represents the workspace of the manipulator.
3. The empty space represents the void

CASE 1: When $l=b$

In this case, workspace of triangle shape starts to form when the lines are moved by a distance greater than $\frac{l}{2\sqrt{3}} - r$ i.e.; ($a > \frac{l}{2\sqrt{3}} - r$) and beyond $\frac{l}{2\sqrt{3}} + r$ i.e., ($a > \frac{l}{2\sqrt{3}} + r$) there is no more workspace, thus in this case ($l = b$) workspace is formed only when $\frac{l}{2\sqrt{3}} - r < a < \frac{l}{2\sqrt{3}} + r$. Three types of workspaces are formed before “ a ” reaches $\frac{l}{2\sqrt{3}} + r$

CASE 1.1: A triangle with its vertex downwards as shown in Fig.3(a) ($\frac{l}{2\sqrt{3}} - r < a$)

$$\text{Area} = \frac{\sqrt{3}}{4} * BC^2 = \sqrt{3}(a+r)^2 + \frac{\sqrt{3}}{4}l^2 - 3l(a+r) \quad (1)$$

CASE 1.2: A triangle with its vertex upwards when $\frac{l}{2\sqrt{3}} + \frac{r}{3} < a < \frac{l}{2\sqrt{3}} + r$ as shown in Fig.3(b)

$$\text{Area} = \frac{\sqrt{3}}{4} * BC^2 = \sqrt{3}(a-r)^2 + l^2 - 3l(a-r) \quad (2)$$

CASE 1.3: In between, when $\frac{l}{2\sqrt{3}} - \frac{r}{3} < a < \frac{l}{2\sqrt{3}} + \frac{r}{3}$, a hexagonal workspace is formed as shown in Fig.3(c). Different points are obtained by solving equations of different lines and the total area is obtained by determining the area of two trapeziums.

$$\text{Area} = 6al + 2lr - 4\sqrt{3}ar - \frac{\sqrt{3}}{2}l^2 - 6\sqrt{3}a^2 + \frac{2r^2}{\sqrt{3}} \quad (3)$$

CASE 2: ($l < b$):

All the workspace shapes under this case are shown in the following table-1.

Drawing workspaces of Table 1

It is to be noted that all the workspaces shown in Table 1 are constant orientation workspace and they are drawn using vertex space method as described in the beginning of Section 2 (refer Fig. 2). Existence of workspace depends upon two conditions:

- $r + a \geq \sqrt{3}b$: This is the geometric condition for the existence of the 3-PRR parallel manipulator. Unless this condition is satisfied, the parallel manipulator cannot be constructed.
- When $a < r$, l : This is the condition where the radius of the mobile platform is very small compared to other dimensions such that when the individual workspaces are advanced following the vertex space method there is no common intersection among the

individual workspace. This results in a no workspace situation.

In Table 1 there are some places where it is mentioned ‘workspace does not exist’ and that refers to either of the above two cases.

Case 3: $l > b$

Although case $l > b$ is physically not preferable, for academic interest this case is presented in Table 2.

DETERMINATION OF CLOSED FORM EXPRESSION OF WORKSPACE AREA BY GEOMETRICAL METHOD

From section 2, it can be inferred that the general shape of the workspace obtained is as shown in Fig. 4.

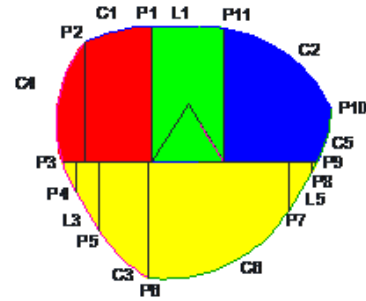


Figure 4: General shape of the workspace

The points P1, P2..., P11 are the points that form the vertices of the workspace. They are obtained by solving the below given combinations using the standard formulae for circle-circle intersection or circle-line intersection:

P1: $C1 \cap L1$	P7: $C6 \cap L5$
P2: $C1 \cap C4$	P8: $C5 \cap L5$
P3: $C4 \cap X_{axis}$	P9: $C5 \cap X_{axis}$
P4: $C4 \cap L2$	P10: $C2 \cap C5$
P5: $C3 \cap L3$	P11: $C2 \cap L1$
P6: $C3 \cap C6$	

The total area is separated into four parts as shown by red, green, blue and yellow colors above. The area of the red colored part is given by

$$\Delta 1 = \int_{Py3}^{Py2} C4(\text{interms of } x = f(y))dy + \int_{Py2}^{Py1} C1(\text{interms of } x = f(y))dy \quad (4)$$

The area of the green colored part is given by:

$$\Delta 2 = \text{area of rectangle} = l(r + a) \quad (5)$$

The area of blue colored part is given by:

$$\Delta 3 = \int_{Px11}^{Px10} C2(\text{interms of } y = f(x))dx - \int_{Px9}^{Px10} C5(\text{interms of } y = f(x))dx \quad (6)$$

The area of the yellow colored part is given by

$$\Delta 4 = \int_{Px3}^{Px4} C4(\text{interms of } y = f(x))dx + \int_{Px4}^{Px5} L3(\text{interms of } y = f(x))dx +$$

$$\int_{P_{x5}}^{P_{x6}} C3(\text{interms of } y = f(x))dx + \int_{P_{x7}}^{P_{x8}} L5(\text{interms of } y = f(x))dx + \int_{P_{x8}}^{P_{x9}} C5(\text{interms of } y = f(x))dx + \int_{P_{x6}}^{P_{x7}} C6(\text{interms of } y = f(x))dx \quad (7)$$

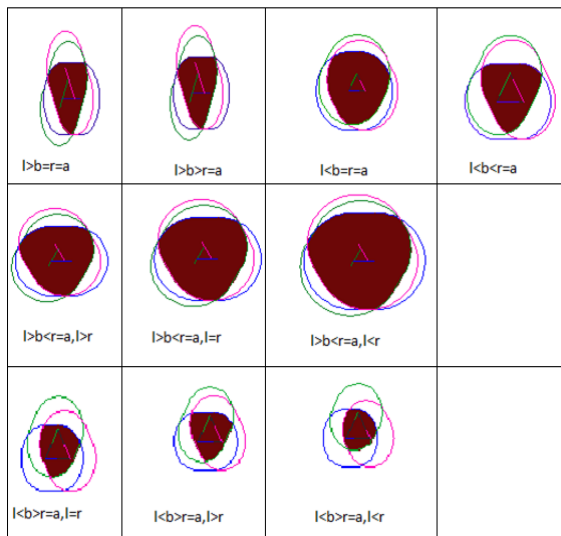
$$\text{Total area is given by: } \Delta = \Delta 1 + \Delta 2 + \Delta 3 + \Delta 4 \quad (8)$$

Now, observing the shapes of the workspace as described in section 2, it can be classified into following categories.

a) Workspace having no void within it

Under this category comes the following cases:

Table 3: Workspace shapes having no voids within it



b) Workspace shapes having voids in it, but the voids do not intersect with each other

This is as shown in Fig.5:

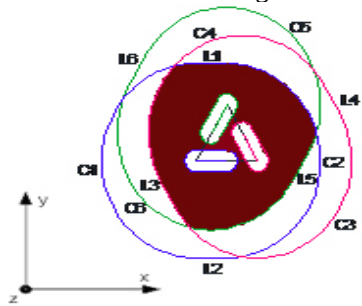


Figure 5: Shape of the workspace having non-intersecting voids

The area of void caused for each serial chain is same for each of the chains. The void area is given by:

$$\Delta' = 3[\pi(r-a)^2 + l(r-a)] \quad (9)$$

So, the workspace area can be determined by subtracting the area as given in Eqn. (9) from that of Eqn. (8).

c) The voids are intersecting-Type 1 (Fig. 6a)

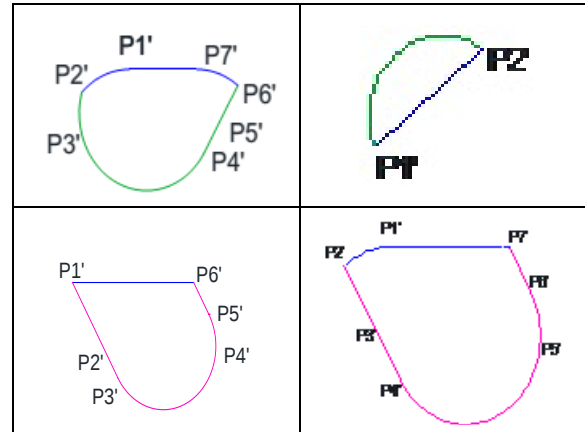
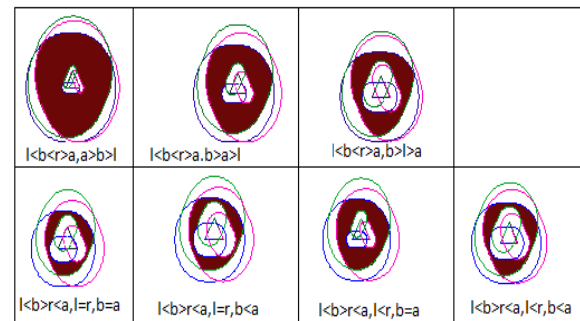


Figure 6: Types of voids when they are intersecting

Table 4: Workspace shape having voids of type 1;



The area of above shape of type 1 void is given

by:

$$\Delta'' = \int_{P_{y2'}}^{P_{y1'}} C1(\text{interms of } x = f(y))dy + \int_{P_{x1'}}^{P_{x7'}} L1'(\text{interms of } y = f(x))dx + \int_{P_{x6'}}^{P_{x7'}} C2'(\text{interms of } y = f(x))dx + \int_{P_{x4'}}^{P_{x7'}} C6'(\text{interms of } y = f(x))dx + \int_{P_{y3'}}^{P_{y2'}} C6'(\text{interms of } y = f(x))dx + \int_{P_{x4'}}^{P_{x5'}} L5'(\text{interms of } y = f(x))dx$$

Now, the total void area is given by:

$$\Delta' = 3[\pi(r-a)^2 + l(r-a)] - 3\Delta''' \quad (10)$$

d) The voids are intersecting of type 2 (Fig. 6(b))

The intersected shape is as shown in Fig.6(b). The area of the shape in Fig.8(b) is given by:

$$\Delta'' = \int_{P_{x1'}}^{P_{x2'}} C4' - L3'dx$$

Total void area is given by:

$$\Delta' = 3[\pi(r-a)^2 + l(r-a)] - 3\Delta''' \quad (11)$$

The cases under this category are shown in table 5.

e) The voids are intersecting of type 3 (Fig. 6(c))

The area of this shape is given by:

$$\Delta'' = \int_{P_{x1'}}^{P_{x6'}} L1'dx + \int_{P_{x6'}}^{P_{x5'}} L4'dx + \int_{P_{x4'}}^{P_{x5'}} C3'dx + \int_{P_{x4'}}^{P_{x3'}} C3'dx + \int_{P_{x2'}}^{P_{x3'}} L3'dx - \int_{P_{x1'}}^{P_{x2'}} L3'dx \quad (12)$$

The cases under this category are shown in table 6:

Table 5: Workspace shape having voids of type 2

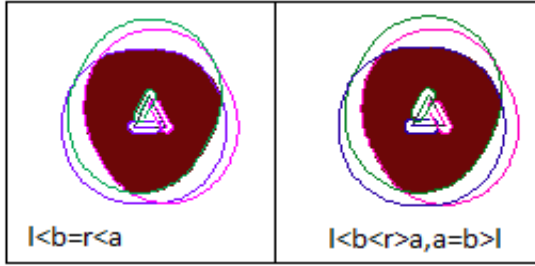
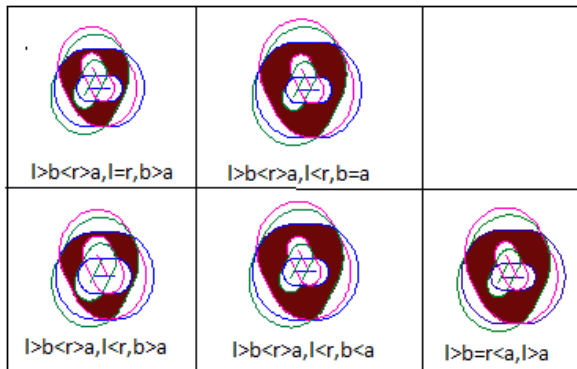


Table 6: Workspace with voids of type 3



Table 7: Workspace with voids of type-4



f) The voids are intersecting of type 4 (Fig. 6(d))

The area of this shape is given by:

$$\Delta'' = \int_{P_{x1'}}^{P_{x7'}} L1'dx + \int_{P_{x7'}}^{P_{x6'}} L4'dx + \int_{P_{x6'}}^{P_{x5'}} C3'dx + \int_{P_{x5'}}^{P_{x4'}} C3'dx + \int_{P_{x4'}}^{P_{x3'}} L3'dx + \int_{P_{x3'}}^{P_{x2'}} C4'dx - \int_{P_{x2'}}^{P_{x3'}} L3'dx \quad (13)$$

The cases under this category are shown in Table 7.

g) The voids are intersecting of type 5 (Fig. 7)

The area of this shape is given by:

$$\Delta'' = \int_{P_{x2'}}^{P_{x1'}} C1'dx + \int_{P_{x1'}}^{P_{x6'}} L1'dx - \int_{P_{x5'}}^{P_{x6'}} L5'dx - \int_{P_{x4'}}^{P_{x5'}} C6'dx + \int_{P_{x3'}}^{P_{x4'}} C6'dx - \int_{P_{x2'}}^{P_{x3'}} C6'dx \quad (14)$$

The case under this category is shown in Fig.8.

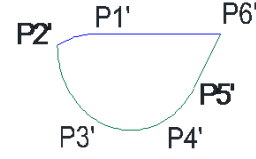


Figure 7: Intercepted shape of the voids-type 5

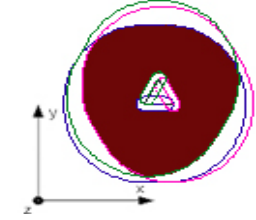


Figure 8: The workspace with void of type 5

h) Void area exceeding the workspace area

Remaining cases in which the void area is exceeding the workspace area are as shown in Fig.9 and Fig. 10.

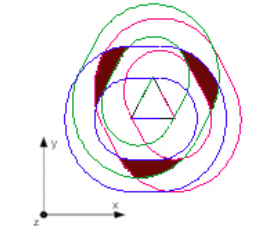


Figure 9: Workspace under case (h) (having 3 symmetrical sections)

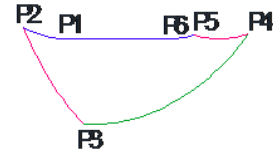


Figure 10: Shape of each section of Fig.9

The workspace area can be divided into three symmetric parts; each part is as shown in Fig.10.

$$\Delta = 3 \left[\int_{P_{x2}}^{P_{x3}} C3'dx + \int_{P_{x3}}^{P_{x4}} C6'dx - \int_{P_{x4}}^{P_{x5}} C3'dx - \int_{P_{x5}}^{P_{x6}} C2'dx - l(r - a) - \int_{P_{x2}}^{P_{x1}} C1'dx \right] \quad (15)$$

The cases under this category are shown in table 8 and 9 in two parts.

Table 8: Workspace under case (h)_Part 1

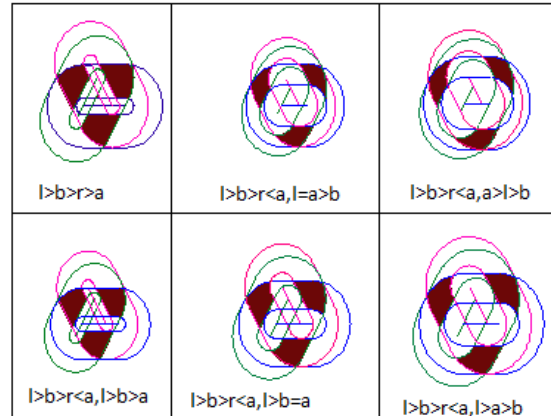
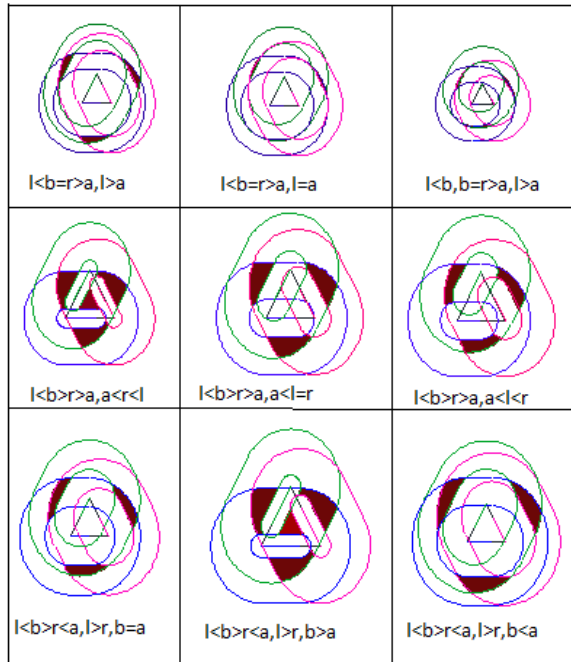


Table 9: Workspace under case (h)_Part 2



PARAMETER OPTIMIZATION

Normally, user comes with a requirement of workspace area and the designer has to give an optimized parallel manipulator which has no/least number of singularities. In section 3, geometrical and closed form expressions of all kinds of shapes of workspace areas are determined and based on that individual parameters can be optimized.

However, it can be decided that workspace shapes in which there is void at the center of the workspace are not much preferable compared to those where there is no void. Considering this, the possible conditions of the parameters, as can be determined from Table 1 and 2 are as follows:

Table 10: Conditions of parameters for no void area

Sl No	Mathematical Condition
1	$l > b = r = a$
2	$l > b > r = a$
3	$l < b < r = a$
4	$l > b < r = a$
5	$l > b < r = a (l = r)$
6	$l > b < r = a (l > r)$
7	$l > b < r = a (l < r)$
8	$l < b > r = a (l = r)$
9	$l < b > r = a (l < r)$
10	$l < b > r = a (l > r)$

For all other cases, there would be voids in the workspace, hence not preferable. This is the great advantage of the *look-up table* looking at which one

can determine which geometrical conditions of the parameters can give the designer a workspace free of voids. The workspace area in all the above cases is determined in the following way (please refer to Fig. 6).

$$\Delta = A1 + A2 + A3 + A4 \quad (16)$$

where

1. $A1 = \int_{P2}^{P1} \sqrt{(r+a)^2 - (y - \frac{\sqrt{3}l}{2})^2 + (b - \frac{l}{2}) + \sqrt{(r+a)^2 + y^2}} dy$
2. $A2 = l(r+a)$
3. $A3 = \int_{P11}^{P10} \frac{(x-l)\sqrt{((r+a)^2 + (x-l)^2)}}{2} dx + \int_{P10}^{P9} \left[\frac{(x-b/2)\sqrt{(r+a)^2 - (x-b/2)^2}}{2} + \frac{(r+a)^2}{2} \sin^{-1} \left(\frac{x-b/2}{r+a} \right) + \frac{\sqrt{3}}{2} bx \right] dx$
4. $A4 = \int_{P7}^{P8} \left[\frac{\sqrt{3}x^2}{2} - 2(r+a)x \right] dx + \int_{P4}^{P5} \left[\sqrt{3}b - 2(r+a)x - \frac{\sqrt{3}x^2}{2} \right] dx + \int_{P8}^{P9} \left[\sqrt{(r+a)^2 - (x-b/2)^2} + \sqrt{3}b/2 \right] dx + \int_{P6}^{P7} \left[\frac{\sqrt{3}(b-l)}{2} x \right] dx + \int_{P5}^{P6} \left[\sqrt{(r+a)^2 - (x-l)^2} \right] dx + \int_{P8}^{P9} \left[\sqrt{(r+a)^2 + (x-b/2)^2} + \frac{\sqrt{3}b}{2} \right] dx$

The algorithm that is followed to determine the optimal parameters of the parallel manipulator suitable for a given workspace area is described in Fig.11.

In this algorithm, the designer first gets the required workspace area from the user. Then he varies the parameters l , r , b and a within certain ranges (different ranges for different parameters). Then he checks which values of these parameters satisfy the conditions mentioned in Table 10. By this he ensures that for these values of the parameters there is no voids inside the workspace. Out of these sets of parameters, he eliminates those cases in which $l > b$ considering feasibility of fabricating such manipulators. Then from the remaining sets of the values, he applies the Eqn. 16 to determine the workspace and selects those values of the parameters for which the workspace area satisfies the required workspace area considering the factor of safety depending upon the type of application, or any other consideration as demanded by the user. These sets of values are the feasible values of the parallel manipulators which will give the required workspace area without any void in the workspace. Then he applies singularity conditions to each one of these parallel manipulators to determine the best parallel manipulator having least or no singularity points within the workspace.

The formulation of the optimization algorithm is as follows:

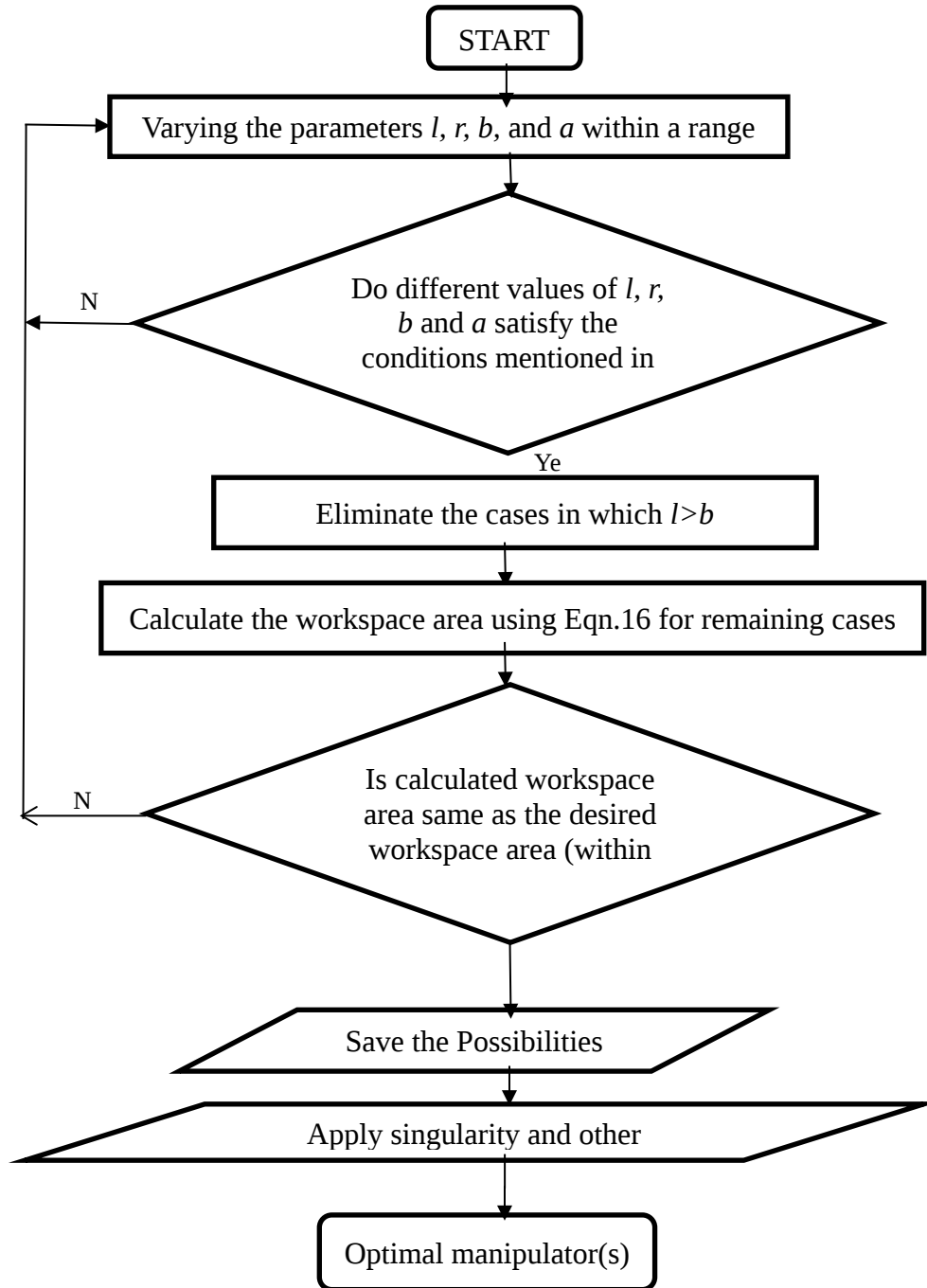


Figure 11: Flowchart describing the algorithm which is followed to determine the optimal parallel manipulator

maximize $f(x) = R_w$
 over $x = [l \ r \ b \ a]^T$
 subject to $g_1: r+a \geq \sqrt{3}b$
 $g_2: 0 < l < b$
 $g_3: \text{Abs}(\det(J)) \geq 0.001$
 $x_{lb} \leq x \leq x_{ub}$, where x_{lb} and x_{ub} are the
 lower and upper bounds of x , respectively
 Here x = set of design variables l, r, b, a as mentioned
 in section 2.
 R_w =Workspace of the manipulator as described in
 Eqn. (16).

The constraints g_1 and g_2 are related to the geometric

conditions required for the assembly of the manipula-
 tor. These are the basic requirements which are
 required for the assembly of the 3-PRR manipulator
 without which it cannot be assembled. Constraint g_3 is
 to eliminate conditions of singularity. And finally the
 design variables vary between set of lower bound and
 upper bound.

CASE STUDY

In the case study, the user wants to 3-PRR planar parallel manipulator having workspace area of 300 sq.

units. The task is thus to give him a 3-PRR planar parallel manipulator having following features:

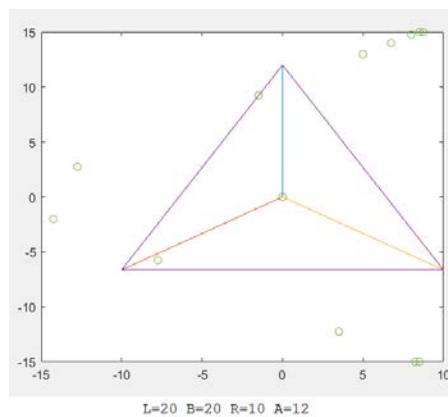
- The workspace area is 300 ± 10 square units (This 10 square units depends upon the user's requirement. If it is a precision application then we may give zero tolerance. The algorithm has the capability to work on such close tolerances)
- All the proposed parallel manipulators do not have any void within the workspace.
- The proposed optimal parallel manipulator(s) has zero or least number of singularities.
- The solution gives us the optimal length of l , r , b and a . They are respectively the maximum displacement of the slider at the base joint, the length of the intermediate link, the base length of the base platform assumed to be a triangle and the radius of the top platform.

Based on the flowchart shown in figure 14, the following sets of parameters satisfy the workspace area constraint and each one of them are having no void within the workspace. Table 11 shows this result.

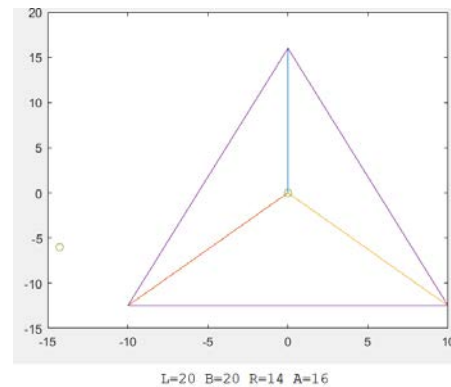
Table 11: Optimal parameters matching with workspace area requirement and having no voids within

Sl No.	l (max slider length in unit)	b (base length of the triangular base platform in unit)	r (link connecting to slider to the mobile platform in unit)	a (radius of the mobile platform in unit)	Workspace area (unit ²)
1	20	20	6	8	301
2	20	20	8	10	301
3	20	20	10	12	301
4	20	20	12	14	301
5	20	20	14	16	301
6	20	20	16	18	301
7	20	20	18	20	301
8	10	50	10	10	281
9	50	50	12	12	307

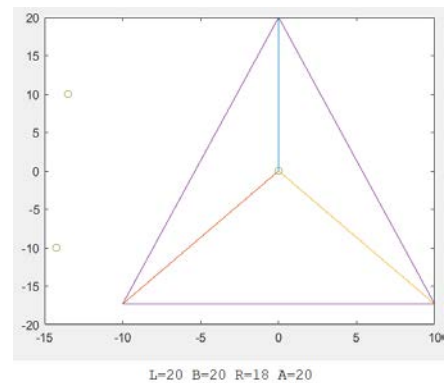
Out of the 9 shown parallel manipulators, serial numbers 3, 5, 7, 8, 9 are chosen for the singularity analysis. The rest are all very close to these dimensions. Singularity analysis for 3-PRR planar parallel manipulators are very much studied in the literature. So based on that singularity analysis is performed and the singularities are plotted in the following figure 12. The circles in the figure represent the forward singularities, the triangle represents the base platform with the three lines connecting the vertices to the centroid of the base platform.



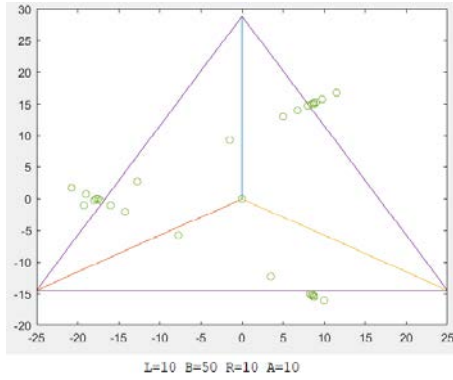
(a) Singularities of manipulator of Sl. No. 3



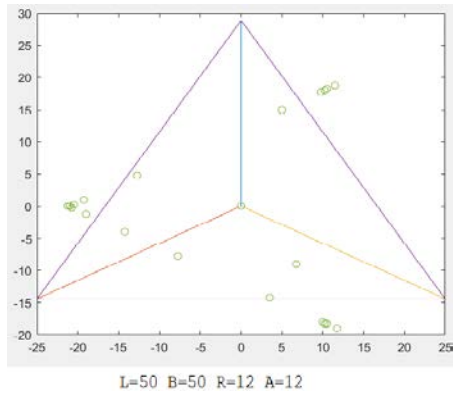
(b) Singularities of manipulator of Sl. No. 5



(c) Singularities of manipulator of Sl. No. 7



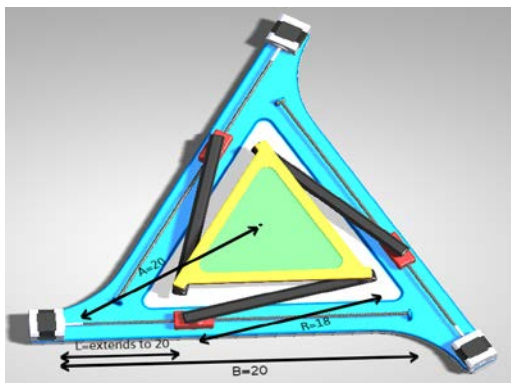
(d) Singularities of manipulator of Sl. No. 8



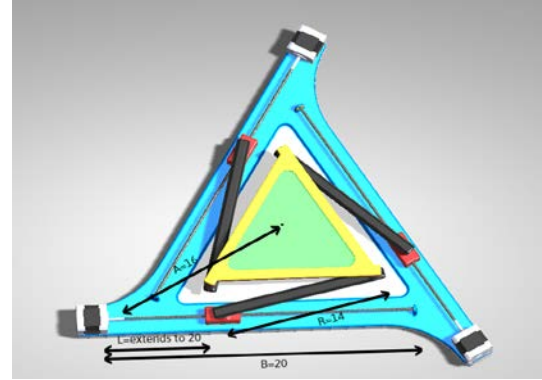
(e) Singularities of manipulator of Sl. No. 9

Figure 12: Singularity distribution of selected manipulators from table 11 (Sl. No 3, 5, 7, 8 and 9)

Considering the number of singularities, except Sl. No. 3, 8 and 9 (Fig. 12 (a), (d), (e)), the other two parallel manipulators can be proposed to the user as suitable parallel manipulators for his application requiring 300 sq. units area. These two manipulators are shown in Fig. 13 (a) and (b) respectively. Out of them Sl. No. 5 (Fig.13(a)) contains two singularities and Sl. No. 7 (Fig.13(b)) contains 3 singularities only. Both these manipulators can be proposed to be optimal parallel manipulators to the user for his required workspace area.



(a) Solid model of the parallel manipulator of serial no. 5 (Table 11)



(b) Solid model of the parallel manipulator of serial no. 7 (Table 11)

Figure 13: The optimal 3-PRR parallel manipulators

CONCLUSIONS

1. This paper presents a geometrical method of determination of the shape of the constant orientation workspace for any kind of variation of its parameters. A *look up table* is also proposed which shows all kinds of shapes of the workspaces for any variation in the parameters of 3-PRR planar parallel manipulator. For each shape of the workspace, geometrical conditions are also mentioned. This helps a lot in choosing geometrical parameters for a required shape of the workspace; it also helps to visualize which geometrical relationship between the parameters results in voids inside the workspace. Thus, the look-up table is of a great help to the designer for optimal design of such parallel manipulators.
2. For each shape of the workspace, closed form mathematical expressions are obtained to determine the area of the workspace. This helps a lot for optimal design; this also helps a lot for applications where precision is of more importance. This is more pronounced in case of parallel manipulators because parallel manipulators are preferred for precision applications.
3. A case study is presented wherein the user gives his requirement of workspace area. A flowchart is presented wherein it is shown, in a step by step method, how to get singularity free (or minimum singularity) optimal parallel manipulators satisfying the workspace area requirement of the user and having no voids within the workspace.
4. For the case study, initially 9 sets of parameters are chosen which satisfy the workspace area requirement and having no voids. Out of them best 5 are selected and they are tested for singularity analysis and out of them 2 parallel manipulators are chosen finally having minimum singularities, satisfying workspace area constraint and having no voids within the workspace. Thus the algorithm is successfully tested for a specific application.

Declarations:

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